Content Summary

One of the most important kinds of thinking in mathematics is proportional reasoning—applying what we know about a figure to larger or smaller figures. In Chapter 11, students have the opportunity to practice this kind of thinking by applying it to similar figures—figures that are enlargements (stretches) or reductions (shrinks) of each other. A transformation that stretches or shrinks a figure by the same factor in all directions is a **dilation**, and the factor is called a **scale factor**.

Similar Polygons

In everyday usage, “similar” means alike in some ways. But in geometry, similar means exactly the same shape (but not necessarily the same size). Two polygons are similar if their corresponding angles are congruent and the lengths of corresponding sides all have the same ratio. This ratio is called the scale factor. Ratios of lengths of other corresponding parts of similar triangles are equal to the scale factor. The book concentrates on similar triangles and—as with congruence—finds shortcuts for determining similarity. These shortcuts can be applied to calculate lengths that can’t be measured directly, such as the height of a flagpole.

Parallel Lines

If lines are drawn between two sides of a triangle and they are parallel to the third side, then each of the lines creates a new triangle that is similar to the original. Therefore, these lines divide proportionately the two sides that they intersect.

Area and Volume

Some of the most important and surprising results involve relationships among the areas or volumes of similar figures. Suppose you double the length and width of a rectangle to form a larger rectangle. You might expect the area to double, but in fact it quadruples, or increases by a factor of 4.

If any two-dimensional figure is dilated by a scale factor of $r$, then its area is multiplied by $r^2$. Similarly, if a three-dimensional figure is dilated by a factor of $r$, then its volume is multiplied by $r^3$. (Surface areas of its faces, which are two-dimensional, are multiplied by $r^2$.) For example, if a sphere is enlarged by a factor of 1.5 (its radius is multiplied by 1.5), then its volume is multiplied by $1.5^3 = 3.375$, and its surface area is multiplied by $1.5^2 = 2.25$. 

(continued)
Chapter 11 • Similarity (continued)

Summary Problem
Draw a pentagon on graph paper, as in Lesson 11.1, Investigation 2. Choose any constant as your scale factor. Multiply the coordinates of each vertex by that constant to locate the vertices of a new pentagon. How do the two pentagons compare?
Questions you might ask in your role as student to your student:
- Are the pentagons similar?
- How many ways can you prove that they’re similar?
- How does the area of the new pentagon compare to the area of the original?
- What scale factors would you use to enlarge the original pentagon? What scale factors would you use to make a smaller pentagon?

Sample Answers
To verify similarity, we have to show that the corresponding sides are proportional and that the corresponding angles are congruent. Your student can verify that the sides are proportional either by measuring the lengths of corresponding sides and writing their ratios or by drawing segments from the origin to the vertices and applying the Extended Parallel/Proportionality Conjecture (finding five pairs of similar triangles). He or she can verify the congruence of corresponding angles by measuring or by applying the SSS similarity shortcut to triangles into which the pentagon can be divided. Once the scale factor is known, it can be squared to determine the ratio of areas. Scale factors larger than 1 will enlarge the figure, whereas scale factors smaller than 1 will reduce the size.
Chapter 11 • Review Exercises

Round your solutions to the nearest tenth of a unit unless otherwise stated.

1. (Lesson 11.1) $ABCD \sim FGHI$. Find $a$ and $b$.

2. (Lesson 11.2) Which similarity shortcut can be used to show that $\triangle ABC \sim \triangle EBD$? Find $a$ and $b$.

3. (Lesson 11.3) Rafael is 1.8 m tall and casts a 0.5 m shadow. If at the same time a nearby tree casts a 1.5 m shadow, how tall is the tree?

4. (Lesson 11.4) Find $x$.

5. (Lesson 11.5) $ABCD \sim EFGH$.
   Area of $ABCD = 15 \text{ cm}^2$
   Area of $EFGH = ?$

6. (Lesson 11.6) Small triangular prism $\sim$ Large triangular prism
   Volume of small prism = 48 in$^3$
   Volume of large prism = ?

7. (Lesson 11.7) $EB \parallel CD$. Find $a$. 

©2008 Kendall Hunt Publishing
1. Because the quadrilaterals are similar, corresponding angles are congruent.

Therefore, \(a = m\angle G = m\angle B = 110^\circ\).

Because the two quadrilaterals are similar, we can set up a proportion to find the missing length.

\[
\frac{13}{7} = \frac{18}{b}
\]

\(13b = 18 \cdot 7\) Multiply both sides by 7; multiply both sides by \(b\).

\(b \approx 9.7\) m Solve for \(b\).

2. Two pairs of corresponding sides are proportional \((\frac{15.5}{4.5} = \frac{9}{3})\) and their included angles are congruent (vertical angles \(\angle ABC\) and \(\angle EBD\)), so the triangles are similar because of the SAS Similarity Conjecture.

To find \(a\), use the Triangle Sum Conjecture to find \(\angle ABC: 180^\circ - 38^\circ - 111^\circ = 31^\circ\). Because \(\angle ABC \equiv \angle EBD\), \(a = m\angle EBD = 31^\circ\). To find \(b\), solve the proportion.

\[
\frac{9}{3} = \frac{7.5}{b}
\]

\(9b = 7.5 \cdot 3\) Multiply both sides by 3; multiply both sides by \(b\).

\(b \approx 2.5\) cm Solve for \(b\).

3. Because the sun's rays all come in at the same angle at a particular time of day, all people or objects and their shadows form similar right triangles. So, Rafael's height and the length of his shadow are proportional to the tree's height and the length of its shadow.

\[
\frac{1.8}{0.5} = \frac{x}{1.5}
\]

\(x = 5.4\) Multiply both sides by 1.5.

The tree is 5.4 m tall.

4. The angle bisector divides the opposite side into segments that are proportional to the other two sides of the triangle. Set up a proportion and solve:

\[
\frac{x}{15} = \frac{4}{9}
\]

\(x \approx 6.7\) cm

5. When two figures are similar, the ratio of their areas is the square of the scale factor.

\[
\frac{AD}{EH} = \frac{3}{7}
\]

The scale factor is the ratio of corresponding sides.

\[
\frac{\text{Area of } ABCD}{\text{Area of } EFGH} = \left(\frac{3}{7}\right)^2 = \frac{9}{49}
\]

The ratio of areas is the square of the scale factor.

\[
\frac{15}{x} = \frac{9}{49}
\]

Substitute values.

\(x \approx 81.7\) cm\(^2\) Solve for \(x\).

6. Scale factor = \(\frac{6}{11}\)

Find the scale factor.

\[
\frac{V_{\text{small prism}}}{V_{\text{large prism}}} = \left(\frac{6}{11}\right)^3
\]

The ratio of the volumes is equal to the cube of the scale factor.

\[
\frac{48}{x} = \frac{216}{1331}
\]

Substitute values.

\(x \approx 295.8\) in\(^3\) Solve for \(x\).

7. Because of parallel proportionality, corresponding sides are proportional.

\[
\frac{AB}{AC} = \frac{EB}{DC}
\]

\[
\frac{12}{12 + a} = \frac{16}{23}
\]

Substitute values.

\(12 \cdot 23 = 16(12 + a)\)

Multiply both sides by \(23(12 + a)\).

\(276 = 192 + 16a\)

Distribute.

\(a = 5.25\)

Solve for \(a\).