Symbolic Representation

In this lesson you will

● write problems in symbolic form and then solve them using algebra

Sometimes the easiest way to solve a problem is to translate it into symbolic expressions and equations and then apply algebraic solution methods. In Examples A and B in your book, a four-step solution process is used to solve two problems. Read these examples carefully and make sure you understand the four problem-solving steps. Then read the example below. Try to solve it yourself before reading the solution.

**EXAMPLE**

The souvenir shop at the Summerville Sox baseball park sells balls, caps, and jerseys with the team logo. Samantha bought a cap and five balls for $51. The four caps Carlos bought cost $12 more than the jersey his brother bought. Mr. Kurowski spent $177 on three balls and three jerseys for his grandchildren. How much does each item cost? (Assume sales tax is included.)

**Solution**

First, list the unknown quantities and assign a variable to each.

Let \( b \) represent the cost of a ball.
Let \( c \) represent the cost of a cap.
Let \( j \) represent the cost of a jersey.

Second, use the information from the problem to write equations.

\[
\begin{align*}
    c + 5b &= 51 & \text{Equation 1: Samantha’s purchases translated into an algebraic equation.} \\
    4c - j &= 12 & \text{Equation 2: Information about Carlos’s and his brother’s purchases.} \\
    3b + 3j &= 177 & \text{Equation 3: Mr. Kurowski’s purchases.}
\end{align*}
\]

Third, solve the system of equations to find the values for the variables.

\[
\begin{align*}
    12c - 3j &= 36 & \text{Multiply both sides of Equation 2 by 3.} \\
    3b + 3j &= 177 & \text{Equation 3.}
\end{align*}
\]

\[
12c + 3b = 213 \quad \text{Add the equations to eliminate the variable } j.
\]

\[
\begin{align*}
    4c + b &= 71 & \text{Divide both sides of the sum above by 3.} \\
    4c + 20b &= 204 & \text{Multiply both sides of Equation 1 by 4.}
\end{align*}
\]

\[-19b = -133 \quad \text{Subtract the equations to eliminate the variable } c.
\]

\[
\begin{align*}
    b &= 7 & \text{Divide both sides by } -19 \text{ to solve for } b.
    c + 5(7) &= 51 & \text{Substitute 7 for } b \text{ in Equation 1.}
    c &= 16 & \text{Solve for } c.
    4(16) - j &= 12 & \text{Substitute 16 for } c \text{ in Equation 2.}
    52 &= j & \text{Solve for } j.
\end{align*}
\]

Finally, interpret your solution. A ball costs $7, a cap costs $16, and a jersey costs $52.

(continued)
Lesson 0.2 • Symbolic Representation (continued)

Investigation: Problems, Problems, Problems
The investigation in your book asks you to solve one of two given problems. Below are a solution to Problem 1 and a hint for solving Problem 2.

Problem 1 Solution
Step 1 Let \( C \) represent the hourly wage for clerks. Let \( S \) represent the hourly wage for supervisors. Let \( B \) represent the daily salary budget.

Step 2 Write equations to represent the information in the problem.

\[
\begin{align*}
16C + 8S &= B \quad \text{Equation 1: Information from Monday.} \\
20C + 4S &= B - 10 \quad \text{Equation 2: Information from Tuesday.} \\
24C + 4S &= B + 20 \quad \text{Equation 3: Information from Wednesday.}
\end{align*}
\]

Step 3 Solve the system of equations.

\[
\begin{align*}
4C &= 30 & \text{Subtract Equation 2 from Equation 3 to eliminate } S \text{ and } B. \\
C &= 7.5 & \text{Divide both sides by 4 to solve for } C.
\end{align*}
\]

\[
\begin{align*}
120 + 8S &= B & \text{Substitute 7.5 for } C \text{ into Equation 1.} \\
150 + 4S &= B - 10 & \text{Substitute 7.5 for } C \text{ into Equation 2.} \\
-30 + 4S &= 10 & \text{Subtract Equation 2 from Equation 1 to eliminate } B.
\end{align*}
\]

\[
\begin{align*}
S &= 10 & \text{Solve for } S.
\end{align*}
\]

\[
\begin{align*}
120 + 80 &= B & \text{Substitute 10 for } S \text{ into Equation 1.} \\
200 &= B & \text{Solve for } B.
\end{align*}
\]

Step 4 Clerks earn $7.50 per hour and supervisors earn $10 per hour. The daily salary budget is $200.

Problem 2 Hint
If there are \( s \) bags of sand, then the weight of the sand can be represented as 50\( s \). Write one equation to express the maximum weight the truck can hold. Write a second equation to represent the ratio of sand to cement. When you interpret your solution, remember that you will want whole numbers of bags of sand and cement.
Organizing Information

In this lesson you will

- organize the information in problems into categories

When you are faced with a difficult problem, it is often best to start by organizing the given information in some way. This can help you see what you know and determine what you need to find out. In Examples A and B, the units of measure are used to link the pieces of information, and a table is used to organize what you know and what you need to know. Example C shows another way to organize and categorize information. Work through all three examples, and then read the example below.

**EXAMPLE**

How many liters of a salt solution with a concentration of 20 grams of salt per liter must be added to 80 liters of a solution with a concentration of 35 grams of salt per liter to create a solution with a concentration of 30 grams of salt per liter?

**Solution**

There are three solutions involved: the 35 g/L solution, the 20 g/L solution, and the combined solution. Call these Solution 1, Solution 2, and Mixture, respectively. Let \( x \) represent the amount of Solution 2 needed. Organize the information by creating a table with the row and column labels shown below. Start by filling in the values you know. (These values are underlined in the table.)

The units indicate that the concentration of a solution is equal to the mass of salt divided by the volume. Use this relationship to fill in the mass of salt for the two solutions. To find the mass and volume of the mixture, add the masses and volumes of Solutions 1 and 2.

<table>
<thead>
<tr>
<th>Concentration of solution (in g/L)</th>
<th>Solution 1</th>
<th>Solution 2</th>
<th>Mixture</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>20</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>Volume of solution (in L)</td>
<td>80</td>
<td>( x )</td>
<td>80 + ( x )</td>
</tr>
<tr>
<td>Mass of salt (in g)</td>
<td>2800</td>
<td>20( x )</td>
<td>2800 + 20( x )</td>
</tr>
</tbody>
</table>

The concentration of the mixture is equal to 30 g/L, but it is also equal to \( \frac{\text{mass of salt in mixture}}{\text{volume of mixture}} = \frac{2800 + 20x}{80 + x} \). Write an equation and solve it for \( x \).

\[
30 = \frac{2800 + 20x}{80 + x}
\]

The concentration equation.

\[
2400 + 30x = 2800 + 20x
\]

Multiply both sides by \( 80 + x \).

\[
10x = 400
\]

Subtract 2400 and 20\( x \) from both sides.

\[
x = 40
\]

Divide both sides by 10.

So, 40 grams of Solution 2 must be added to create the mixture.

(continued)
Lesson 0.3 • Organizing Information (continued)

Investigation: Who Owns the Zebra?

Read the investigation in your book. There are many ways to approach this problem. One method involves making a table showing all the possible house colors, cars, pets, newspapers, and plants for each household.

<table>
<thead>
<tr>
<th>Position</th>
<th>1 (left)</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5 (right)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>House color</strong></td>
<td>Red</td>
<td>Red</td>
<td>Red</td>
<td>Red</td>
<td>Red</td>
</tr>
<tr>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
<td>Green</td>
</tr>
<tr>
<td>White</td>
<td>White</td>
<td>White</td>
<td>White</td>
<td>White</td>
<td>White</td>
</tr>
<tr>
<td>Yellow</td>
<td>Yellow</td>
<td>Yellow</td>
<td>Yellow</td>
<td>Yellow</td>
<td>Yellow</td>
</tr>
<tr>
<td>Blue</td>
<td>Blue</td>
<td>Blue</td>
<td>Blue</td>
<td>Blue</td>
<td>Blue</td>
</tr>
<tr>
<td><strong>Car</strong></td>
<td>Station wagon</td>
<td>Station wagon</td>
<td>Station wagon</td>
<td>Station wagon</td>
<td>Station wagon</td>
</tr>
<tr>
<td>SUV</td>
<td>SUV</td>
<td>SUV</td>
<td>SUV</td>
<td>SUV</td>
<td>SUV</td>
</tr>
<tr>
<td>Van</td>
<td>Van</td>
<td>Van</td>
<td>Van</td>
<td>Van</td>
<td>Van</td>
</tr>
<tr>
<td>Sports car</td>
<td>Sports car</td>
<td>Sports car</td>
<td>Sports car</td>
<td>Sports car</td>
<td>Sports car</td>
</tr>
<tr>
<td>Compact</td>
<td>Compact</td>
<td>Compact</td>
<td>Compact</td>
<td>Compact</td>
<td>Compact</td>
</tr>
</tbody>
</table>

Make a table showing all the possibilities. Then use the clues to cross out the possibilities you know are wrong and circle the ones you know are right. You will have to go through the clues several times; some clues will not be useful right away. Here are some steps to get you started:

- The first clue that is immediately useful is the eighth clue, which indicates that the family in the middle house reads the *Times*. So under position 3, circle “*Times*.” Cross out the other newspapers under position 3 and cross out “*Times*” under all the other positions.
- Using the ninth clue, you can circle “Compact” under position 1, cross out all the other cars for position 1, and cross out “Compact” under all the other positions.
- Using the twelfth clue, you can cross out “Beets” under position 3.
- Using the thirteenth clue, you can cross out “Okra” under position 1.
- Using the fourteenth clue, you can circle “Blue” under position 2, cross out the other house colors under position 2, and cross out “Blue” under the other positions.
- Using the first clue, you can cross out “Station wagon” under position 2 and cross out “Red” under position 1.
- Using the second clue, you can cross out “Dog” under position 1.

Continue reading through the clues and marking the information you know. Every so often, stop and look carefully at your table. You may find that you have crossed out all but one possibility in a particular section or row. You can then circle the remaining possibility. When you are finished, you should find that the family in the fourth green house owns the zebra.