Lesson 5.1 • Polygon Sum Conjecture

In Exercises 1 and 2, find each lettered angle measure.

1. \(a = \_\_\_, \quad b = \_\_\_, \quad c = \_\_\_, \quad d = \_\_\_, \quad e = \_\_\_\)  
2. \(a = \_\_\_, \quad b = \_\_\_, \quad c = \_\_\_, \quad d = \_\_\_, \quad e = \_\_\_, \quad f = \_\_\_\)

3. One exterior angle of a regular polygon measures 10°. What is the measure of each interior angle? How many sides does the polygon have?

4. The sum of the measures of the interior angles of a regular polygon is 2340°. How many sides does the polygon have?

5. \(ABCD\) is a square. \(ABE\) is an equilateral triangle.  
   \(x = \_\_\_\_\_\)  

6. \(ABCDE\) is a regular pentagon. \(ABFG\) is a square.  
   \(x = \_\_\_\_\_\)  

7. Use a protractor to draw pentagon \(ABCDE\) with \(m\angle A = 85°\), \(m\angle B = 125°\), \(m\angle C = 110°\), and \(m\angle D = 70°\). What is \(m\angle E\)? Measure it, and check your work by calculating.
Lesson 5.2 • Exterior Angles of a Polygon

1. How many sides does a regular polygon have if each exterior angle measures 30°?

2. How many sides does a polygon have if the sum of the measures of the interior angles is 3960°?

3. If the sum of the measures of the interior angles of a polygon equals the sum of the measures of its exterior angles, how many sides does it have?

4. If the sum of the measures of the interior angles of a polygon is twice the sum of its exterior angles, how many sides does it have?

In Exercises 5–7, find each lettered angle measure.

5. \( a = \), \( b = \)

6. \( a = \), \( b = \)

7. \( a = \), \( b = \), \( c = \)

8. Find each lettered angle measure.

\( a = \)
\( b = \)
\( c = \)
\( d = \)

9. Construct an equiangular quadrilateral that is not regular.
**Lesson 5.3 • Kite and Trapezoid Properties**

In Exercises 1–4, find each lettered measure.

1. Perimeter = 116. \( x = \) _____

2. \( x = \) _____, \( y = \) _____

3. \( x = \) _____, \( y = \) _____

4. \( x = \) _____, \( y = \) _____

5. Perimeter \( PQRS = 220. \) \( PS = \) _____

6. \( b = 2a + 1. \) \( a > \) _____

In Exercises 7 and 8, use the properties of kites and trapezoids to construct each figure. Use patty paper or a compass and a straightedge.

7. Construct an isosceles trapezoid given base \( AB, \angle B, \) and distance between bases \( XY. \)

8. Construct kite \( ABCD \) with \( AB, BC, \) and \( BD. \)

9. Write a paragraph or flowchart proof of the Converse of the Isosceles Trapezoid Conjecture. *Hint: Draw \( AE \) parallel to \( TP \) with \( E \) on \( TR. \)*

**Given:** Trapezoid \( TRAP \) with \( \angle T \equiv \angle R 

**Show:** \( TP \equiv RA \)
Lesson 5.4 • Properties of Midsegments

In Exercises 1–3, each figure shows a midsegment.

1. \( a = \underline{\quad}, \quad b = \underline{\quad}, \quad c = \underline{\quad} \)
2. \( x = \underline{\quad}, \quad y = \underline{\quad}, \quad z = \underline{\quad} \)
3. \( x = \underline{\quad}, \quad y = \underline{\quad}, \quad z = \underline{\quad} \)

4. \( X, \quad Y, \quad \) and \( Z \) are midpoints. Perimeter \( \triangle PQR = 132, RQ = 55, \) and \( PZ = 20. \)
   
   Perimeter \( \triangle XYZ = \underline{\quad} \)
   
   \( PQ = \underline{\quad} \)
   
   \( ZX = \underline{\quad} \)

5. \( MN \) is the midsegment. Find the coordinates of \( M \) and \( N. \) Find the slopes of \( AB \) and \( MN. \)

6. Explain how to find the width of the lake from \( A \) to \( B \) using a tape measure, but without using a boat or getting your feet wet.

7. \( M, \quad N, \) and \( O \) are midpoints. What type of quadrilateral is \( AMNO? \) How do you know? Give a flowchart proof showing that \( \triangle ONC \equiv \triangle MBN. \)

8. Give a paragraph or flowchart proof.
   
   **Given:** \( \triangle PQR \) with \( PD = DF = FH = HR \) and \( QE = EG = GI = IR \)
   
   **Show:** \( HI \parallel FG \parallel DE \parallel PQ \)
Lesson 5.5 • Properties of Parallelograms

In Exercises 1–7, \( ABCD \) is a parallelogram.

1. Perimeter \( ABCD = \) \( \)  
   \[ \begin{array}{c}
   D \\
   26 \text{ cm} \\
   15 \text{ cm} \\
   A \\
   \end{array} \]

2. \( AO = 11 \), and \( BO = 7 \).
   \[ \begin{array}{c}
   D \\
   C \\
   O \\
   B \\
   A \\
   \end{array} \]

3. Perimeter \( ABCD = 46 \).
   \[ \begin{array}{c}
   A \\
   B \\
   C \\
   D \\
   \end{array} \]

4. \( a = \), \( b = \), \( c = \)  
   \[ \begin{array}{c}
   D \\
   C \\
   A \\
   B \\
   \end{array} \]

5. Perimeter \( ABCD = 119 \), and \( BC = 24 \).
   \[ \begin{array}{c}
   A \\
   B \\
   C \\
   D \\
   \end{array} \]

6. \( a = \), \( b = \), \( c = \)  
   \[ \begin{array}{c}
   A \\
   B \\
   C \\
   D \\
   \end{array} \]

7. Perimeter \( ABCD = 16x - 12 \).
   \[ \begin{array}{c}
   A \\
   63 \\
   B \\
   \end{array} \]

8. Ball B is struck at the same instant by two forces, \( \vec{F}_1 \) and \( \vec{F}_2 \). Show the resultant force on the ball.

9. Find each lettered angle measure.
   \[ \begin{array}{c}
   a = \) \( , \) \( g = \) \( \\
   b = \) \( , \) \( h = \) \( \\
   c = \) \( , \) \( i = \) \( \\
   d = \) \( , \) \( j = \) \( \\
   e = \) \( , \) \( k = \) \( \\
   f = \) \( \\
   \end{array} \]

10. Construct a parallelogram with diagonals \( \overline{AC} \) and \( \overline{BD} \).
    Is your parallelogram unique? If not, construct a different (noncongruent) parallelogram.

\[ \begin{array}{c}
A \\
\rightarrow \ \\
C \\
\rightarrow \ \ \ \ B \\
\rightarrow \ \ \ \ D \\
\end{array} \]
Lesson 5.6 • Properties of Special Parallelograms

Name ___________________________  Period _______  Date ________________

1. PQRS is a rectangle and OS = 16.
   
   \[ OQ = \] \[ m\angle QRS = \] \[ PR = \]

![Diagram of rectangle PQRS with points O, S, Q, R]

2. KLMN is a square and NM = 8.
   
   \[ m\angle OKL = \] \[ m\angle MOL = \] \[ Perimeter KLMN = \]

![Diagram of square KLMN with points O, K, L, M]

3. ABCD is a rhombus, AD = 11, and DO = 6.
   
   \[ OB = \] \[ BC = \] \[ m\angle AOD = \]

![Diagram of rhombus ABCD with points O, A, B, D]

In Exercises 4–11, match each description with all the terms that fit it.

<table>
<thead>
<tr>
<th>Description</th>
<th>Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Trapezoid</td>
<td>b. Isosceles triangle</td>
</tr>
<tr>
<td>e. Kite</td>
<td>f. Rectangle</td>
</tr>
<tr>
<td>4. Diagonals bisect each other.</td>
<td>5. Diagonals are perpendicular.</td>
</tr>
<tr>
<td>8. Opposite sides are congruent.</td>
<td>9. Opposite angles are congruent.</td>
</tr>
</tbody>
</table>

In Exercises 12 and 13, graph the points and determine whether \( ABCD \) is a trapezoid, parallelogram, rectangle, or none of these.

12. \( A(-4, -1), B(0, -3), C(4, 0), D(-1, 5) \)

![Graph with points A, B, C, D]

13. \( A(0, -3), B(-1, 2), C(-3, 4), D(-2, -1) \)

![Graph with points A, B, C, D]

14. Construct rectangle \( ABCD \) with diagonal \( \overline{AC} \) and \( \angle CAB \).
Lesson 5.7 • Proving Quadrilateral Properties

Write or complete each flowchart proof.

1. **Given:** $ABCD$ is a parallelogram and $AP \equiv QC$
   **Show:** $AC$ and $PQ$ bisect each other

   **Flowchart Proof**
   
   ![Flowchart Proof Diagram](image)

2. **Given:** Dart $ABCD$ with $AB \equiv BC$ and $CD \equiv AD$
   **Show:** $\angle A \equiv \angle C$

3. Show that the diagonals of a rhombus divide the rhombus into four congruent triangles.
   **Given:** Rhombus $ABCD$
   **Show:** $\triangle ABO \equiv \triangle CBO \equiv \triangle CDO \equiv \triangle ADO$

4. **Given:** Parallelogram $ABCD$, $BY \perp AC$, $DX \perp AC$
   **Show:** $DX \equiv BY$
7. (See flowchart proof at bottom of page 102.)

8. Flowchart Proof

\[ \overrightarrow{CD} \text{ is a median} \]
\[ \text{Given} \]
\[ \overrightarrow{AC} = \overrightarrow{BC} \]
\[ \text{Given} \]
\[ \overrightarrow{AD} = \overrightarrow{BD} \]
\[ \text{Definition of median} \]
\[ \overrightarrow{CD} = \overrightarrow{CD} \]
\[ \text{Same segment} \]
\[ \triangle ADC \cong \triangle BDC \]
\[ \text{SSS Conjecture} \]
\[ \angle ACD = \angle BCD \]
\[ \text{CPCTC} \]
\[ \overrightarrow{CD} \text{ bisects } \angle ACB \]
\[ \text{Definition of bisector} \]

LESSON 5.1 • Polygon Sum Conjecture

1. \( a = 103^\circ, b = 103^\circ, c = 97^\circ, d = 83^\circ, e = 154^\circ \)

2. \( a = 92^\circ, b = 44^\circ, c = 51^\circ, d = 85^\circ, e = 44^\circ, f = 136^\circ \)

LESSON 5.2 • Exterior Angles of a Polygon

1. 12 sides
2. 24 sides
3. 4 sides
4. 6 sides
5. \( a = 64^\circ, b = 138^\circ \)
6. \( a = 102^\circ, b = 9^\circ \)
7. \( a = 156^\circ, b = 132^\circ, c = 108^\circ \)
8. \( a = 135^\circ, b = 40^\circ, c = 105^\circ, d = 135^\circ \)

Lesson 4.7, Exercises 1, 2, 3

1.

\[ \overrightarrow{PQ} = \overrightarrow{SR} \]
\[ \text{Given} \]
\[ \overrightarrow{PQ} \parallel \overrightarrow{SR} \]
\[ \text{Given} \]
\[ \angle PQS = \angle RSQ \]
\[ \text{AIA Conjecture} \]
\[ \triangle PQS \cong \triangle RSQ \]
\[ \text{SAS Conjecture} \]
\[ SP = QR \]
\[ \text{CPCTC} \]

2.

\[ \overrightarrow{KT} = \overrightarrow{KT} \]
\[ \text{Given} \]
\[ \overrightarrow{KE} = \overrightarrow{KT} \]
\[ \text{Given} \]
\[ KITE \text{ is a kite} \]
\[ \text{Definition of kite} \]
\[ \overrightarrow{TE} = \overrightarrow{TT} \]
\[ \text{Same segment} \]
\[ \triangle KET = \triangle KIT \]
\[ \text{SSS Conjecture} \]
\[ \angle EKT = \angle KIT \]
\[ \text{CPCTC} \]
\[ \angle ETK = \angle ITK \]
\[ \text{CPCTC} \]
\[ \overrightarrow{KT} \text{ bisects } \angle EKI \text{ and } \angle ETI \]
\[ \text{Definition of bisector} \]

3.

\[ \overrightarrow{AB} \parallel \overrightarrow{CD} \]
\[ \text{Definition of parallel lines} \]
\[ \overrightarrow{AB} \parallel \overrightarrow{CD} \]
\[ \text{Definition of parallel lines} \]
\[ \angle ABD = \angle CDB \]
\[ \text{AIA Conjecture} \]
\[ \overrightarrow{BD} = \overrightarrow{DB} \]
\[ \text{Same segment} \]
\[ \triangle BDA \cong \triangle DBC \]
\[ \text{ASA Conjecture} \]
\[ \angle A = \angle C \]
\[ \text{CPCTC} \]
**LESSON 5.3 • Kite and Trapezoid Properties**

1. \( x = 30 \)  
2. \( x = 124^\circ, y = 56^\circ \)  
3. \( x = 64^\circ, y = 43^\circ \)  
4. \( x = 12^\circ, y = 49^\circ \)  
5. \( PS = 33 \)  
6. \( a > 11 \)

7. AMNO is a parallelogram. By the Triangle Midsegment Conjecture, \( ON \parallel AM \) and \( MN \parallel AO \).

**Flowchart Proof**

1. \( OC = \frac{1}{2} AC \)
2. \( MN = \frac{1}{2} AC \)
3. \( MB = \frac{1}{2} AB \)
4. \( ON = \frac{1}{2} AB \)
5. \( \angle CON = \angle A \)
6. \( \angle NMB = \angle A \)
7. \( \angle CON = \angle NMB \)
8. \( \angle A \)
9. \( \triangle MNB = \triangle MNB \)

**Lesson 4.8, Exercise 7**

1. \( a = 89^\circ, b = 54^\circ, c = 91^\circ \)  
2. \( x = 21, y = 7, z = 32 \)  
3. \( x = 17, y = 11, z = 6.5 \)  
4. Perimeter \( \triangle XYZ = 66, PQ = 37, ZX = 27.5 \)
5. \( M(12, 6), N(14.5, 2); AB = -1.6, \) slope \( MN = -1.6 \)
6. Pick a point \( P \) from which \( A \) and \( B \) can be viewed over land. Measure \( AP \) and \( BP \) and find the midpoints \( M \) and \( N \), \( AB = 2MN \).
**LESSON 5.5 • Properties of Parallelograms**

1. Perimeter $ABCD = 82$ cm
2. $AC = 22, BD = 14$
3. $AB = 16, BC = 7$
4. $a = 51^\circ, b = 48^\circ, c = 70^\circ$
5. $AB = 35.5$
6. $a = 41^\circ, b = 86^\circ, c = 53^\circ$
7. $AD = 75$
8. 

**9.** $a = 38^\circ, b = 142^\circ, c = 142^\circ, d = 38^\circ, e = 142^\circ, f = 38^\circ, g = 52^\circ, h = 12^\circ, i = 61^\circ, j = 81^\circ, k = 61^\circ$
10. No

**LESSON 5.6 • Properties of Special Parallelograms**

1. $OQ = 16, m\angle QRS = 90^\circ, PR = 32$
2. $m\angle OKL = 45^\circ, m\angle MOL = 90^\circ$, perimeter $KLMN = 32$
3. $OB = 6, BC = 11, m\angle AOD = 90^\circ$

**Lesson 5.7, Exercise 1**
4. Flowchart Proof

LESSON 6.1 • Tangent Properties

1. \( w = 126^\circ \)
2. \( m\angle BQX = 65^\circ \)
3. a. \( m\angle NQP = 90^\circ, m\angle MPQ = 90^\circ \)
   b. Trapezoid. Possible explanation: MP and NQ are both perpendicular to PQ, so they are parallel to each other. The distance from M to PQ is MP, and the distance from N to PQ is NQ. But the two circles are not congruent, so MP \neq NQ. Therefore, MN is not a constant distance from PQ and they are not parallel. Exactly one pair of sides is parallel, so MNQP is a trapezoid.
4. \( y = \frac{1}{3}x + 10 \)
5. Possible answer: Tangent segments from a point to a circle are congruent. So, \( PA \equiv PB, PB \equiv PC, \) and \( PC \equiv PD. \) Therefore, \( PA \equiv PD. \)
6. a. 4.85 cm
   b. 11.55 cm
7. 

LESSON 6.2 • Chord Properties

1. \( a = 95^\circ, b = 85^\circ, c = 47.5^\circ \)
2. \( y \) cannot be determined, \( w = 90^\circ \)
3. \( z = 45^\circ \)
4. \( w = 100^\circ, x = 50^\circ, y = 110^\circ \)
5. \( w = 49^\circ, x = 122.5^\circ, y = 65.5^\circ \)
6. \( x = 16 \text{ cm}, y \) cannot be determined
7. Kite. Possible explanation: \( OM \equiv ON \) because congruent chords \( AB \) and \( AC \) are the same distance from the center. \( AM \equiv AN \) because they are halves of congruent chords. So, \( AMON \) has two pairs of adjacent congruent sides and is a kite.
8. The perpendicular segment from the center of the circle bisects the chord, so the chord has length 12 units. But the diameter of the circle is 12 units, and the chord cannot be as long as the diameter because it doesn’t pass through the center of the circle.
9. \( P(0,1), M(4,2) \)
10. \( m\angle AB = 49^\circ, m\angle BC = 253^\circ, m\angle AC = 156^\circ, m\angle CB = 311^\circ \)
11. Possible answer: Fold and crease to match the endpoints of the arc. The crease is the perpendicular bisector of the chord connecting the endpoints. Fold and crease so that one endpoint falls on any other point on the arc. The crease is the perpendicular bisector of the chord between the two matching points. The center is the intersection of the two creases.

1. \( m\angle XNM = 40^\circ, m\angle XN = 180^\circ, m\angle MN = 100^\circ \)
2. \( x = 120^\circ, y = 60^\circ, z = 120^\circ \)
3. \( a = 90^\circ, b = 55^\circ, c = 35^\circ \)
4. \( a = 50^\circ, b = 60^\circ, c = 70^\circ \)
5. \( x = 140^\circ \)
6. \( m\angle A = 90^\circ, m\angle AB = 72^\circ, m\angle C = 36^\circ, m\angle CB = 108^\circ \)
7. \( m\angle AD = 140^\circ, m\angle D = 30^\circ, m\angle AB = 60^\circ, m\angle DAB = 200^\circ \)
8. \( p = 128^\circ, q = 87^\circ, r = 58^\circ, s = 87^\circ \)
9. \( a = 50^\circ, b = 50^\circ, c = 80^\circ, d = 50^\circ, e = 130^\circ, f = 90^\circ, g = 50^\circ, h = 50^\circ, j = 90^\circ, k = 40^\circ, m = 80^\circ, n = 50^\circ \)