Lesson 2.1 • Inductive Reasoning

For Exercises 1–8, use inductive reasoning to find the next two terms in each sequence.

1. 4, 8, 12, 16, _____, _____
2. 400, 200, 100, 50, 25, _____, _____
3. $\frac{1}{8}$, $\frac{2}{7}$, $\frac{4}{5}$, _____, _____
4. $-5$, $-3$, $-2$, $1$, $-1$, $0$, _____, _____
5. 360, 180, 120, 90, _____, _____
6. 1, 3, 9, 27, 81, _____, _____
7. 1, 5, 17, 53, 161, _____, _____
8. 1, 5, 14, 30, 55, _____, _____

For Exercises 9–12, use inductive reasoning to draw the next two shapes in each picture pattern.

9. ![Shape Pattern]
10. ![Shape Pattern]
11. ![Shape Pattern]
12. ![Shape Pattern]

For Exercises 13–15, use inductive reasoning to test each conjecture. Decide if the conjecture seems true or false. If it seems false, give a counterexample.

13. Every odd whole number can be written as the difference of two squares.
14. Every whole number greater than 1 can be written as the sum of two prime numbers.
15. The square of a number is larger than the number.
1. \( \triangle ABC \) is equilateral. Is \( \triangle ABD \) equilateral? What type of reasoning, inductive or deductive, do you use when solving this problem?

2. If \( 6 \neq 8, 10 \neq 3 = 6\frac{1}{2} \), and \( -3 \neq -2 = -2.5 \), then
   \[ 4 \neq 8 = \quad -5 \neq 0 = \quad 2 \neq 2 = \]
   What type of reasoning, inductive or deductive, do you use when solving this problem?

3. \( \angle A \) and \( \angle D \) are complementary. \( \angle A \) and \( \angle E \) are supplementary. What can you conclude about \( \angle D \) and \( \angle E \)? What type of reasoning, inductive or deductive, do you use when solving this problem?

4. What type of reasoning, inductive or deductive, do you use when solving this problem?

5. Solve each equation for \( x \). Give a reason for each step in the process.
   a. \( 4x + 3(2 - x) = 8 - 2x \)
   b. \( \frac{19 - 2(3x - 1)}{5} = x + 2 \)
   What type of reasoning, inductive or deductive, do you use when solving these problems?

6. A sequence is generated by the function \( f(n) = 5 - n^2 \). Give the first five terms in the sequence. What type of reasoning, inductive or deductive, do you use when solving this problem?

7. A sequence begins \( -4, 1, 6, 11 \ldots \)
   a. Give the next two terms in the sequence. What type of reasoning, inductive or deductive, do you use when solving this problem?
   b. Find a rule that generates the sequence. Then give the 50th term in the sequence. What type of reasoning, inductive or deductive, do you use when solving this problem?

8. Choose any 3-digit number. Multiply it by 7. Multiply the result by 11. Then multiply the result by 13. Do you notice anything? Try a few other 3-digit numbers and make a conjecture. Use deductive reasoning to explain why your conjecture is true.

Which are whatnots?

Not whatnots

Whatnots
Lesson 2.3 • Finding the $n$th Term

For Exercises 1–4, tell whether or not the rule is a linear function.

1. $n$ | 1 | 2 | 3 | 4 | 5  
   ---|---|---|---|---|---
   $f(n)$ | 8 | 15 | 22 | 29 | 36

2. $n$ | 1 | 2 | 3 | 4 | 5  
   ---|---|---|---|---|---
   $g(n)$ | 14 | 11 | 8 | 5 | 2

3. $n$ | 1 | 2 | 3 | 4 | 5  
   ---|---|---|---|---|---
   $h(n)$ | -9 | -6 | -2 | 3 | 9

4. $n$ | 1 | 2 | 3 | 4 | 5  
   ---|---|---|---|---|---
   $j(n)$ | $-\frac{3}{2}$ | -1 | $-\frac{1}{2}$ | 0 | $\frac{1}{2}$

For Exercises 5 and 6, complete each table.

5. $n$ | 1 | 2 | 3 | 4 | 5 | 6  
   ---|---|---|---|---|---|---
   $f(n) = 7n - 12$ | | | | | | 

6. $n$ | 1 | 2 | 3 | 4 | 5 | 6  
   ---|---|---|---|---|---|---
   $g(n) = -8n - 2$ | | | | | | 

For Exercises 7–9, find the function rule for each sequence. Then find the 50th term in the sequence.

7. $n$ | 1 | 2 | 3 | 4 | 5 | 6 | ... | $n$ | ... | 50  
   ---|---|---|---|---|---|---|---|---|---|---|---
   $f(n)$ | 9 | 13 | 17 | 21 | 25 | 29 | | | | | |

8. $n$ | 1 | 2 | 3 | 4 | 5 | 6 | ... | $n$ | ... | 50  
   ---|---|---|---|---|---|---|---|---|---|---|---
   $g(n)$ | 6 | 1 | -4 | -9 | -14 | -19 | | | | | |

9. $n$ | 1 | 2 | 3 | 4 | 5 | 6 | ... | $n$ | ... | 50  
   ---|---|---|---|---|---|---|---|---|---|---|---
   $h(n)$ | 6.5 | 7 | 7.5 | 8 | 8.5 | 9 | | | | | |

10. Find the rule for the number of tiles in the $n$th figure. Then find the number of tiles in the 200th figure.

![Diagram of triangles]

11. Sketch the next figure in the sequence. Then complete the table.

![Diagram of squares]

| $n$ | 1 | 2 | 3 | 4 | ... | $n$ | ... | 50 |
|-----|---|---|---|---|---|---|---|---|---|
| Number of segments and lines | 2 | 6 | | | | | | | |
| Number of regions of the plane | 4 | | | | | | | | |
Lesson 2.4 • Mathematical Modeling

1. If you toss a coin, you will get a head or a tail. Copy and complete the geometric model to show all possible results of four consecutive tosses.

H
   
   T

How many sequences of results are possible? How many sequences have exactly one tail? Assuming a head or a tail is equally likely, what is the probability of getting exactly one head in four tosses?

2. If there are 12 people sitting around a table, how many different pairs of people can have conversations during dinner, assuming they can all talk to each other? What geometric figure can you use to model this situation?

3. Tournament games and results are often displayed using a geometric model. Two examples are shown below. Sketch a geometric model for a tournament involving 4 teams and a tournament involving 6 teams. Each team must have the same chance to win. Try to have as few games as possible in each tournament. Show the total number of games in each tournament. Name the teams a, b, c . . . and number the games 1, 2, 3 . . . .

a
   b
   c

3 teams, 3 games (round robin)

a
   b
   c
   d

4 teams, 3 games (single elimination)
Lesson 2.5 • Angle Relationships

For Exercises 1–8, find each lettered angle measure without using a protractor.

1. \[ \angle a = 112^\circ, \quad \angle b = c \]
2. \[ \angle a = 38^\circ, \quad \angle b = \angle c \]
3. \[ \angle d = 40^\circ, \quad \angle e = 70^\circ, \quad \angle f \]
4. \[ \angle a = 42^\circ, \quad \angle b = \angle c \]
5. \[ \angle a = 132^\circ, \quad \angle b = 138^\circ \]
6. \[ \angle a = 70^\circ, \quad \angle b = \angle c \]
7. \[ \angle a = 110^\circ, \quad \angle b = 100^\circ, \quad \angle c = 25^\circ \]
8. \[ \angle a = \angle b \]

For Exercises 9–14, tell whether each statement is always (A), sometimes (S), or never (N) true.

9. _____ The sum of the measures of two acute angles equals the measure of an obtuse angle.
10. _____ If \( \angle XAY \) and \( \angle PAQ \) are vertical angles, then either \( X, A, \) and \( P \) or \( X, A, \) and \( Q \) are collinear.
11. _____ The sum of the measures of two obtuse angles equals the measure of an obtuse angle.
12. _____ The difference between the measures of the supplement and the complement of an angle is 90°.
13. _____ If two angles form a linear pair, then they are complementary.
14. _____ If a statement is true, then its converse is true.

For Exercises 15–19, fill in each blank to make a true statement.

15. If one angle of a linear pair is obtuse, then the other is ____________.
16. If \( \angle A \cong \angle B \) and the supplement of \( \angle B \) has measure 22°, then \( m\angle A = \) ________________.
17. If \( \angle P \) is a right angle and \( \angle P \) and \( \angle Q \) form a linear pair, then \( m\angle Q \) is ________________.
18. If \( \angle S \) and \( \angle T \) are complementary and \( \angle T \) and \( \angle U \) are supplementary, then \( \angle U \) is a(n) ________________ angle.
19. Switching the “if” and “then” parts of a statement changes the statement to its ________________.
Lesson 2.6 • Special Angles on Parallel Lines

For Exercises 1–11, use the figure at right.

For Exercises 1–5, find an example of each term.

1. Corresponding angles
2. Alternate interior angles
3. Alternate exterior angles
4. Vertical angles
5. Linear pair of angles

For Exercises 6–11, tell whether each statement is always (A), sometimes (S), or never (N) true.

6. _____ \( \angle 1 \cong \angle 3 \)  
7. _____ \( \angle 3 \cong \angle 8 \)
8. _____ \( \angle 2 \) and \( \angle 6 \) are supplementary.  
9. _____ \( \angle 7 \) and \( \angle 8 \) are supplementary.
10. _____ \( m\angle 1 \neq m\angle 6 \)  
11. _____ \( m\angle 5 = m\angle 4 \)

For Exercises 12–14, use your conjectures to find each angle measure.

12. 

13. 

14.

For Exercises 15–17, use your conjectures to determine whether or not \( \ell_1 \parallel \ell_2 \), and explain why. If not enough information is given, write “cannot be determined.”

15. 

16. 

17.

18. Find each angle measure.