CONDENSED LESSON 4.1

Triangle Sum Conjecture

In this lesson you will

- State a conjecture about the sum of the measures of the angles in a triangle
- Complete a paragraph proof of the **Triangle Sum Conjecture**

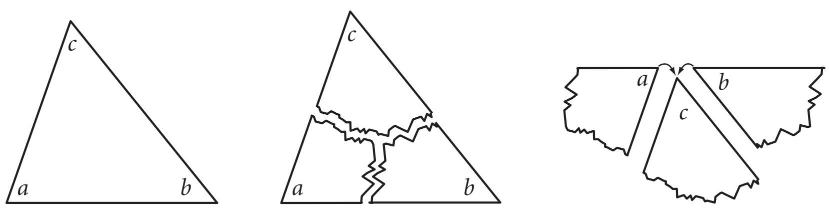
In this chapter you will focus on properties of triangles. To start, you will look at the angle measures of triangles.

Investigation: The Triangle Sum

Draw two large acute triangles with very different shapes and two large obtuse triangles with very different shapes.

For each triangle, measure the three angles as accurately as possible, and then find the sum of the three measures. You should find that the angle sum for each triangle is the same. What is the angle sum?

To check this sum, write the letters *a*, *b*, and *c* in the interiors of the three angles of one of the acute triangles, and carefully cut out the triangle. Then tear off the three angles of the triangle and arrange them so that their vertices meet at a point.

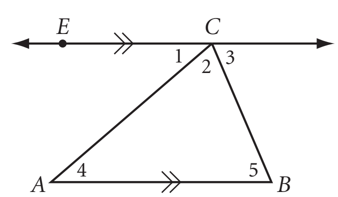


How does the arrangement verify the angle sum you found above?

Your work in this investigation leads to the following conjecture.

Triangle Sum Conjecture The sum of the measures of the angles in every triangle is 180°. **C-17**

Next you will write a **paragraph proof** to show why the Triangle Sum Conjecture is true. In your proof, you can use conjectures, definitions, and properties to support your argument. Look at the figure at right. $\triangle ABC$ is any triangle. $\overleftrightarrow{EC}$ is drawn parallel to $\overline{AB}$. *Note:* $\overleftrightarrow{EC}$ is an **auxiliary line** (or helping line) because it is an extra line added to the figure to help with the proof. Consider the questions on page 201 of your book.



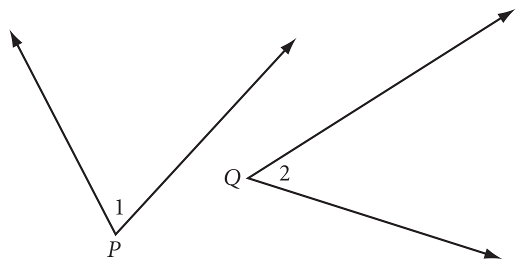
Copy the diagram and mark angle relationships that might help with your proof. Then use the diagram and your answers to these questions to write a paragraph proof explaining why the Triangle Sum Conjecture is true.

After you have finished, compare your paragraph proof with the one on page 202 of your book.

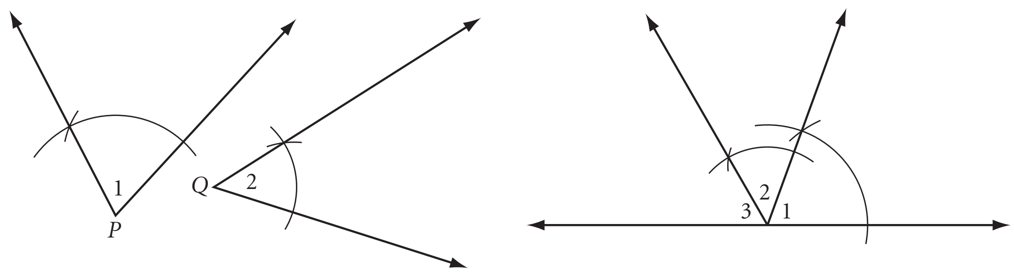
(continued)

The Triangle Sum Conjecture allows you to construct the third angle of a triangle if you are given two of the angles. Work through the example in your book. The following example shows a slightly different method. Try it to see whether one method is easier. Can you find another method that works?

EXAMPLE A Given $\angle P$ and $\angle Q$ of $\triangle PQR$, construct $\angle R$.

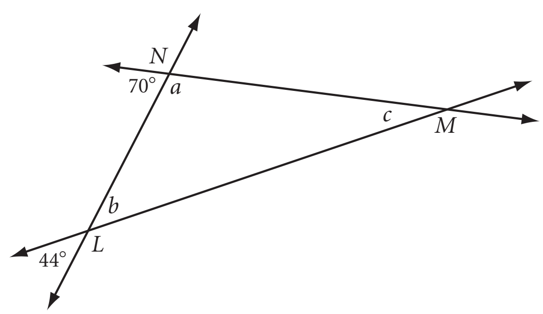


► **Solution** Draw a line and construct $\angle P$ opening to the right on this line. Construct $\angle Q$ so that it shares both the vertex of $\angle P$ and the side of $\angle P$ that is not on the line. The angle labeled 3 on the diagram is $\angle R$, because the sum of the measures of the three angles is 180°.



The following example applies what you have learned in this lesson.

EXAMPLE B Find the lettered angle measures.



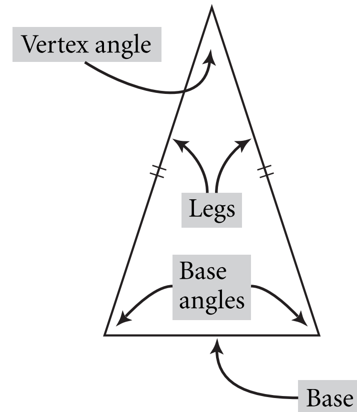
► **Solution** The angle labeled a and the 70° angle form a linear pair, so $a + 70° = 180°$. Therefore, $a = 110°$. The angle labeled b and the 44° angle are vertical angles, so $b = 44°$. By the Triangle Sum Conjecture, $110° + 44° + c = 180°$, so $c = 26°$.

CONDENSED LESSON

4.2 Properties of Isosceles Triangles

In this lesson you will

- Discover how the angles of an isosceles triangle are related
- Make a conjecture about triangles that have two congruent angles



An *isosceles triangle* is a triangle with at least two congruent sides. The angle between the congruent sides is called the *vertex angle*. The other two angles are called the *base angles*. The side between the base angles is called the *base*. The other two sides are called the *legs*.

Investigation 1: Base Angles in an Isosceles Triangle

Draw an acute angle C on patty paper. Then follow Steps 2 and 3 in your book to construct an isosceles triangle, $\triangle ABC$.

Because $\overline{CA}$ and $\overline{CB}$ are the congruent sides, $\angle C$ is the vertex angle and $\angle A$ and $\angle B$ are base angles. Use your protractor to measure the base angles. How do the measures compare? Confirm your answer by folding your patty paper, keeping $\overline{AB}$ aligned with itself. Is $\angle A \cong \angle B$?

Now draw two more isosceles triangles, one with an obtuse vertex angle and one with a right vertex angle. Compare the base angles in each triangle. Are your findings the same as for the isosceles acute triangle?

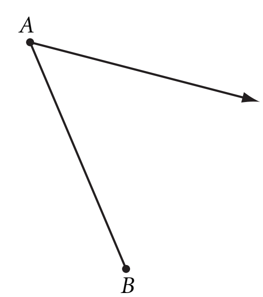
Your observations should lead to the following conjecture.

Isosceles Triangle Conjecture If a triangle is isosceles, then its base angles are congruent. **C-18**

Equilateral triangles are also isosceles triangles, because at least two of their sides are congruent. How do you think the Isosceles Triangle Conjecture applies to equilateral triangles?

As you know, reversing the "if" and "then" parts of a conjecture gives the *converse* of the conjecture. Is the converse of the Isosceles Triangle Conjecture true? In other words, if a triangle has two congruent angles, is it isosceles? To test this statement, you need to draw a triangle with two congruent angles.

Investigation 2: Is the Converse True?



Draw a segment, $\overline{AB}$, on your paper. Draw an acute angle at point A. In the finished triangle, $\angle A$ and $\angle B$ will be the congruent angles.

Notice that $\angle A$ must be acute. If it were right or obtuse, the sum of the measures of $\angle A$ and $\angle B$ would be greater than or equal to 180°, and, as you know, the sum of all *three* angle measures must be 180°.

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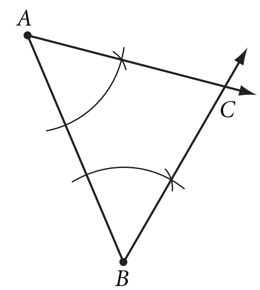
Now copy $\angle A$ at point B on the same side of $\overline{AB}$. If necessary, extend the sides of the angles until they intersect. Label the point of intersection C.



Use your compass to compare the lengths of sides $\overline{AC}$ and $\overline{BC}$. Do they appear to be the same length? Check your answer using patty paper. Draw at least one more triangle with two congruent angles, and compare the side lengths. Your findings should provide evidence that the converse of the Isosceles Triangle Conjecture is true.

Converse of the Isosceles Triangle Conjecture If a triangle has two congruent angles, then it is an isosceles triangle. **C-19**

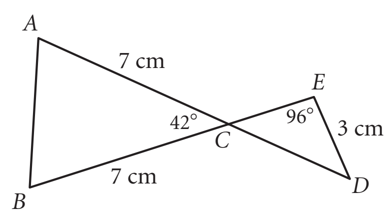
The following example gives you practice applying what you have learned.

EXAMPLE

$m\angle A =$ ______

$m\angle D =$ ______

$EC =$ ______



▶ ***Solution***

By the Triangle Sum Conjecture, $m\angle A + m\angle B + 42° = 180°$, so $m\angle A + m\angle B = 138°$. Because $\angle A$ and $\angle B$ are the base angles of an isosceles triangle, they are congruent. So, $m\angle A = \frac{1}{2}(138°) = 69°$.

Because $\angle ACB$ and $\angle ECD$ are vertical angles, they are congruent. So, $m\angle ECD = 42°$. By the Triangle Sum Conjecture, $42° + 96° + m\angle D = 180°$. Solving for $m\angle D$ gives $m\angle D = 42°$.

Because $\angle ECD \cong \angle D$, $\triangle CDE$ is isosceles by the Converse of the Isosceles Triangle Conjecture. Therefore, the legs are congruent, so $EC = ED = 3$ cm.

CONDENSED
LESSON

4.3 Triangle Inequalities

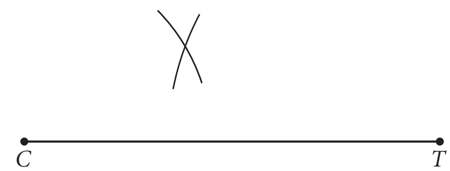
In this lesson you will

- Determine whether you can form a triangle from any three segments
- Discover a relationship between the side lengths and angle measures of a triangle
- Look for a relationship between the measure of the **exterior angle** of a triangle and the measures of the corresponding **remote interior angles**

If you are given three segments, will you always be able to form a triangle with those segments as sides? In the following investigation, you will explore this question.

Investigation 1: What Is the Shortest Path from *A* to *B*?

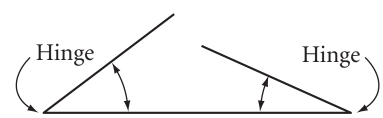
In Step 1 of the investigation, you are given two sets of three segments to use as side lengths of triangles. Consider the first set of segments. To construct $\triangle CAT$, first copy $\overline{CT}$. To construct the other two sides of the triangle, swing an arc of length AC centered at point C and an arc of length AT centered at point T. Point A is where the two arcs intersect.



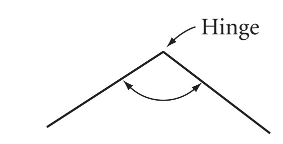
Now try to use the second set of segments to construct $\triangle FSH$. Are you able to do it? Why or why not?

You should have found that the arcs that you made using the lengths of two of the sides did not intersect, so it was not possible to construct $\triangle FSH$. In general, for three segments to form a triangle, the sum of the lengths of any two segments must be greater than the length of the third segment. Here are two ways to visualize this.

Imagine two of the segments connected to the endpoints of the third segment by hinges. To form a triangle, you need to be able to swing the segments so that their unhinged endpoints meet but do not lie completely flat. This is possible only if the combined length of the two segments is greater than the length of the third segment.



Imagine two segments connected by a hinge. To form a triangle, you need to be able to adjust the opening between these sides so that the unhinged endpoints meet the endpoints of the third segment, without lying completely flat. This is possible only if the combined length of the two hinged segments is greater than the length of the third segment.

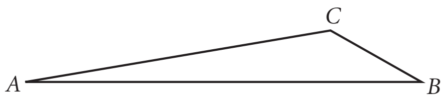


You can state this idea as a conjecture.

Triangle Inequality Conjecture The sum of the lengths of any two sides of a triangle is greater than the length of the third side. **C-20**

(continued)

You can think of the triangle conjecture in a different way: The shortest distance between two points is along the segment connecting them. In other words, the distance from *A* to *C* to *B* cannot be shorter than the distance from *A* to *B*.

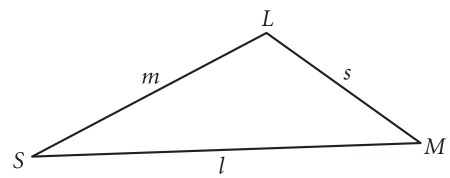


No matter where *C* is,
$AC + CB \geq AB$.

Investigation 2: Where Are the Largest and Smallest Angles?

Draw a scalene obtuse triangle. Follow Steps 1 and 2 in your book to label the angles and sides according to their size. Then answer the questions in Step 3.

As in the example at right, you should find that the longest side is opposite the angle with the largest measure, the second longest side is opposite the side with the second largest measure, and the shortest side is opposite the side with the smallest measure.

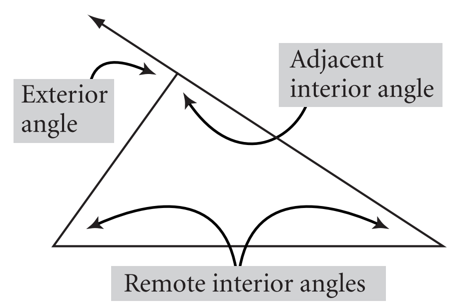


Draw a scalene acute triangle and follow Steps 1–3 again. Are your findings the same?

State your findings as a conjecture. Here is one possible way to word the conjecture.

> **Side-Angle Inequality Conjecture** In a triangle, if one side is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side. **C-21**

So far, you have been focusing on the *interior angles* of triangles. Triangles also have *exterior angles.* To construct an **exterior angle,** extend one side beyond the vertex. Each exterior angle of a triangle has an **adjacent interior angle** and a pair of **remote interior angles.**



Investigation 3: Exterior Angles of a Triangle

In this investigation you will look for a relationship between the measure of an exterior angle and the measure of the two associated remote interior angles. Follow Steps 1–3 in your book for at least two different triangles. You can state your findings as a conjecture.

> **Triangle Exterior Angle Conjecture** The measure of an exterior angle of a triangle is equal to the sum of the measures of the remote interior angles. **C-22**

CONDENSED LESSON 4.4

Are There Congruence Shortcuts?

In this lesson you will

- Look for shortcuts for determining whether two triangles are congruent

If the three sides and three angles of one triangle are congruent to the three sides and three angles of another triangle, then you know the triangles are congruent. But, to determine whether two triangles are congruent, do you really need to make all six comparisons?

In this lesson and the next, you will search for shortcuts that allow you to determine whether two triangles are congruent by making only three comparisons. Page 221 of your book illustrates the six different ways that three parts of one triangle can be congruent to three parts of another. Note that the order in which the parts are listed is important. For example, Side-Angle-Side (SAS) refers to two sides and the angle included *between* the sides, while Side-Side-Angle (SSA) refers to two sides and an angle that is *not between* them.

Investigation 1: Is SSS a Congruence Shortcut?

In this investigation you will explore the following question: If the three sides of one triangle are congruent to the three sides of another triangle, must the triangles be congruent? In other words, is Side-Side-Side (SSS) a congruence shortcut?

Follow Step 1 in your book to construct a triangle using the three given segments. Now try to construct a *different* triangle using the three segments as sides. Are you able to do it? (To determine whether two triangles are the same or different, you can place one on top of the other to see whether they coincide.)

You should find that you can make only one triangle from the three segments. In fact, if you are given *any* three segments (that satisfy the triangle inequality), you will be able to make only one triangle. That is, any two triangles with the same side lengths must be congruent. You can state this observation as a conjecture.

SSS Congruence Conjecture If the three sides of one triangle are congruent to the three sides of another triangle, then the triangles are congruent. **C-23**

Investigation 2: Is SAS a Congruence Shortcut?

Next you will consider the Side-Angle-Side (SAS) case. If two sides and the included angle of one triangle are congruent to two sides and the included angle of another, must the triangles be congruent?

Follow Step 1 in your book to construct a triangle from the three given parts. Now try to construct a *different* triangle from the same three parts. Are you able to do it? (Remember, the angle must be included *between* the sides.)

(continued)

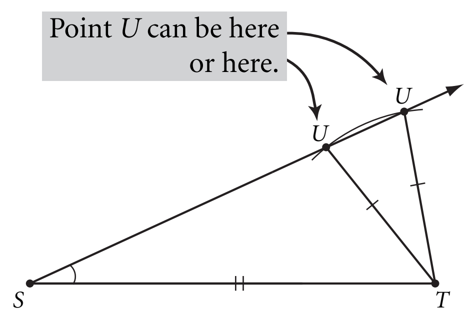
You should find that you can make only one triangle from the given parts. In fact, if you are given *any* two sides and an included angle, you will be able to make only one triangle. You can state this observation as a conjecture.

SAS Congruence Conjecture If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent. **C-24**

Investigation 3: Is SSA a Congruence Shortcut?

In this investigation you will explore the Side-Side-Angle (SSA) case. If two sides and a non-included angle of one triangle are congruent to the corresponding sides and angle of another triangle, must the triangles be congruent?

Follow Step 1 in your book to construct a triangle from the three given parts. Now try to construct a *different* triangle using the same two sides and non-included angle. Are you able to construct two different triangles using the same parts?

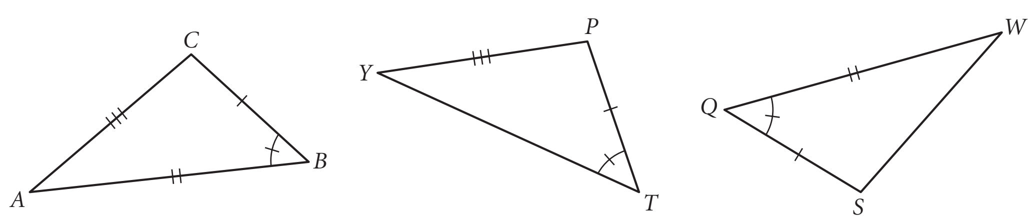


Once you construct $\overline{ST}$ on a side of $\angle S$, there are two possible locations for point U on the other side of the angle.

You can state this observation in a conjecture: If two sides and a non-included angle of one triangle are congruent to the corresponding two sides and non-included angle of another triangle, then the triangles are not necessarily congruent.

Here is an example that uses the new conjectures from this lesson.

EXAMPLE Using *only* the information given, determine which triangles below are congruent and state which congruence shortcut you used.



▶ ***Solution*** Because $\overline{AB} \cong \overline{WQ}$, $\angle B \cong \angle Q$, and $\overline{BC} \cong \overline{QS}$, $\triangle ABC \cong \triangle WQS$ by SAS.

Although $\overline{BC} \cong \overline{TP}$, $\overline{PY} \cong \overline{CA}$, and $\angle B \cong \angle T$, you cannot conclude that $\triangle ABC \cong \triangle TPY$ because SSA is not a congruence shortcut.

For $\triangle TPY$ and $\triangle QSW$, you know only that $\angle Q \cong \angle T$ and $\overline{QS} \cong \overline{TP}$. This is not enough information to conclude that the triangles are congruent.

CONDENSED LESSON

4.5 Are There Other Congruence Shortcuts?

In this lesson you will

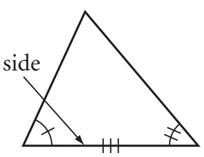
- Look for more shortcuts for determining whether two triangles are congruent

In Lesson 4.4, you saw that there are six ways in which three parts of one triangle can be congruent to three parts of another, and you investigated three of these cases. You learned the following congruence shortcuts.

- SSS: If three sides of one triangle are congruent to the three sides of another triangle, then the triangles are congruent.
- SAS: If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.

You also learned that if two sides and a non-included angle of one triangle are congruent to two sides and a non-included angle of another, then the triangles are *not* necessarily congruent. Now you will explore the three remaining cases.

A side that is between two given angles of a triangle is called an **included side,** as shown in the diagram at right.



Investigation 1: Is ASA a Congruence Shortcut?

In this investigation you explore the Angle-Side-Angle (ASA) case. If two angles and the included side of one triangle are congruent to two angles and the included side of another, must the triangles be congruent?

Follow Step 1 in your book to construct a triangle using the three given parts. Now try to construct a *different* triangle using the three parts. Are you able to do it? (Remember, to determine whether two triangles are the same or different, you can place one on top of the other to see if they coincide.)

You should find that you can make only one triangle from the three given parts. In fact, if you are given any two angles and an included side, you will be able to make only one triangle. You can state this fact as a conjecture.

ASA Congruence Conjecture If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent. **C-25**

Investigation 2: Is SAA a Triangle Congruence Shortcut?

Now consider Side-Angle-Angle (SAA), where the side is not included between the two angles.

Follow Step 1 in your book to construct a triangle from the three given parts. Now try to construct a *different* triangle using the same three parts. Are you able to do it?

(continued)

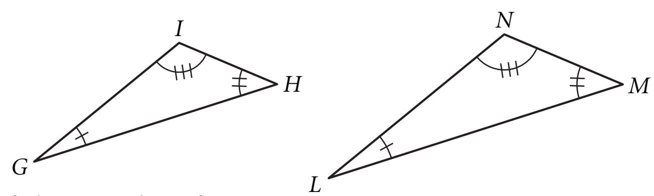
You should find that you can make only one triangle when given two angles and a non-included side. You can now state the following conjecture:

SAA Congruence Conjecture If two angles and a non-included side of one triangle are congruent to the corresponding angles and side of another triangle, then the triangles are congruent. **C-26**

Investigation 3: Is AAA a Triangle Congruence Shortcut?

Finally, you will explore the Angle-Angle-Angle (AAA) case. Construct a triangle from the three angles given in your book. Now try to construct a *different* triangle using the same three angles. Are you able to do it?

Because no side lengths are given, you can make the first side any length you like. By using a different length in your second triangle, you get a triangle of a different size.



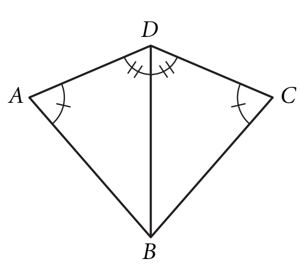
You can now state the following conjecture: If three angles of one triangle are congruent to the three angles of another triangle, then the two triangles are not necessarily congruent.

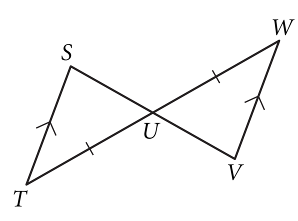
In the last two lessons, you have found that SSS, SAS, ASA, and SAA are all congruence shortcuts. Add them to your conjecture list. Here is an example.

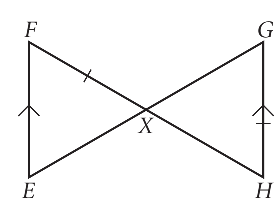
EXAMPLE

Complete each statement and tell which congruence shortcut you used to determine that the triangles are congruent. If the triangles cannot be shown to be congruent, write "cannot be determined."

a. $\triangle ADB \cong \triangle$ ________ **b.** $\triangle STU \cong \triangle$ ________ **c.** $\triangle EFX \cong \triangle$ ________







Solution

a. Because $\angle A \cong \angle C$, $\angle ADB \cong \angle CDB$, and $\overline{BD} \cong \overline{BD}$, $\triangle ADB \cong \triangle CDB$ by SAA.

b. Because $\overline{ST}$ and $\overline{WV}$ are parallel, $\angle S \cong \angle V$ and $\angle T \cong \angle W$. It is given that $\overline{TU} \cong \overline{WU}$. Therefore, $\triangle STU \cong \triangle VWU$ by SAA. You could also reason that $\angle SUT \cong \angle VUW$ because they are vertical angles, so $\triangle STU \cong \triangle VWU$ by ASA.

c. Cannot be determined. Because $\overline{EF}$ and $\overline{GH}$ are parallel, $\angle E \cong \angle G$ and $\angle F \cong \angle H$. However, the congruent sides $\overline{FX}$ and $\overline{GH}$ are not corresponding. So, there is not enough information to show that the two triangles are congruent.

CONDENSED LESSON

4.6 Corresponding Parts of Congruent Triangles

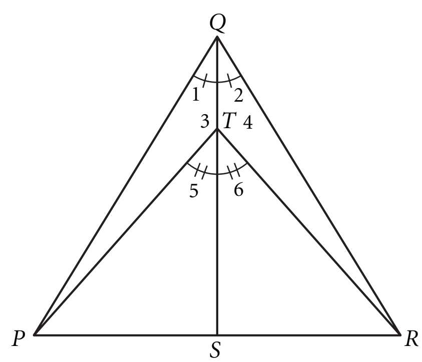
In this lesson you will

- Use the fact that corresponding parts of congruent triangles are congruent to prove statements
- Learn techniques for keeping track of information when you are writing a proof

In Lessons 4.4 and 4.5, you discovered four shortcuts for showing that two triangles are congruent—SSS, SAS, ASA, and SAA. Once you have established that two triangles are congruent, you know that their corresponding parts are congruent. We will abbreviate the statement *corresponding parts of congruent triangles are congruent* as CPCTC.

Example A in your book uses CPCTC to prove that two segments are congruent. Read this example carefully. Notice that the argument first explains why the *triangles* AMD and BMC are congruent, and then uses CPCTC to explain why the *sides* $\overline{AD}$ and $\overline{BC}$ are congruent. Below is another example.

EXAMPLE A Give a deductive argument to explain why $\overline{PT} \cong \overline{RT}$.



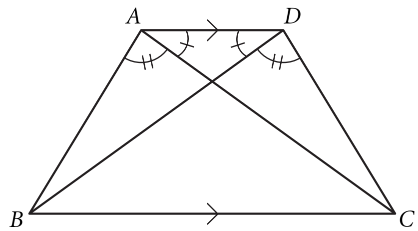
▶ Solution First, you'll show that $\triangle PQT \cong \triangle RQT$ and then use CPCTC to show that $\overline{PT} \cong \overline{RT}$. You are given that $\angle 1 \cong \angle 2$. You also know that $\overline{QT} \cong \overline{QT}$ because they are the same segment. Now, because $\angle 3$ and $\angle 5$ are a linear pair, $m\angle 3 + m\angle 5 = 180°$, or equivalently, $m\angle 3 = 180° - m\angle 5$. Because $\angle 5 \cong \angle 6$, you can substitute $m\angle 6$ for $m\angle 5$ to get $m\angle 3 = 180° - m\angle 6$.

However, $\angle 4$ and $\angle 6$ form a linear pair, so $m\angle 4$ is also equal to $180° - m\angle 6$. Therefore, $m\angle 3 = m\angle 4$. That is, $\angle 3 \cong \angle 4$. So, by ASA, $\triangle PQT \cong \triangle RQT$. Because the triangles are congruent, $\overline{PT} \cong \overline{RT}$ by CPCTC.

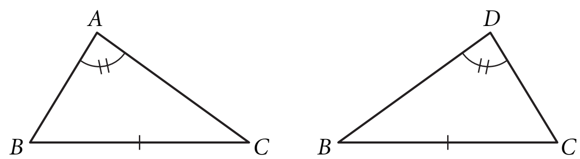
When you are trying to prove that triangles are congruent, it can be hard to keep track of the information. Be sure to mark all the information on the figure. If the triangles are hard to see, you can draw them with different colors or redraw them separately. These techniques are demonstrated in Example B in your book. Read that example and make sure you understand it. Then read the example on the next page.

(continued)

EXAMPLE B Is $\overline{AC} \cong \overline{DB}$? Write a paragraph proof explaining why.



► Solution You can draw triangles ABC and DCB separately to see them more clearly. As you discover more information, mark it on the original diagram and the separated triangles.



Paragraph Proof: Show that $\overline{AC} \cong \overline{DB}$.

$\angle BAC \cong \angle CDB$. Also, $\overline{BC} \cong \overline{CB}$ because they are the same segment. Because $\overline{AD} \parallel \overline{BC}$, the alternate interior angles are congruent. Therefore, $\angle ACB \cong \angle DAC$ and $\angle ADB \cong \angle DBC$. Because it is given that $\angle DAC \cong \angle ADB$, it must also be true that $\angle ACB \cong \angle DBC$. (Mark this information on the diagrams.) $\triangle ABC \cong \triangle DCB$ by SAA. By CPCTC, $\overline{AC} \cong \overline{DB}$.

CONDENSED LESSON

4.7 Flowchart Thinking

In this lesson you will

- Write **flowchart proofs**

So far, you have been writing explanations as deductive arguments or paragraph proofs. Example A in your book shows a paragraph proof. Read this example and make sure you understand the proof.

When a logical argument is complex or includes many steps, a paragraph proof may not be the clearest way to present the steps. In such cases, it is often helpful to organize the steps in the form of a *flowchart*. A **flowchart** is a visual way to organize all the steps in a complicated procedure in their proper order. The steps in the procedure are written in boxes. Arrows connect the boxes to show how facts lead to conclusions.

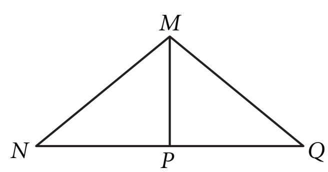
Creating a **flowchart proof** makes your logic visible so that others can follow your reasoning. Example B in your book presents the argument from Example A in flowchart form. Read the proof carefully. Notice that each statement is written in a box and that the logical reason for each step is written beneath its box.

More flowchart proofs are given in the examples below. In each example, try to write a proof yourself before looking at the solution. Remember, there are often several ways to prove a statement. Your proof may not be the same as the one given.

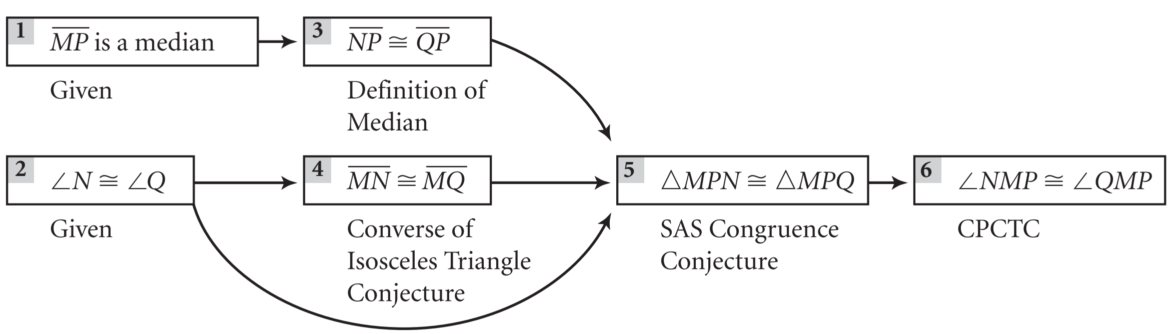
EXAMPLE A

Given: $\overline{MP}$ is a median
$\angle N \cong \angle Q$

Show: $\angle NMP \cong \angle QMP$



► ***Solution*** **Flowchart Proof**



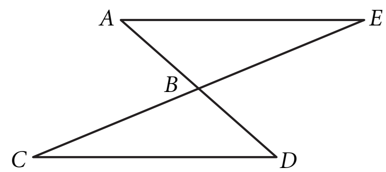
(continued)

EXAMPLE B

Given: $\overline{AE} \parallel \overline{CD}$

B is the midpoint of $\overline{CE}$

Show: $\overline{AB} \cong \overline{DB}$



▶ ***Solution*** **Flowchart Proof**

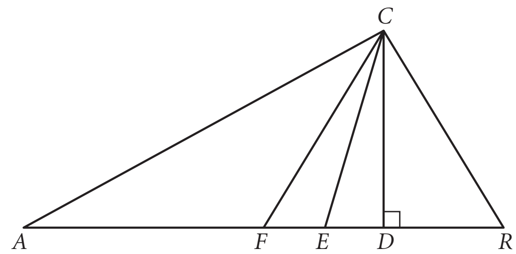
1. $\overline{AE} \parallel \overline{CD}$ — Given → 3
2. B is the midpoint of $\overline{CE}$ — Given → 4
3. $\angle E \cong \angle C$ — AIA Conjecture → 6
4. $\overline{BC} \cong \overline{BE}$ — Definition of midpoint → 6
5. $\angle ABE \cong \angle DBC$ — Vertical Angles Conjecture → 6
6. $\triangle ABE \cong \triangle DBC$ — ASA Congruence Conjecture → 7
7. $\overline{AB} \cong \overline{DB}$ — CPCTC

CONDENSED LESSON

4.8 Proving Special Triangle Conjectures

In this lesson you will

- Make a conjecture about the bisector of the vertex angle in an isosceles triangle
- Make and prove a conjecture about equilateral triangles
- Learn about **biconditional** conjectures



In $\triangle ARC$, $\overline{CD}$ is the altitude to the base $\overline{AR}$, $\overline{CE}$ is the angle bisector of $\angle ACR$, and $\overline{CF}$ is the median to side $\overline{AR}$. This example illustrates that the angle bisector, the altitude, and the median can all be different segments. Is this always true? Can they all be the same segment? You will explore these questions in the investigation.

Investigation: The Symmetry Line in an Isosceles Triangle

Construct a large isosceles triangle on a sheet of unlined paper. Label it *ARK*, with *K* as the vertex angle.

Construct angle bisector $\overline{KD}$ with point *D* on $\overline{AR}$. Compare $\triangle ADK$ with $\triangle RDK$. Do they look congruent?

Use your compass to compare $\overline{AD}$ and $\overline{RD}$. Are they congruent? If so, then *D* is the midpoint of $\overline{AR}$, and therefore $\overline{KD}$ is the median to $\overline{AR}$. Notice that $\angle ADK$ and $\angle RDK$ are a linear pair and are therefore supplementary. Now compare $\angle ADK$ and $\angle RDK$. Are the angles congruent? If so, what must the measure of each angle be? What does this tell you about $\overline{KD}$ and $\overline{AR}$?

Your findings should lead to the following conjecture.

Vertex Angle Bisector Conjecture In an isosceles triangle, the bisector of the vertex angle is also the altitude and the median to the base. **C-27**

In Chapter 3, you discovered that if a triangle is equilateral, then each angle measures 60°. Therefore, if a triangle is equilateral, then it is equiangular. This is called the Equilateral Triangle Conjecture. A proof of this statement is given on page 245 of your book. Read this proof carefully and make sure you understand each step. The *converse* of this statement is also true. That is, if a triangle is equiangular, then it is equilateral. The following conjecture combines these ideas.

Equilateral/Equiangular Triangle Conjecture Every equilateral triangle is equiangular, and, conversely, every equiangular triangle is equilateral. **C-28**

(continued)

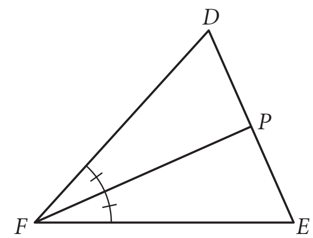
A **biconditional** conjecture is a conjecture in which one condition cannot be true unless the other is also true. In other words, both the statement and its converse are true. The Equilateral/Equiangular Triangle Conjecture is biconditional, so it can be written: A triangle is equilateral *if and only if* it is equiangular.

Here is an example using the new conjectures.

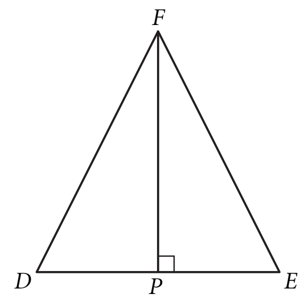
EXAMPLE

$\triangle DEF$ is isosceles with $\overline{DF} \cong \overline{EF}$.

a. $m\angle D = 67°$
$DE = 15$ cm
$m\angle DFP =$ ______
$DP =$ ______



b. $m\angle DFE = 54°$
$DP = 7$ cm
$m\angle DFP =$ ______
$DE =$ ______



▶ ***Solution***

a. If $m\angle D = 67°$, then $m\angle E = 67°$ because $\angle D$ and $\angle E$ are base angles of an isosceles triangle. Therefore, $m\angle DFE = 180° - (67° + 67°) = 46°$. Because $\overline{FP}$ bisects $\angle DFE$, $m\angle DFP = \frac{1}{2}(46°) = 23°$.

Because $\overline{FP}$ bisects vertex angle DFE, it must also be the median to $\overline{DE}$. Therefore, $DP = \frac{1}{2}DE = \frac{1}{2}(15 \text{ cm}) = 7.5$ cm.

b. Because $\overline{FP}$ is the altitude to $\overline{DE}$, it must also be the bisector of vertex angle DFE. Therefore, $m\angle DFP = \frac{1}{2}(m\angle DFE) = \frac{1}{2}(54°) = 27°$.

Because $\overline{FP}$ bisects vertex angle DFE, it must also be the median to $\overline{DE}$. Therefore, $DE = 2DP = 2(7 \text{ cm}) = 14$ cm.