Overview

In Chapter 3, students use equations to model linear growth and graphs of straight lines and learn the balancing method for solving equations. This chapter builds toward the concept of function, which is formalized in Chapter 8.

Lesson 3.1 begins the development of linear growth with the study of recursive sequences. In Lesson 3.2, students encounter linear plots. These two ideas are combined through the notion of “walking instructions” to study motion in Lesson 3.3. The ideas of starting value and rate of change are formalized in Lesson 3.4 with the intercept form of a line.

In Lesson 3.5, students study rates of change further, using input-output tables that foreshadow the study of functions. In the Activity Day, Lesson 3.7, students model real-world data with their linear equations.

The Mathematics

Linearity

Having a constant rate of change is a primary characteristic of linearity. You start somewhere and advance by the same amount at each step. This kind of change is represented by a recursive sequence, easily generated on a calculator.

\[-15\quad \text{ENTER}\]
\[\text{Ans} + 10 \quad \text{ENTER}\]
\[\text{ENTER}, \quad \text{ENTER}\]

Start with \(-15\).
Add 10 to the answer.
Continue to add 10 to each answer.

A constant rate of change produces linear growth, though the values will be shrinking instead of growing if the rate of change is negative.

A second way to think about linearity is through equations that relate variables. Students begin to use, write, and make sense of the intercept form of the equation of a line, \(y = a + bx\). Seeing and using multiple representations help students connect the recursive sequence start value with \(a\) and its constant rate of change with \(b\). The calculator steps for the recursive sequence above are equivalent to the equation \(y = -15 + 10x\), when the initial \(x\)-value is 0.

The traditional slope-intercept form, \(y = mx + b\), is mentioned in Lesson 4.2. In Lesson 4.3, students will see the point-slope form, \(y = y_1 + b(x - x_1)\).

A third way to think about linearity is through graphs. Indeed, the term linearity comes from the fact that the associated graphs are (straight) lines. Students have seen linear graphs before—in data points, the graph of \(y = x\) for comparing estimates with actual distances in Chapter 1, and direct variations \(y = kx\) in Chapter 2.

The new forms of equations of a line indicate new ways of thinking of the graph. For example, the intercept form \(y = a + bx\) allows students to graph by starting at point \((0, a)\) and moving vertically \(b\) units for each unit they move across from left to right. This process reflects the constant rate of change of linear growth and the recursive sequence. Later, in Chapter 8, students will discover that the point-slope equation \(y = y_1 + b(x - x_1)\) represents a vertical shift of \(y_1\) and a horizontal shift of \(x_1\).

Most data sets from real-world situations with a linear trend aren’t exactly linear. Lesson 3.7 provides an activity for finding an equation that models a set of data points that lie close to but not on a straight line. Students will learn more about lines of fit in Chapter 4.

Solving Equations

Many real-life situations call for predicting when linear growth will reach a certain value. Ways of making that prediction reflect the three ways of thinking about linearity—constant rate of change, equations that relate variables, and graphs.

From the constant-rate-of-change perspective, you can run the recursive routine until it reaches the desired output, counting input steps as you go. To mount a flagpole 75 ft up on a building, what floor would you go to if the building’s basement floor is 15 ft below the ground and its floors are 10 ft apart?
Just run the sequence \(-15 + 10; \) \(\ldots \) until you get to 75 ft.

To undo those steps and get back to the number of floors, you can subtract \(-15\) and divide by 10, the distance between floors.

This undoing process took place with equations in Chapter 2: If \(10x - 15 = 75\), you can “get back to” \(x\) by adding 15 to 75 and then dividing by 10. The equation can also be solved using the metaphor of an equation as a pan balance. To keep it balanced, you do the same thing to both sides.

The third approach to linearity, graphs, also provides a means of solving equations. You can graph the data points or the equation on a graphing calculator and then use the trace feature to approximate the input value for the desired output value. Using the calculator’s table feature is another way to approximate a solution.

**Using This Chapter**

Lesson 3.1 is essential because recursive sequences will be used throughout the book. Solving by undoing is emphasized until the introduction of the balancing method in Lesson 3.6. If you must skip a lesson, you could skip the activity day, Lesson 3.7. No new material is presented, and the rope tying is done again in Lesson 5.2.

**Resources**

**Discovering Algebra Resources**

Teaching and Worksheet Masters
- Lessons 3.2, 3.5, 3.6

Calculator Notes 0D, 1J, 2A, 2C, 3A, 3B, 3C, 3D

Sketchpad Demonstrations
- Lessons 3.1, 3.5, 3.6

Fathom Demonstrations
- Lessons 3.1, 3.2, 3.4, 3.7

CBL 2 Demonstration
- Lesson 3.5

Dynamic Algebra Explorations online
- Lessons 3.1, 3.2

Assessment Resources
- Quiz 1 (Lessons 3.1–3.3)
- Quiz 2 (Lessons 3.4–3.6)
- Chapter 3 Test
- Chapter 3 Constructive Assessment Options
- Chapters 1 to 3 Exam

More Practice Your Skills for Chapter 3

Condensed Lessons for Chapter 3

**Other Resources**

*Play It Again Sam: Recurrence Equations and Recursion in Mathematics and Computer Science* by Rochelle Wilson Meyer and Walter Meyer.

For complete references to this and other resources see www.keypress.com/DA.

**Pacing Guide**

<table>
<thead>
<tr>
<th>day 1</th>
<th>day 2</th>
<th>day 3</th>
<th>day 4</th>
<th>day 5</th>
<th>day 6</th>
<th>day 7</th>
<th>day 8</th>
<th>day 9</th>
<th>day 10</th>
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<tr>
<td>standard</td>
<td>3.1</td>
<td>3.2</td>
<td>3.2</td>
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<td>quiz, 3.4</td>
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<td>review, assessment</td>
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<tr>
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<td>3.2</td>
<td>3.2</td>
<td>3.3, project</td>
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<td>3.5</td>
<td>3.6</td>
<td>3.7</td>
<td>review, TAL, assessment</td>
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<table>
<thead>
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<th>day 11</th>
<th>day 12</th>
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<tbody>
<tr>
<td>standard</td>
<td>mixed review, exam</td>
</tr>
<tr>
<td>enriched</td>
<td>mixed review, exam</td>
</tr>
</tbody>
</table>

**Materials**

- boxes of toothpicks
- graph paper
- colored pencils
- 4 m measuring tapes, metersticks, or ropes
- motion sensors
- stopwatches or watches with second hands
- 300 pennies or other markers
- washers, optional
- paper cups (three per group)
- pieces of rope of different lengths (around 1 m) and thickness (two per group)
Weavers repeat steps when they make baskets and mats, creating patterns of repeating shapes. This process is not unlike recursion. In the top photo, a mat weaver in Myanmar creates a traditional design with palm fronds. The bottom photo shows bowls crafted by Native American artisans.

By keeping the amount of woven material constant, the weavers produce consistent patterns of lines and shapes. To make the mat in the chevron-like pattern, the weaver first lays the warp pieces lengthwise in a repeating pattern of two light and one dark. Then he weaves the woof (or weft) pieces horizontally with the same sequence of two light and one dark. Each woof piece alternately hops over three of the warp pieces and goes under three warp pieces.

Students can try this themselves with colored paper strips and write instructions for how to start the woof piece at the edge.

Coiled baskets begin with a spiral at the base, in contrast to baskets that begin with a wagon-wheel-like pattern at the base. As the basket diameter widens, the space widens between elements that cross the coil. Help students notice that some of the designs, when the baskets are viewed from top or bottom, have reflective (mirror) symmetry, but others have radial symmetry.

Myanmar (Burma) is located between India and Thailand near the Bay of Bengal. The Native American baskets are a Pima coiled tray, an Apache coiled jar, two Yokuts coiled bowls, a Mono coiled bowl, a Maidu coiled bowl, a Washo coiled bowl, and a Pomo coiled gift basket.
Recursive Sequences

The Empire State Building in New York City has 102 floors and is 1250 ft high. How high up are you when you reach the 80th floor? You can answer this question using a recursive sequence. In this lesson you will learn how to analyze geometric patterns, complete tables, and find missing values using numerical sequences.

A recursive sequence is an ordered list of numbers defined by a starting value and a rule. You generate the sequence by applying the rule to the starting value, then applying it to the resulting value, and repeating this process.

The table shows heights above and below ground at different floor levels in a 25-story building. Write a recursive routine that provides the sequence of heights that corresponds to the building floor numbers 0, 1, 2, . . . . Use this routine to find each missing value in the table.

### Example A

<table>
<thead>
<tr>
<th>Floor number</th>
<th>Basement (0)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>...</th>
<th>10</th>
<th>...</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (ft)</td>
<td>−4</td>
<td>9</td>
<td>22</td>
<td>35</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>217</td>
<td>...</td>
</tr>
</tbody>
</table>

**Solution**

The starting value is −4 because the basement is 4 ft below ground level. Each floor is 13 ft higher than the floor below it, so the rule for finding the next floor height is “add 13 to the current floor height.”

The calculator screen shows how to enter this recursive routine into your calculator. Press \( -4 \) (ENTER) to start your number sequence. Press \( +13 \) (ENTER). The calculator automatically displays \( \text{Ans} + 13 \) and computes the next value. Simply pressing (ENTER) again applies the rule for finding successive floor heights. [See Calculator Note OD.]

You can see that the 4th floor is at 48 ft.

How high up is the 10th floor? Count the number of times you press (ENTER) until you reach 10. Which floor is at a height of 217 ft? Keep counting until you see that value on your calculator screen. What’s the height of the 25th floor? Keep applying the rule by pressing (ENTER) and record the values in your table.

The 10th floor is at 126 ft, the 17th floor is at 217 ft, and the 25th floor is at 321 ft.

### Example A

[ELL] Floor, or story, means the level of a building, usually beginning with 1. The height of a floor could mean the floor-to-ceiling distance of one floor, but here it

### Lesson Objectives

- Review or become familiar with the concept of recursion
- Investigate recursive (arithmetic) sequences using the calculator

### NCTM Standards

<table>
<thead>
<tr>
<th>CONTENT</th>
<th>PROCESS</th>
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<tbody>
<tr>
<td>Number</td>
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<td>Reasoning</td>
</tr>
<tr>
<td>Geometry</td>
<td>Communication</td>
</tr>
<tr>
<td>Measurement</td>
<td>Connections</td>
</tr>
<tr>
<td>Data/Probability</td>
<td>Representation</td>
</tr>
</tbody>
</table>

158 CHAPTER 3 Linear Equations
Investigation
Recursive Toothpick Patterns

In this investigation you will learn to create and apply recursive sequences by modeling them with puzzle pieces made from toothpicks.

Consider this pattern of triangles.

![Pattern of triangles]

Step 1
Make Figures 1–3 of the pattern using as few toothpicks as possible. How many toothpicks does it take to reproduce each figure? How many toothpicks lie on the perimeter of each figure?

Step 2
Copy the table with enough rows for six figures of the pattern. Make Figures 4–6 from toothpicks by adding triangles in a row and complete the table.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Number of toothpicks</th>
<th>Perimeter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Step 3
What is the rule for finding the number of toothpicks in each figure? What is the rule for finding the perimeter? Use your calculator to create recursive routines for these rules. Check that these routines generate the numbers in your table.

Step 4
Now make Figure 10 from toothpicks. Count the number of toothpicks and find the perimeter. Does your calculator routine give the same answers? Find the number of toothpicks and the perimeter for Figure 25. Figure 10: 21 toothpicks with a perimeter of 12; Figure 25: 51 toothpicks with a perimeter of 27

Next you’ll see what sequences you can generate with a new pattern.

Step 5
Design a pattern using a row of squares, instead of triangles, with your toothpicks. Repeat Steps 1–4 and answer all the questions with the new design.

Step 6
Choose a unit of measurement and explain how to calculate the area of a square made from toothpicks. How does your choice of unit affect calculations for the areas of each figure?

Guiding the Investigation

If you think your students will not handle toothpicks appropriately, you can have them draw line segments on paper.

One Step
Draw the three figures showing the growth of a triangle pattern or ask students to look at page 159. Point out how the perimeter is changing. Ask students to design their own toothpick pattern with a changing feature, such as perimeter, and to write a recursive routine to make their calculator generate the sequence of numbers. As you interact, be sure students see the idea of a common difference between consecutive terms.

Step 1 If students do not use just one toothpick per side, their answers will be multiples of 3.

Step 2 Be sure students extend their patterns of shapes horizontally. If they add shapes in several directions, their sequences will not be arithmetic.

The table is organized so that the calculated sequences are displayed vertically in columns. Calculators will also display the results of recursive routines vertically on the home screen.
Step 5  Toothpicks: Add 3 to the previous number. Perimeter: Add 2 to the previous perimeter. Press 4 \( \text{Enter} \) and then \( \text{Ans} + 3 \) \( \text{Enter} \) to find the number of toothpicks. To find the perimeter, press 4 \( \text{Enter} \) and then \( \text{Ans} + 2 \) \( \text{Enter} \). Figure 10: 31 toothpicks with 22 on the perimeter; Figure 25: 76 toothpicks with 52 on the perimeter.

Step 6  Students might measure a toothpick in centimeters, in inches, or as a unit 1 toothpick long. For a unit of 1 toothpick, the area is equal to the number of squares in the figure. Otherwise, the number of squares must be multiplied by the unit area of each square to calculate the area of the entire figure.

Step 7  Students might be reluctant to use figures whose areas are difficult to find in terms of the edges. Suggest that they can use the area of the first figure as 1 square unit even if their basic shape is not a square.

**SHARING IDEAS**

Ask several students to present their questions from Step 8. For one of them in which the perimeter increases by a different number from the number of toothpicks, ask why. Usually one or more toothpicks in the perimeter at the previous step are no longer in the perimeter.

You might begin to use the term rate of change to describe what’s happening—for example, the amount being added to the perimeter is the rate of change of the perimeter, in toothpicks per stage.

If you ask some students to put up their sequences with terms missing and have the class guess the missing terms, you can anticipate Example B.

**Assessing Progress**

Watch for systematic data collection, familiarity with geometric shapes and terms such as perimeter and area, and ability to contribute to group work.

**Example B**

This example is good for students who may not have grasped the nature of a sequence with common differences of consecutive terms. Many students enjoy creating and solving problems like this. Encourage differing approaches to part c. You can find the common difference by looking down the list to the first consecutive pair or by finding half the difference between the third term and the first term: 

\[
\frac{1}{2} \left[ 29 - (-7) \right] = -11.
\]

Hidden numbers in the illustration include 0, 1, 2, 3, 4, 7, 8, and 11.
EXERCISES

Practice Your Skills

1. Evaluate each expression without using your calculator. Then check your result with your calculator.
   a. \(-2(5 - 9) + 7\) \(= 15\)
   b. \(-\frac{4(-8)}{3 - 5}\) \(= -16\)
   c. \(\frac{5 + (-6)(-5)}{-7}\) \(= -5\)

2. Consider the sequence of figures made from a row of pentagons.

   - a. Copy and complete the table for five figures.
   - b. Write a recursive routine to find the perimeter of each figure. Assume each side is 1 unit long.
   - c. Find the perimeter of Figure 10.
   - d. Which figure has a perimeter of 47? Figure 15

3. Find the first six values generated by the recursive routine
   

4. Write a recursive routine to generate each sequence. Then use your routine to find the 10th term of the sequence.
   a. \(3, 9, 15, 21, \ldots\)
   b. \(1.7, 1.2, 0.7, 0.2, \ldots\)
   c. \(-3, 6, -12, 24, \ldots\)
   d. \(384, 192, 96, 48, \ldots\)

Closing the Lesson

You generate a recursive sequence on the calculator by entering a starting number and then designating how to operate on one number to get the next number in the sequence.

BUILDING UNDERSTANDING

Students practice writing recursive routines to generate sequences, most of which have a constant difference between consecutive terms.

ASSIGNING HOMEWORK

- Essential 1–4, 6
- Performance assessment 5, 7, 8
- Portfolio 6
- Journal 10, 11
- Group 6, 9, 11, 12
- Review 13, 14

Helping with the Exercises

For more practice with recursive geometric sequences, you can use the Sketchpad demonstration Patterns and Recursion.

Exercise 4 In 4d, students see a recursive sequence defined by division for the first time. The Fathom demonstration Recursive Sequences can be used to replace this exercise.

4a. Start with 3, then apply the rule \(\text{Ans} + 6\); 10th term = 57.
4b. Start with 1.7, then apply the rule \(\text{Ans} - 0.5\); 10th term = -2.8.
4c. Start with -3, then apply the rule \(\text{Ans} \cdot -2\); 10th term = 1536.
4d. Start with 384, then apply the rule \(\text{Ans}/2\) or \(\text{Ans} \cdot 0.5\); 10th term = 0.75.

LESSON 3.1 Recursive Sequences
Exercise 5 Students may have difficulty calculating the heights of the floors. Encourage them to draw a picture to see that there are 16 floors, numbered 86 through 101, spanning a distance of 174 ft, and that the lower 85 floors cover a distance of 1050 ft. If students notice the height here of 1224 and the height in the introduction of 1250, say that the height 1224 is to the floor of the 102nd floor and 1250 includes the height of the antenna.

5a. The recursive routine is 0 and then \( \text{Ans} + 12.35 \). The starting value is 0, the height of ground level (the first floor). Add the average floor height for the next 85 floors: 12.35 ft.

5b. The recursive routine is 1050 and then \( \text{Ans} + 10.875 \). The starting value is the height of the 86th floor. Add 10.875, the average floor height of floors 86 through 101.

5c. When you are 531 ft high, you are 43 floors up and thus on the 44th floor.

6a. Possible explanation: The smallest square has an area of 1. The next larger white square has an area of 4, which is 3 more than the smallest square. The next larger gray square has an area of 9, which is 5 more than the 4-unit white square.

6b. The recursive routine is 1, \( \text{Ans} + 1 \), and so on.

6c. 17, the value of the 9th term in the sequence

6d. The first term in the sequence is 1, and the second is 3. Which term is the number 95? Explain how you found your answer. The 48th term is 95; students might press 48 times or compute \( 2(48) - 1 \).

7a. The table for six figures of the L-shaped puzzle pieces is

<table>
<thead>
<tr>
<th>Figure</th>
<th>Toothpicks</th>
<th>Perimeter</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>14</td>
<td>12</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>16</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>26</td>
<td>20</td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td>32</td>
<td>24</td>
<td>15</td>
</tr>
<tr>
<td>6</td>
<td>38</td>
<td>26</td>
<td>18</td>
</tr>
</tbody>
</table>

7b. To find the number of toothpicks, press 8 and then \( \text{Ans} + 6 \). To find the perimeter, press 8 and then \( \text{Ans} + 4 \). For the area, press 3 and then \( \text{Ans} + 3 \).

7c. Figure 10 has 62 toothpicks, a perimeter of 44, and an area of 30.

7d. Figure 25, made from 152 toothpicks, has a perimeter of 104 and an area of 75.

6a. Possible explanation: The smallest square has an area of 1. The next larger white square has an area of 4, which is 3 more than the smallest square. The next larger gray square has an area of 9, which is 5 more than the 4-unit white square.

6b. The recursive routine is 1, \( \text{Ans} + 1 \), and so on.

6c. 17, the value of the 9th term in the sequence

6d. The first term in the sequence is 1, and the second is 3. Which term is the number 95? Explain how you found your answer. The 48th term is 95; students might press 48 times or compute \( 2(48) - 1 \).

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<td>2</td>
<td>14</td>
<td>12</td>
<td>6</td>
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<tr>
<td>3</td>
<td>20</td>
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<td>15</td>
</tr>
<tr>
<td>6</td>
<td>38</td>
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7c. Figure 10 has 62 toothpicks, a perimeter of 44, and an area of 30.

7d. Figure 25, made from 152 toothpicks, has a perimeter of 104 and an area of 75.
8. **APPLICATION** The table gives some floor heights in a building.

<table>
<thead>
<tr>
<th>Floor</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>9</th>
<th>10</th>
<th>12</th>
<th>15</th>
<th>17</th>
<th>19</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (m)</td>
<td>...</td>
<td>-3</td>
<td>1</td>
<td>5</td>
<td>9</td>
<td>...</td>
<td>37</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

a. How many meters are between the floors in this building? 4 m

b. Write a recursive routine that will give the sequence of floor heights if you start at the 25th floor and go to the basement (floor 0). Which term in your sequence represents the height of the 7th floor? What is the height?

c. How many terms are in the sequence in 8b? 26 terms

d. Floor “-1” corresponds to the first level of the parking substructure under the building. If there are five parking levels, how far underground is level 5? 19 m

9. Consider the sequence __ , 4, 8, __ , 32, . . . .

a. Find two different recursive routines that could generate these numbers. Most numbers are needed to uniquely determine a recursive routine.

b. For each routine, what are the missing numbers? What are the next two numbers?

c. If you want to generate this number sequence with exactly one routine, what more do you need?

10. Positive multiples of 7 are generally listed as 7, 14, 21, 28, . . . .

a. If 7 is the 1st multiple of 7 and 14 is the 2nd multiple, then what is the 17th multiple? 107 · 7, or 119

b. How many multiples of 7 are between 100 and 200? 14

c. Compare the number of multiples of 7 between 100 and 200 with the number between 200 and 300. Does the answer make sense? Do all intervals of 100 have this many multiples of 7? Explain.

d. Describe two different ways to generate a list containing multiples of 7.

11. Some babies gain an average of 1.5 lb per month during the first 6 months after birth.

a. Write a recursive routine that will generate a table of monthly weights for a baby weighing 6.8 lb at birth. Press 6.8 ENTER and then Ans + 1.5 ENTER, ENTER, ....

b. Write a recursive routine that will generate a table of monthly weights for a baby weighing 7.2 lb at birth. Press 7.2 ENTER, and then Ans + 1.5 ENTER, ENTER, ....

c. How are the routines in 11a and 11b the same? How are they different? The starting terms differ; the rule itself is the same.

d. Copy and complete the table of data for this situation.

<table>
<thead>
<tr>
<th>Age (mo)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight of Baby A (lb)</td>
<td>6.8</td>
<td>8.3</td>
<td>9.8</td>
<td>11.3</td>
<td>12.8</td>
<td>14.3</td>
<td>15.8</td>
</tr>
<tr>
<td>Weight of Baby B (lb)</td>
<td>7.2</td>
<td>8.7</td>
<td>10.2</td>
<td>11.7</td>
<td>13.2</td>
<td>14.7</td>
<td>16.2</td>
</tr>
</tbody>
</table>

e. How are the table values for the two babies the same? How do they differ? Each baby always increases by 1.5 lb, and the difference between the babies’ weights is always 0.4 lb; the starting values are different.

Exercise 8b In other words, let the 25th floor height be the first term, the 24th floor height be the second term, and so on.

8b. Press 101 ENTER and then Ans - 4 ENTER. The 19th term represents the height of the 7th floor. The height is 29 m.

9a. One routine is press -16 ENTER and then Ans + 12 ENTER. Another is press 2 ENTER and then Ans - 2 ENTER.

9b. Two possible sequences are {-16, -4, 8, 20, 32, 44, 56, . . . } and {2, -4, 8, -16, 32, -64, 128, . . . }.

Exercise 10 If appropriate, ask about multiples of 7 that aren’t positive. Mention nonnegative multiples (which include 0) as well as negative multiples.

10c. Possible answer: There are 14 multiples between 100 and 200. There are also 14 multiples of 7 between 200 and 300, but there are 15 between 300 and 400.

10d. Possible answer: The 4th multiple of 7 is 4 · 7, or 28; the 5th multiple of 7 is 5 · 7, or 35; and so on. Recursively, you start with 7 and then continue adding 7.
12. Write recursive routines to help you answer 12a–d.
   a. Find the 9th term of 1, 3, 9, 27, . . .
   b. Find the 123rd term of 5, 5, 5, 5, . . .
   c. Find the term number of the first positive term of the sequence −16.2, −14.8, −13.4, −12, . . .
   d. Which term is the first to be either greater than 100 or less than −100 in the sequence 1, 2, 4, 8, 16, . . .?

Review

13. The table gives the normal monthly precipitation for three cities in the United States.

   a. Display the data in three box plots, one for each city, and use them to compare the precipitation for the three cities.
   b. What information do you lose by displaying the data in a box plot? What type of graph might be more helpful for displaying the data?

Precipitation for Three Cities

<table>
<thead>
<tr>
<th>Month</th>
<th>Portland, Oregon</th>
<th>San Francisco, California</th>
<th>Seattle, Washington</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>5.4</td>
<td>4.1</td>
<td>5.4</td>
</tr>
<tr>
<td>February</td>
<td>3.9</td>
<td>3.0</td>
<td>4.0</td>
</tr>
<tr>
<td>March</td>
<td>3.6</td>
<td>3.1</td>
<td>3.8</td>
</tr>
<tr>
<td>April</td>
<td>2.4</td>
<td>1.3</td>
<td>2.5</td>
</tr>
<tr>
<td>May</td>
<td>2.1</td>
<td>0.3</td>
<td>1.8</td>
</tr>
<tr>
<td>June</td>
<td>1.5</td>
<td>0.2</td>
<td>1.6</td>
</tr>
<tr>
<td>July</td>
<td>0.7</td>
<td>0.0</td>
<td>0.9</td>
</tr>
<tr>
<td>August</td>
<td>1.1</td>
<td>0.1</td>
<td>1.2</td>
</tr>
<tr>
<td>September</td>
<td>1.8</td>
<td>0.3</td>
<td>1.9</td>
</tr>
<tr>
<td>October</td>
<td>2.7</td>
<td>1.3</td>
<td>3.3</td>
</tr>
<tr>
<td>November</td>
<td>5.3</td>
<td>3.2</td>
<td>5.7</td>
</tr>
<tr>
<td>December</td>
<td>6.1</td>
<td>3.1</td>
<td>6.0</td>
</tr>
</tbody>
</table>


San Francisco has the least precipitation and is the only city in which there is a month with no precipitation. One indicator that the weather is much drier in San Francisco is that the month with no precipitation is not an outlier.

13b. You lose information about what time of year is wettest; a bar graph or scatter plot would show trends over the months of the year more clearly.

2.8 14. Create an undo table and solve the equation listed by undoing the order of operations. $x = −2.6$

### Equation: $8 + 3(x − 5) = −14.8$

<table>
<thead>
<tr>
<th>Description</th>
<th>Undo</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pick $x$.</td>
<td></td>
<td>−2.6</td>
</tr>
<tr>
<td>$− (5)$</td>
<td>$+ (5)$</td>
<td>$7.6$</td>
</tr>
<tr>
<td>$× (3)$</td>
<td>$/ (3)$</td>
<td>$22.8$</td>
</tr>
<tr>
<td>$+ (8)$</td>
<td>$− (8)$</td>
<td>$14.8$</td>
</tr>
</tbody>
</table>
Linear Plots

In this lesson you will learn that the starting value and the rule of a recursive sequence take on special meaning in certain real-world situations. When you add or subtract the same number each time in a recursive routine, consecutive terms change by a constant amount. Using your calculator, you will see how the starting value and rule let you generate data for tables quickly. You will also plot these data sets and learn that the starting value and rule relate to characteristics of the graph.

You walk into an elevator in the basement of a building. Its control panel displays “0” for the floor number. As you go up, the numbers increase one by one on the display, and the elevator rises 13 ft for each floor. The table shows the floor numbers and their heights above ground level.

- Write recursive routines for the two number sequences in the table. Enter both routines into calculator lists.
- Define variables and plot the data in the table for the first few floors of the building. Does it make sense to connect the points on the graph?
- What is the highest floor with a height less than 200 ft? Is there a floor that is exactly 200 ft high?

Solution

The starting value for the floor numbers is 0, and the rule is to add 1. The starting value for the height is −4, and the rule is to add 13. You can generate both number sequences on the calculator using lists.

- Press \( \{0, -4\} \) and press \( \text{ENTER} \) to input both starting values at the same time. To use the rules to get the next term in the sequence, press \( \{\text{Ans}(1) + 1, \text{Ans}(2) + 13\} \) \( \text{ENTER} \). See Calculator Note 3A.

These commands tell the calculator to add 1 to the first term in the list and to add 13 to the second number. Press \( \text{ENTER} \) again to compute the next floor number and its corresponding height as the elevator rises.

In this routine, the calculator displays a new list of numbers horizontally every time you press \( \text{ENTER} \), but the terms for the sequences of floor numbers and heights appear vertically aligned on the screen. Be sure students understand the important difference between the use of braces and parentheses on the calculator.

Emphasize the use of dimensions: height and floor. On the Elevator Table transparency, you might make marks between consecutive terms in the right column to show the common differences.

Ask why the term linear relationship was chosen for two quantities in which each unit increase in one results in a constant increase in the other. An answer to this question could wait until Sharing.

Many elevators use Braille symbols. This alphabet for the blind was developed by Louis Braille (1809–1852). For more information about Braille, see the links at www.keymath.com/DA.
b. Let \( x \) represent the floor number and \( y \) represent the floor’s height in feet. Mark a scale from 0 to 5 on the \( x \)-axis and –10 to 50 on the \( y \)-axis. Plot the data from the table. The graph starts at \((0, -4)\) on the \( y \)-axis. The points appear to be in a line. It does not make sense to connect the points because it is not possible to have a decimal or fractional floor number.

c. The recursive routine generates the points \((0, -4), (1, 9), (2, 22), \ldots, (15, 191), (16, 204), \ldots\). The height of the 15th floor is 191 ft. The height of the 16th floor is 204 ft. So the 15th floor is the highest floor with a height less than 200 ft. No floor is exactly 200 ft high.

Notice that to get to the next point on the graph from any given point, move right 1 unit on the \( x \)-axis and up 13 units on the \( y \)-axis. The points you plotted in the example showed a linear relationship between floor numbers and their heights.

In what other graphs have you seen linear relationships?

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**Investigation**

**On the Road Again**

A green minivan starts at the Mackinac Bridge and heads south for Flint on Highway 75. At the same time, a red sports car leaves Saginaw and a blue pickup truck leaves Flint. The minivan travels 72 mi/h. The pickup travels 66 mi/h. The sports car travels 48 mi/h.

When and where will they pass each other on the highway? In this investigation you will learn how to use recursive sequences to answer questions like these.

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**LESSON OBJECTIVES**

- Graph scatter plots of recursive sequences
- Continue to explore the connection between graphs and tables and how they can be used to solve problems
- Build toward an introduction of the intercept form of a line

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**NCTM STANDARDS**

<table>
<thead>
<tr>
<th>CONTENT</th>
<th>PROCESS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>Problem Solving</td>
</tr>
<tr>
<td>Algebra</td>
<td>Reasoning</td>
</tr>
<tr>
<td>Geometry</td>
<td>Communication</td>
</tr>
<tr>
<td>Measurement</td>
<td>Connections</td>
</tr>
<tr>
<td>Data/Probability</td>
<td>Representation</td>
</tr>
</tbody>
</table>
**Step 3** Make a table to record the highway distance from Flint for each vehicle. After you complete the first few rows of data, change your recursive routines to use 10 min intervals for up to 4 h.

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Minivan (mi)</th>
<th>Sports car (mi)</th>
<th>Pickup (mi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>220</td>
<td>35</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>218.8</td>
<td>35.8</td>
<td>1.1</td>
</tr>
<tr>
<td>2</td>
<td>217.6</td>
<td>36.6</td>
<td>2.2</td>
</tr>
<tr>
<td>5</td>
<td>211</td>
<td>39</td>
<td>3.5</td>
</tr>
<tr>
<td>10</td>
<td>208</td>
<td>43</td>
<td>11</td>
</tr>
<tr>
<td>20</td>
<td>196</td>
<td>51</td>
<td>22</td>
</tr>
</tbody>
</table>

**Procedure Note**

After you enter the recursive routine into the calculator, press \( \{0, 220, 0, 35\} \). Copy the data displayed on your calculator screen onto your table. Repeat this process.

**Step 4** Let \( x \) represent the time in minutes since the vehicles started their trips; let \( y \) represent the distance from Flint in highway miles.

**Step 5** Define variables and plot the information from the table onto a graph. Mark and label each axis in 10-unit intervals, with time on the horizontal axis. Using a different color for each vehicle, plot its \((\text{time}, \text{distance})\) coordinates.

On the graph, do the points for each vehicle seem to fall on a line? Does it make sense to connect each vehicle’s points in a line? If so, draw the line. If not, explain why not: Yes, a line through the points for each vehicle represents every possible instant of time.

Use your graph and table to find the answers for Steps 6–10.

**Step 6** On the \( y \)-axis representing each vehicle’s initial distance from Flint; the rules affect the steepness and direction of each line.

**Step 7** Where does the starting value for each routine appear on the graph? How does the recursive rule for each routine affect the points plotted?

**Step 8** Which line represents the minivan? How can you tell?

**Step 9** Where are the vehicles when the minivan meets the first one headed north?

**Step 10** How can you tell by looking at the graph whether the pickup or the sports car is traveling faster? When and where does the pickup pass the sports car?

Which vehicle arrives at its destination first? How many minutes pass before the second and third vehicles arrive at their destinations? How can you tell by looking at the graph?

What assumptions about the vehicles are you making when you answer the questions in the previous steps?

**Steps 8 and 9** Students may have difficulty keeping in mind both variables being graphed. Students may say, or at least believe, something like “They’re at the same place but not at the same time.” You can find very good approximations for the answers to the questions using either the table or the graph. Ask students to save their results if you are going to assign Lesson 5.2, Exercise 11.

**Step 2** As needed, discuss how the different directions the vehicles are traveling affect whether you add or subtract in the recursive routine. Also discuss the difference between miles apart on the highway and miles apart in a straight line. The distances are all miles apart on the highway. Be ready to remind students how to enter recursive data into calculator lists. (See Calculator Note 0D.)

**Step 3** To input the starting values in a calculator list for the times and the distances of the minivan, pickup, and sports car, respectively, press \( \{0, 220, 0, 35\} \). To apply the rule, press \( \{\text{Ans}(1) + 1, \text{Ans}(2) - 1.2, \text{Ans}(3) + 1.1, \text{Ans}(4) + 0.8\} \). The starting values represent the time in minutes and each vehicle’s distance from Flint. The rule is to add or subtract the speed in miles per minute, depending on the vehicle’s direction.

**Step 4** Each student can choose a different car and generate the related sequence recursively. Remind students of the instant replay function (Calculator Note 2C). To modify the recursive routines to use 10 min intervals, you can recall the last entry and change it to read \( \{\text{Ans}(1) + 10, \text{Ans}(2) - 12, \text{Ans}(3) + 11, \text{Ans}(4) + 8\} \).

**Step 5** This is the most important phase of the investigation. Using different colors is important for differentiating the vehicles.

**Step 7** The line going down from left to right; it starts 220 units above the origin on the \( y \)-axis and gets closer to the \( x \)-axis as time passes and the minivan gets closer to Flint.

**Step 7** Be sure students go beyond an answer like “It’s colored green.” Ask: “How did you know which line to color green?”

**Step 11** Students may have difficulty finding very good approximations for the answers to the questions using either the table or the graph. Ask students to save their results if you are going to assign Lesson 5.2, Exercise 11.
Consider how to model this situation more realistically. What if the vehicles are traveling at different speeds? What if one driver stops to get gas or a bite to eat? What if the vehicles’ speeds are not constant? Discuss how these questions affect the recursive routines, tables of data, and their graphs. If speeds are not constant, the points will not lie in a line and you would not be adding or subtracting the same number in the recursive routine. The lines would have horizontal pieces if drivers stop.

You can use the Dynamic Algebra Exploration found at www.keymath.com/DA to further explore the situation described in the investigation. ❖

### **Exercises**

**Practice Your Skills**

**You will need your graphing calculator for Exercises 4–7 and 9.**

1. Decide whether each expression is positive or negative without using your calculator. Then check your answer with your calculator.
   - a. \(-35(44) + 23\) negative; \(-1517\)
   - b. \((-14)(-36) - 32\) positive; \(472\)
   - c. \(25 - \frac{152}{12}\) positive; \(12\frac{3}{3}\)
   - d. \(50 - 23(-12)\) positive; \(326\)
   - e. \(-\frac{12 - 38}{15}\) negative; \(-3\frac{3}{3}\)
   - f. \(24(15 - 76)\) negative; \(-1464\)

2. List the terms of each number sequence of \(y\)-coordinates for the points shown on each graph. Then write a recursive routine to generate each sequence.
   - a. \([0, 5], [1, 6], [2, 7], [3, 8], [4, 9]\); \(0.5, \text{ Ans } + 0.5\)
   - b. \([4, 3, 2, 1, 0]; 4, \text{ Ans } - 1\)
   - c. \([-1, -0.75, -0.5, -0.25, 0, 0.25]; -1, \text{ Ans } + 0.25\)
   - d. \([-1.5, 0, 1.5, 3]; -1.5, \text{ Ans } + 1.5\)

3. Make a table listing the coordinates of the points shown on 2b and 2d.

4. Plot the first five points represented by each recursive routine in 4a and b on separate graphs. Then answer 4c and d.
   - a. \([0, 5] \text{ Enter} \)
     
     \(\text{Ans}(1) + 1, \text{ Ans}(2) + 7 \text{ Enter} ; \text{ Enter} \ldots\)
   - b. \([0, -3] \text{ Enter} \)
     
     \(\text{Ans}(1) + 1, \text{ Ans}(2) - 6 \text{ Enter} ; \text{ Enter} \ldots\)

### **Sharing Ideas**

If some students had the idea that the graphs showed the paths of the vehicles, ask the class to critique that notion. This is a good place to ask questions beginning with “Are you saying ...?” in order to clarify ideas without having to tell much.

Have students present Steps 9–12. “Are you saying that the relationship is linear because the rate of change is constant?” Elicit the idea that the amount being added at each step of the recursive routine is the rate of change.

If you have time, “How do linear relationships relate to directly proportional quantities and direct variation?” [Directly proportional quantities have a linear relationship, but the opposite is not necessarily true. The graph of a direct variation is a straight line through the origin. The graph of a linear relationship between variables is also a straight line, but not necessarily through the origin.]

As a synonym for steepness, you might use the word slope, though its formal definition will not come until Chapter 4.
c. On which axis does each starting point lie? What is the x-coordinate of each starting point? the y-axis: 0

d. As the x-value increases by 1, what happens to the y-coordinates of the points in each sequence in 4a and b? In 4a, the y-coordinates increase by 7. In 4b, the y-coordinates decrease by 6.

5. The direct variation $y = 2.54x$ describes the relationship between two standard units of measurement where $y$ represents centimeters and $x$ represents inches.
   a. Write a recursive routine that would produce a table of values for any whole number of inches. Use a calculator list.
   b. Use your routine to complete the missing values in this table.

\[
\begin{array}{|c|c|}
\hline
\text{Inches} & \text{Centimeters} \\
\hline 0 & 0 \\
1 & 2.54 \\
2 & 5.08 \\
14 & 35.56 \\
17 & 43.18 \\
\hline
\end{array}
\]

Reason and Apply

6. APPLICATION A car is moving at a speed of 68 mi/h from Dallas toward San Antonio. Dallas is about 272 mi from San Antonio.
   a. Write a recursive routine to create a table of values relating time to distance from San Antonio for 0 to 5 h in 1 h intervals.
   b. Graph the information in your table.
   c. What is the connection between your plot and the starting value in your recursive routine?
   d. What is the connection between the coordinates of any two consecutive points in your plot and the rule of your recursive routine?
   e. Draw a line through the points of your plot. What is the real-world meaning of this line? What does the line represent that the points alone do not?
   f. When is the car within 100 mi of San Antonio? Explain how you got your answer.
   g. How long does it take the car to reach San Antonio? Explain how you got your answer.

7. APPLICATION A long-distance telephone carrier charges $1.38 for international calls of 1 minute or less and $0.36 for each additional minute.
   a. Write a recursive routine using calculator lists to find the cost of a 7-minute phone call. The recursive routine keeps track of time and cost for each minute. Apply the routine until you get {7, 3.54}. A 7 min call costs $3.54.
   b. Without graphing the sequence, give a verbal description of the graph showing the costs for calls that last whole numbers of minutes. Include in your description all the important values you need in order to draw the graph.

7a. Possible answer: {1, 1.38} \( \text{ENTER} \), \{Ans(1) + 1, Ans(2) + 0.36\} \( \text{ENTER} \), \( \text{ENTER} \), .....

7b. Possible answer: The graph should consist of points that lie on a line. It should include the point (1, 1.38). Each subsequent point should be 1 unit to the right and $0.36 higher than the point before it.
Exercise 8 Discuss how measuring depth on the bow, bridge, or stern of a submarine as it is surfacing affects data.

8a. [Graph showing data points]

8b. The points for each submarine appear to lie on a line; the USS Dallas surfaces at a faster rate.

Exercise 9 Suggest that students calculate the perimeter by treating each edge as 1 unit long.

9b. Number of tiles: The starting value is 1; the rule is add 1.
Triangle: The starting value is 3; the rule is add 1.
Rhombus: The starting value is 4; the rule is add 2.
Pentagon: The starting value is 5; the rule is add 3.
Hexagon: The starting value is 6; the rule is add 4.

To generate the sequences for all tiles simultaneously, enter \{1, 3, 4, 5, 6\} and \{Ans(1) + 1, Ans(2) + 1, Ans(3) + 2, Ans(4) + 3, Ans(5) + 4\}.

9c. triangle: 52; rhombus: 102; pentagon: 152; hexagon: 202

9d. Yes; each line means that any time in this range corresponds to depth below the surface.
No; there must be a whole number of tiles and a whole number of edges.

The submarine’s nose rises slightly above the water when surfacing.

9e. The points of each graph appear to lie on a line, and each graph starts at 1; the graphs increase in steepness from the triangle tile to the hexagon tile.
10. A bicyclist, 1 mi (5280 ft) away, pedals toward you at a rate of 600 ft/min for 3 min.
   The bicyclist then pedals at a rate of 1000 ft/min for the next 5 min.
   a. Describe what you think the plot of (time, distance from you) will look like.
   b. Graph the data using 1 min intervals for your plot.
   c. Invent a question about the situation, and use your graph to answer the question.

11b. Consider the expression

\[
\frac{5.4 + 3.2(x - 2.8)}{1.2} - 2.3
\]

a. Use the order of operations to find the value of the expression if \( x = 7.2 \).
   \[ x = 3.4 \]

b. Set the expression equal to 3.8. Solve for \( x \) by undoing the sequence of operations
   you listed in 11a.
   \[ x = 3.4 \]

c. Explain how to tell which quadrant a point will be in by looking at the coordinates.

12. Isaac learned a way to convert from degrees Celsius to Fahrenheit. He adds 40 to the
   Celsius temperature, multiplies by 9, divides by 5, and then subtracts 40.
   a. Write an expression for Isaac’s conversion method.
   b. Write the steps to convert from Fahrenheit to Celsius by undoing Isaac’s method.
   c. Write an expression for the conversion in 12b.

13. APPLICATION Karen is a U.S. exchange student in Austria. She wants to make her favorite pizza recipe for her
    host family, but she needs to convert the quantities
    to the metric system. Instead of using cups for
    flour and sugar, her host family measures dry
    ingredients in grams and liquid ingredients
    in liters. Karen has read that 4 cups of flour
    weigh 1 pound.
    a. Karen's recipe calls for \( \frac{1}{2} \) cup water and
       \( \frac{1}{2} \) cup flour. Convert these quantities to
       metric units.
       b. Karen's recipe says to bake the pizza at 425°F. Convert
       this temperature to degrees Celsius. Use your work
       in Exercise 12 to help you.

14. Draw and label a coordinate plane with each axis scaled from -10 to 10.
    a. Represent each point named with a dot, and label it using its letter name.
       \( A(3, -2) \), \( B(-8, 1.5) \), \( C(9, 0) \), \( D(-9.5, -3) \), \( E(7, -4) \).
       \( F(1, -1) \), \( G(0, -6.5) \), \( H(2.5, 3) \), \( I(-6, 7.5) \), \( J(-5, -6) \).
    b. List the points in Quadrant I, Quadrant II, Quadrant III, and Quadrant IV.
    c. Explain how to tell which quadrant a point will be in by looking at the coordinates.
    d. Explain how to tell if a point lies on one of the axes.

14b. Quad I: \( H \); Quad II: \( B \); Quad III: \( D \); Quad IV: \( A \), \( E \), \( F \); x-axis: \( C \); y-axis: \( G \)

14c. Sample answer: If the coordinates are both 0, then the point is on the origin. If the x-coordinate
    is 0, then the point is on the y-axis. If the y-coordinate is 0, then the point is on
    the x-axis.

If the first coordinate is positive, then the point will
be in Quadrant I or IV. To tell which quadrant, look
at the y-coordinate. If the y-coordinate is positive,
the point is in Quadrant I. If the y-coordinate is
negative, the point is in Quadrant IV.

If the first coordinate is negative, then the point
will be in Quadrant II or III. To tell which quad-
rant, look at the y-coordinate. If the y-coordinate
is positive, the point is in Quadrant II. If the
y-coordinate is negative, the point is in
Quadrant III.
Time-Distance Relationships

Modeling time-distance relationships is one very useful application of algebra. You began working with this topic in Lesson 3.2. In this lesson you will explore time-distance relationships in more depth by considering various walking scenarios. You’ll learn how the starting position, speed, direction, and final position of a walker influence a graph and an equation.

The \((time, distance)\) graphs below provide a lot of information about the “walks” they picture. The fact that the lines are straight and increasing means that both walkers are moving away from the motion sensor at a steady rate. The first walker starts 0.5 meter from the sensor, whereas the second walker starts 1 meter from the sensor. The first graph pictures a walker moving 4.5 \(\text{meters} / \text{4 seconds}\), or 1 meter per second. The second walker covers 3 \(\text{meters} / \text{4 seconds}\), or 0.5 meter per second.

In this investigation you’ll analyze time-distance graphs, and you’ll use a motion sensor to create your own graphs.

Investigation

Walk the Line

Imagine that you have a 4-meter measuring tape positioned on the floor. A motion sensor measures your distance from the tape’s 0-mark as you walk, and it graphs the information. On the calculator graphs shown here, the horizontal axis shows time from 0 to 6 seconds and the vertical axis shows distance from 0 to 4 meters.

LESSON OBJECTIVES

- Explore time-distance relationships
- Write walking instructions or act out walks for a given graph
- Sketch graphs based on given walking instructions or table data
- Use an electronic data collection device, motion sensor, and graphing calculator to collect and graph data

TEACHER’S EDITION
Step 1 Write a set of walking instructions for each graph. Tell where the walk begins, how fast the person walks, and whether the person walks toward or away from the motion sensor located at the 0-mark.

Step 2 Graph a 6-second walk based on each set of walking instructions or data.

a. Start at the 2.5-meter mark and stand still.
b. Start at the 3-meter mark and walk toward the sensor at a constant rate of 0.4 meter per second.
c. | Time (s) | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
   | Distance (m) | 0.8 | 1.0 | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 |

Step 3 Write a recursive routine for the table in Step 2c. The starting value is 0.8; the rule is add 0.2.

For the next part of the investigation, you will need a graphing calculator and a motion sensor. Your group will need a space about 4 meters long and 1.5 meters wide (13 feet by 5 feet). Tape to the floor a 4-meter measuring tape or four metersticks end-to-end. Assign these tasks among your group members: walker, motion-sensor holder, coach, and timer.

Step 4 Your group will try to create the graph shown in Step 1, graph a. Remember that you wrote walking directions for this graph. Use your motion sensor to record the walker’s motion. Use Calculator Note 3B for help using the motion sensor. After each walk, discuss what you could have done to better replicate the graph. Repeat the walk until you have a good match for graph a. Rotate jobs, and repeat Step 4 to model graphs b and c from Step 1 and the three descriptions from Step 2.

Using motion-sensor technology in the investigation, you were able to actually see how accurately you duplicated a given walk. The next examples will provide more practice with time-distance relationships.

**EXAMPLE A**

a. Graph a walk from the set of instructions “Start at the 0.3-meter mark and walk at a steady 0.25 meter per second for 6 seconds.”
b. Write a set of walking instructions based on the table data, and then sketch a graph of the walk.

| Time (s) | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| Distance (m) | 4.0 | 3.6 | 3.2 | 2.8 | 2.4 | 2.0 | 1.6 |

tells the walker to start. The coach makes helpful comments to guide the walker.

Step 5 Groups should discuss what they could have done to match more accurately the given graph, table, or instructions.

**EXAMPLE A**

This example gives students more practice in graphing walking instructions, including some derived from a data table. As in the investigation, the rates are constant and the graphs are straight lines, slanting both upward and downward.

If you’re not using motion sensors, the holder records in a table the walker’s distance from the 0 m mark each second. The timer begins timing when the coach tells the walker to start and counts the seconds aloud.

If you are using motion sensors, the holder should hold the calculator and start collecting data on the coach’s command. (See Calculator Note 3B.) The holder should hold the motion sensor chest high, keeping it level and aimed directly at the walker. Timing begins when the coach...
Think about where the walker starts and how much distance he or she will cover in a given amount of time.

a. Walking at a steady rate of 0.25 meter per second for 6 seconds means the walker will move $0.25 \text{ m/s} \times 6 \text{ s} = 1.5 \text{ m}$. The walker starts at 0.5 m and ends at $0.5 + 1.5 = 2 \text{ m}$.

b. Walking instructions: “Start at the 4-meter mark and walk toward the sensor at 0.4 meter per second.” You can graph this walk by plotting the data points given.

EXERCISES

Practice Your Skills

1. Write a recursive routine for the table in Example A, part b. $\{0, 4.0\}$ and $\{\text{Ans}(1) + 1, \text{Ans}(2) - 0.4\}$

2. Sketch a graph of a walk starting at the 1-meter mark and walking away from the sensor at a constant rate of 0.5 meter per second.

3. Write a set of walking instructions and sketch a graph of the walk described by $\{0, 0.8\}$ and $\{\text{Ans}(1) + 1, \text{Ans}(2) + 0.2\}$.

4. Describe the walk shown in each graph. Include where it started and how quickly and in what direction the walker moved.

   a. The walker starts 2.5 m away from the motion sensor and walks away at a rate of 1 m in 6 s.

   b. The walker starts 1 m away from the motion sensor and walks away from it at a rate of 2.5 m in 6 s.

Closing the Lesson

The main point of this lesson is that motion at a constant speed is graphed with a straight line. If the walker is moving away from the motion sensor, so that distance is increasing, the line rises from left to right; if the walker is moving toward the sensor, so that distance is decreasing, the line falls from left to right; if the walker is motionless, the line is horizontal.

The speed of the walker is related to the steepness of the line and to the additive constant in a recursive routine. The higher the speed, the steeper the graph and the farther the additive constant is from 0.
5. Describe the walk represented by the data in each table.

a. | Time (s) | Distance (m) |
   | 0      | 6           |
   | 1      | 5.8         |
   | 2      | 5.6         |
   | 3      | 5.4         |
   | 4      | 5.2         |
   | 5      | 5.0         |
   | 6      | 4.8         |

The walker starts 6 m away from the motion sensor and walks toward it at a rate of 0.2 m/s for 6 s.

b. | Time (s) | Distance (m) |
   | 0      | 1           |
   | 1      | 1.6         |
   | 2      | 2.2         |
   | 3      | 2.8         |
   | 4      | 3.4         |
   | 5      | 4.0         |
   | 6      | 4.6         |

The walker starts 1 m away from the motion sensor and walks away from it at a rate of 0.6 m/s for 6 s.

6. Which graph better represents a walk in which the walker starts 2 m from the motion sensor and walks away from it at a rate of 0.25 m/s for 6 s? Explain.

The first graph, which shows a line, because the walk is a continuous process; the walker is somewhere at every possible time in the 6 s.

7. At what rate in ft/s would you walk so that you were moving at a constant speed of 1 mi/h?

7e. \[ \frac{1 \text{ mi}}{60 \text{ s}} = \frac{5280 \text{ ft}}{1 \text{ mi}} \times \frac{1 \text{ s}}{60 \text{ min}} = \frac{1.4666 \text{ ft}}{1 \text{ s}} \]

or approximately 1.46 ft/s.

8. The time-distance graph shows Carol walking at a steady rate. Her partner used a motion sensor to measure her distance from a given point.

a. According to the graph, how much time did Carol spend walking? 4 s

b. Was Carol walking toward or away from the motion sensor? Explain your thinking. \[ \text{Away} \] the distance is increasing.

c. Approximately how far away from the motion sensor was she when she started walking? approximately 0.5 m

d. If you know Carol is 2.9 m away from the motion sensor after 4 s, how fast was she walking? \[ \frac{2.9}{4} = 0.6 \text{ m/s} \]

e. If the equipment will measure distances only up to 6 m, how many seconds of data can be collected if Carol continues walking at the same rate? 0.5 s

f. Looking only at the graph, how do you know that Carol was neither speeding up nor slowing down during her walk? The graph is a straight line.

9. Draw a scatter plot on your paper picturing \((\text{time}, \text{distance})\) at 1 s intervals if you start timing Carol’s walk as she walks toward her partner starting at a distance of 5.9 m and moving at a constant speed of 0.6 m/s.
10. Describe how the rate affects the graph of each situation.
   a. The graph of a person walking toward a motion sensor.  The rate is negative, so the line slopes down to the right.
   b. The graph of a person standing still.  The rate is neither negative nor positive, it is zero, so the line is horizontal.
   c. The graph of a person walking slowly.
      The line is not very steep.

11. Match each calculator Answer routine to a graph.
   i. Ans $+ 0.5$, $\ldots$
   a. 2.5
   ii. Ans $+ 1.0$, $\ldots$
   b. 1.0
   iii. Ans $+ 1.0$, $\ldots$
   c. 2.0
   iv. Ans $- 0.5$, $\ldots$
   d. 2.5

12. Describe how you would instruct someone to walk the line $y = x$, where $x$ is measured in seconds and $y$ is measured in feet. Describe how to walk the line $y = x$, where $x$ is measured in seconds and $y$ is measured in meters. Which line represents a faster rate? Explain.

13. For each situation, determine if it is possible to collect such walking data and either describe how to collect it or explain why it is not possible.

   a. Not possible; the walker would have to be at more than one distance from the sensor at the 3 s mark.
   b. Possible; the walker simply stands still about 2.5 m from the sensor.
   c. Not possible; the walker can’t be in two places at any given time.
2.1 14. Solve each proportion for $x$.
   \[ \frac{x}{3} = \frac{7}{2}, \quad \frac{x}{5} = \frac{2}{3}, \quad \frac{x}{c} = \frac{9}{11}, \quad \frac{x}{e} = \frac{d}{c} \]
   \[ \frac{x}{1} = \frac{22}{9}, \quad \frac{x}{e} = 2.4 \]

2.3 15. On his Man in Motion World Tour in 1987, Canadian Rick Hansen wheeled himself 24,901.55 miles to support spinal cord injury research and rehabilitation, and wheelchair sport. He covered 4 continents and 34 countries in two years, two months, and two days. Learn more about Rick’s journey with the link at www.keymath.com/DA.

   a. Find Rick’s average rate of travel in miles per day. (Assume there are 365 days in a year and 30.4 days in a month.) \[ \text{rate} = \frac{24,901.55}{365 	imes 2.67} \]
   b. How much farther would Rick have traveled if he had continued his journey for another 1½ years?
   c. If Rick continued at this same rate, how many days would it take him to travel 60,000 miles? How many years is that?

2.3 16. **APPLICATION** Nicholai’s car burns 13.5 gallons of gasoline every 175 miles.
   a. What is the car’s fuel consumption rate? \[ 13.5 \text{ gal} \div 175 \text{ mi} = 0.077 \text{ gal/mi} \]
   b. At this rate, how far will the car go on 5 gallons of gas?
   c. How many gallons does Nicholai’s car need to go 100 miles?

---

**PASCAL’S TRIANGLE**

The first five rows of Pascal’s triangle are shown.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

The triangle can be generated recursively. The sides of the triangle are 1’s, and each number inside the triangle is the sum of the two diagonally above it.

Complete the next five rows of Pascal’s triangle. Research its history and practical application. What is the connection between Sierpiński’s triangle and Pascal’s triangle? Can you find the sequence of triangular numbers in Pascal’s triangle? What is its connection to the Fibonacci number sequence? Present your findings in a paper or a poster.

---

**Supporting the project**

**MOTIVATION**

This pattern of numbers is named for a 17th-century mathematician. He used this pattern of numbers extensively in his study of probability. Was that the beginning of the history of this number pattern? (See above for the next four rows.)

**OUTCOMES**

- The recursive rule includes a starting 1 and this rule: If a number is at the end of a row, it’s 1, and if it’s not, then it’s the sum of the two numbers diagonally above it.
- The report includes history going back to the ancient Chinese civilization.
- The triangular numbers (1, 3, 6, 10, 15, …) are in the third diagonal.
- The Fibonacci numbers (1, 1, 2, 3, 5, 8, …) are sums of the numbers on diagonals described by “start with a 1 on the left, go over one and a half numbers and up to the next row, follow that diagonal, adding the numbers.” (For example, 1 + 3 + 1 = 5; 1 + 4 + 3 = 8; 1 + 5 + 6 + 1 = 13.)
- Sierpiński’s triangle (see above)

---

**Exercise 15 [Alert]** Students may be misled by the extra information about continents and countries. The rate needed for 15c is the reciprocal of the rate needed for 15a and 15b.

15a. \[ \frac{24,901.55 \text{ mi}}{(2 \times 365 + 2 \times 30.4 + 2) \text{ days}} = 31.4 \text{ mi/day} \]
15b. \[ 31.4 \text{ mi/day} \times 60,000 \text{ mi} = \frac{17,191.5 \text{ mi}}{1 \text{ day}} \]
15c. \[ \frac{31.4 \text{ mi}}{1 \text{ day}} \times 60,000 \text{ mi} = 1,911 \text{ days, or more than 5 yr} \]

---

**Pascal’s Triangle Project**

The next four rows are:

- $1, 6, 15, 21, 15, 6, 1$
- $1, 7, 21, 35, 35, 21, 7, 1$
- $1, 8, 28, 56, 70, 56, 28, 8, 1$
- $1, 9, 36, 84, 126, 126, 84, 36, 9, 1$

**Fibonacci Numbers**

- $1, 3, 3, 1$
- $1, 4, 6$
- $1, 5, 10, 5, 1$
- $1, 15, 20, 15, 6, 1$
- $1, 21, 35, 21, 7, 1$

**Sierpiński’s Triangle**

- If the odd numbers in Pascal’s triangle are colored in, the even numbers are left uncolored, and the triangle is extended infinitely, then it becomes a Sierpiński triangle.
LESSON
3.4

PLANNING

LESSON OUTLINE

One day:
25 min Investigation
5 min Sharing
10 min Examples
5 min Closing
5 min Exercises

MATERIALS

• Calculator Note 1J
• Fathom demonstration Working Out, optional

TEACHING

(Language) Define metabolism as the physical and chemical processes that maintain the body, for instance, turning food into energy.

In this lesson students make the transition from using recursive routines to writing linear equations in intercept form. The intercept form is like the well-known slope-intercept form except that the order is different: $y = ax + bx$. This order emphasizes starting with the number $a$ and adding the number $b$ repeatedly ($x$ times).

Investigation

Working Out with Equations

Manisha starts her exercise routine by jogging to the gym. Her trainer says this activity burns 215 calories. Her workout at the gym is to pedal a stationary bike. This activity burns 3.8 calories per minute.

First you'll model this scenario with your calculator.

**Step 1** Use calculator lists to write a recursive routine to find the total number of calories Manisha has burned after each minute she pedals the bike. Include the 215 calories she burned on her jog to the gym.

**Step 2** Copy and complete the table using your recursive routine.

**Step 3** After 20 minutes of pedaling, how many calories has Manisha burned? How long did it take her to burn 443 total calories?

 Tongue the Investigation

The Fathom demonstration Working Out can replace this investigation.

One Step

Pose this problem: “Manisha starts her exercise routine by jogging to the gym, which burns 215 calories. At the gym she pedals a stationary bike, burning 3.8 calories per minute. How long will it take her to burn a total of 538 calories?” Encourage students to

LESSON OBJECTIVES

• Write a linear equation in intercept form given a recursion routine, a graph, or data
• Learn the meaning of $y$-intercept for a linear equation in intercept form

NCTM STANDARDS

<table>
<thead>
<tr>
<th>CONTENT</th>
<th>PROCESS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>Problem Solving</td>
</tr>
<tr>
<td>Algebra</td>
<td>Reasoning</td>
</tr>
<tr>
<td>Geometry</td>
<td>Communication</td>
</tr>
<tr>
<td>Measurement</td>
<td>Connections</td>
</tr>
<tr>
<td>Data/Probability</td>
<td>Representation</td>
</tr>
</tbody>
</table>
Next you’ll learn to write an equation that gives the same values as the calculator routines.

Step 4 215 + 3.8(20) = 291

Step 5 215 + 3.8(38) = 359.4 calories; you don’t need to calculate the previous terms in the sequence or create a table to find the answer.

Step 6 Let x represent the pedaling time in minutes, and let y represent the total number of calories Manisha burns. Write an equation relating time to total calories burned. \( y = 215 + 3.8x \)

Step 7 Sample checks:
215 + 3.8(1) = 218.8;
215 + 3.8(60) = 443

Step 8 The x-axis represents every instant of time, and the y-axis represents every fraction of calories burned. This graph models a continuous linear relationship. See graph below.

Step 9 Plot the points from your table on your calculator. Then enter your equation into the Y= menu. Graph your equation to check that it passes through the points. Give two reasons why drawing a line through the points realistically models this situation. [Ask] See Calculator Note 1J to review how to plot points and graph an equation. –

Substitute 358 for y in your equation to find the elapsed time required for Manisha to burn a total of 358 calories. Explain your solution process. Check your result. 358 = 215 + 3.8x; x = 85. Check: 215 + 3.8(85) = 358.

How do the starting value and the rule of your recursive routine show up in your graph? When is the starting value of the recursive routine also the value where the graph crosses the y-axis?

The equation for Manisha’s workout shows a linear relationship between the total calories burned and the number of minutes pedaling on the bike. You probably wrote this linear equation as

\[ y = 215 + 3.8x \quad \text{or} \quad y = 3.8x + 215 \]

The form \( y = ax + b \) is the intercept form. The value of \( a \) is the \( y \)-intercept, which is the value of \( y \) when \( x \) is zero. The intercept gives the location where the graph crosses the \( y \)-axis. The number multiplied by \( x \) is \( b \), which is called the coefficient of \( x \).

Step 10 In the equation \( y = 215 + 3.8x \), the starting value is 215 and the rule to add shows up as the coefficient of \( x \). The starting value is the \( y \)-intercept. The rule add 3.8 gives the steepness of the line. The starting value of the recursive routine is the \( y \)-intercept only when the starting value of \( x \) is zero.

SHARING IDEAS

Ask several students to share their solution methods for Step 9. At an appropriate time, introduce the term \( \text{intercept form} \) for the equation \( y = a + bx \) and the related terms \( y \)-intercept and coefficient of \( x \). [Ask] “Is the equation \( y = bx + a \) equivalent to the intercept form?” [Yes] “Are equations \( y = ax + b \) and \( y = mx + b \) (often called the \( \text{slope-intercept form} \)) also equivalent?” [Yes] Substitute numbers for \( a \), \( b \), and \( m \) as needed. These questions may allow students to see that the letters \( a \), \( b \), and \( m \) represent constants in particular equations, whereas the letters \( x \) and \( y \) represent variables. [Ask] “Is the equation \( y = a - bx \) equivalent to \( y = bx - a \)” [No; \( y = a - bx \) is equivalent to \( y = -bx + a \). It is very important to keep the signs consistent.]

[Ask] “How are linear equations related to other equations you have seen in this course?” [The first, in Chapter 1, was \( y = x \). It’s a special case of a direct variation \( y = kx \), with \( k = 1 \). And direct variations are special cases of the intercept form \( a + bx \), with \( a = 0 \) and \( b = k \).]
In the equation $y = 215 + 3.8x$, 215 is the value of $a$. It represents the 215 calories Manisha burned while jogging before her workout. The value of $b$ is 3.8. It represents the rate her body burned calories while she was pedaling. What would happen if Manisha chose a different physical activity before pedaling on the stationary bike?

You can also think of direct variations in the form $y = kx$ as equations in intercept form. For instance, Sam's trainer tells him that swimming will burn 7.8 calories per minute. When the time spent swimming is 0, the number of calories burned is 0, so $a$ is 0 and drops out of the equation. The number of calories burned is proportional to the time spent swimming, so you can write the equation $y = 7.8x$.

The constant of variation $k$ is 7.8, the rate at which Sam's body burns calories while he is swimming. It plays the same role as $b$ in $y = ax + bx$.

**EXAMPLE A**

Suppose Sam has already burned 325 calories before he begins to swim for his workout. His swim will burn 7.8 calories per minute.

a. Create a table of values for the calories Sam will burn by swimming 60 minutes and the total calories he will burn after each minute of swimming.

b. Define variables and write an equation in intercept form to describe this relationship.

c. On the same set of axes, graph the equation for total calories burned and the direct variation equation for calories burned by swimming.

d. How are the graphs similar? How are they different?

**Solution**

a. The total numbers of calories burned appear in the third column of the table. Each entry is 325 plus the corresponding entry in the middle column.

<table>
<thead>
<tr>
<th>Swimming time (min)</th>
<th>Calories burned by swimming</th>
<th>Total calories burned</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>325</td>
</tr>
<tr>
<td>1</td>
<td>7.8</td>
<td>332.8</td>
</tr>
<tr>
<td>2</td>
<td>15.6</td>
<td>340.6</td>
</tr>
<tr>
<td>20</td>
<td>156</td>
<td>481</td>
</tr>
<tr>
<td>30</td>
<td>234</td>
<td>559</td>
</tr>
<tr>
<td>45</td>
<td>351</td>
<td>676</td>
</tr>
<tr>
<td>60</td>
<td>468</td>
<td>793</td>
</tr>
</tbody>
</table>

b. Let $y$ represent the total number of calories burned, and let $x$ represent the number of minutes Sam spends swimming.

$$y = 325 + 7.8x$$
c. The direct variation equation is \( y = 7.8x \). Enter it into Y1 on your calculator. Enter the equation \( y = 325 + 7.8x \) into Y2. Check to see that these equations give the same values as the table by looking at the calculator table.

d. The lower line shows the calories burned by swimming and is a direct variation. The upper line shows the total calories burned. It is 325 units above the first line because, at any particular time, Sam has burned 325 more calories. Both graphs have the same value of \( b \), which is 7.8 calories per minute. The graphs are similar because both are lines with the same steepness. They are different because they have different \( y \)-intercepts.

What will different values of \( a \) in the equation \( y = a + bx \) do to the graph?

**EXAMPLE B**

A minivan is 220 mi from its destination, Flint. It begins traveling toward Flint at 72 mi/h.

a. Define variables and write an equation in intercept form for this relationship.

b. Use your equation to calculate the location of the minivan after 2.5 h.

c. Use your equation to calculate when the minivan will be 130 mi from Flint.

d. Graph the relationship and locate the points that are the solutions to parts b and c.

e. What is the real-world meaning of the rate of change in this relationship? What does the sign of the rate of change indicate?

**Solution**

a. Let the independent variable, \( x \), represent the time in hours since the beginning of the trip. Let \( y \) represent the distance in miles between the minivan and Flint. The equation for the relationship is \( y = 220 - 72x \).

b. Substitute the time, 2.5 h, for \( x \).

\[
y = 220 - 72 \cdot 2.5 = 40
\]

So the minivan is 40 mi from Flint.

c. Substitute 130 mi for \( y \) and solve the equation \( 220 - 72x = 130 \).

\[
\begin{align*}
220 + 72x &= 130 & \text{Original equation. The subtraction of 72x is written as addition of } -72x. \\
72x &= -90 & \text{Subtract 220 to undo the addition.} \\
x &= 1.25 & \text{Divide by } -72 \text{ to undo the multiplication.}
\end{align*}
\]

The minivan will be 130 mi from Flint after 1.25 h. You can change 0.25 h to minutes using dimensional analysis. \( 0.25 \text{ h} \cdot \frac{60 \text{ min}}{1 \text{ h}} = 15 \text{ min} \), so you can also write the answer as 1 h 15 min.
**Closing the Lesson**

A linear equation in intercept form, \( y = a + bx \), reflects the recursive routine used to generate a sequence of data values with a constant rate of change. Such a routine begins with \( a \) and adds \( b \) repeatedly. (The value of either \( a \) or \( b \) may be negative.)

**Building Understanding**

Students practice writing, graphing, and exploring linear equations, primarily in intercept form.

**Assigning Homework**

<table>
<thead>
<tr>
<th>Assignment Type</th>
<th>Problems</th>
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<tbody>
<tr>
<td>Essential</td>
<td>1, 2 or 3, 6, 7, 10</td>
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<tr>
<td>Performance assessment</td>
<td>9, 10</td>
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<tr>
<td>Portfolio</td>
<td>6</td>
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<tr>
<td>Journal</td>
<td>5, 7, 8</td>
</tr>
<tr>
<td>Group</td>
<td>7</td>
</tr>
<tr>
<td>Review</td>
<td>4, 5, 11–15</td>
</tr>
</tbody>
</table>

**Helping with the Exercises**

**Exercise 1** If students have difficulty relating recursive routines to the explicit equations, suggest that they make tables of data.

**Exercises 2 and 3** These are the first equations in a while that don’t use just \( x \) and \( y \) for variable names. Encourage students to write the dimensions of each number and variable, especially the rate.

2c. 24 represents the initial number of miles the driver is from his or her destination.

2d. 45 means the driver is driving at a speed of 45 mi/h.

**Exercises**

You will need your graphing calculator for Exercises 2, 3, 6, and 9.

### Practice Your Skills

1. Match the recursive routine in the first column with the equation in the second column.

<table>
<thead>
<tr>
<th>Routine</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>ii. ( a. ) 3  ( \text{ENTER} ) ( \text{ENTER} ) ( \text{ENTER} ) ( \text{ENTER} ) ( \text{ENTER} ) ( \text{ENTER} )</td>
<td>i. ( y = 4 - 3x )</td>
</tr>
<tr>
<td>iv. ( b. ) 4  ( \text{ENTER} ) ( \text{ENTER} ) ( \text{ENTER} ) ( \text{ENTER} )</td>
<td>ii. ( y = 3 + 4x )</td>
</tr>
<tr>
<td>iii. ( c. ) ( -3 )  ( \text{ENTER} ) ( \text{ENTER} ) ( \text{ENTER} )</td>
<td>iii. ( y = -3 - 4x )</td>
</tr>
<tr>
<td>i. ( d. ) 4  ( \text{ENTER} ) ( \text{ENTER} ) ( \text{ENTER} ) ( \text{ENTER} )</td>
<td>iv. ( y = 4 + 3x )</td>
</tr>
</tbody>
</table>

2. You can use the equation \( d = 24 - 45t \) to model the distance from a destination for someone driving down the highway, where distance \( d \) is measured in miles and time \( t \) is measured in hours. Graph the equation and use the trace function to find the approximate time for each distance given in 2a and b.

<table>
<thead>
<tr>
<th>Distance ( d )</th>
<th>Time ( t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. ( d = 16 \text{ mi} )</td>
<td>( t \approx 0.18 \text{ h} )</td>
</tr>
<tr>
<td>b. ( d = 3 \text{ mi} )</td>
<td>( t \approx 0.47 \text{ h} )</td>
</tr>
</tbody>
</table>

3. What is the real-world meaning of 24? 2d. What is the real-world meaning of 45? 3e. Solve the equation \( 24 - 45t = 16 \). \( t = \frac{8}{45} \), or 0.17
3. You can use the equation \( d = 4.7 + 2.8t \) to model a walk in which the distance from a motion sensor \( d \) is measured in feet and the time \( t \) is measured in seconds. Graph the equation and use the trace function to find the approximate distance from a motion sensor for each time value given in 3a and b.
   a. \( t = 12 \text{ s} \quad d \approx 38.3 \text{ ft} \)
   b. \( t = 7.4 \text{ s} \quad d \approx 25.42 \text{ ft} \)
   c. What is the real-world meaning of 4.7?  
   d. What is the real-world meaning of 2.8?

The walker started 4.7 ft away from the motion sensor. The walker was walking at a rate of 2.8 ft/s.

4. Undo the order of operations to find the x-value in each equation.
   a. \( 3(x - 5.2) + 7.8 = 14 \quad \Rightarrow \quad x \approx 7.267 \)
   b. \( \frac{x - 8}{4} = 2.8 \quad \Rightarrow \quad x = 11.2 \)

5. The equation \( y = 35 + 0.8x \) gives the distance a sports car is from Flint after \( x \) minutes.
   a. How far is the sports car from Flint after 25 minutes? \( 35 + 0.8(25) = 55 \text{ mi} \)
   b. How long will it take until the sports car is 75 miles from Flint? How to find the solution using two different methods. 50 min; students might use a graph or the undo method.

### Reason and Apply

6. **APPLICATION** Louis is beginning a new exercise workout. His trainer shows him the calculator table with \( x \)-values showing his workout time in minutes. The \( Y_1 \)-values are the total calories Louis burned while running, and the \( Y_2 \)-values are the number of calories he wants to burn.
   a. Find how many calories Louis has burned before beginning to run, how many he burns per minute running, and the total calories he wants to burn.
   b. Write a recursive routine that will generate the values listed in \( Y_1 \).
   c. Use your recursive routine to write a linear equation in intercept form. Check that your solution generates the table values listed in \( Y_1 \).
   d. Write a recursive routine that will generate the values listed in \( Y_2 \).
   e. Use the trace function to find the approximate coordinates of the point where the lines meet. What is the real-world meaning of this point?
   f. Graph the equations in \( Y_1 \) and \( Y_2 \) on your calculator. Your window should show a time of up to 30 minutes. What is the real-world meaning of the \( y \)-intercept of \( Y_1 \)?
   g. Suggest that they write the appropriate dimensions.

7. Jo mows lawns after school. She finds that she can use the equation \( P = -300 + 15N \) to calculate her profit.
   a. Give some possible real-world meanings for the numbers –300 and 15 and the variable \( N \).
   b. Invent two questions related to this situation and then answer them.
   c. Solve the equation \( P = -300 + 15N \) for the variable \( N \).
   d. What does the equation in 7c tell you? It tells you the number of lawns you have to mow to make a certain amount of profit.

---

**Exercise 6** In Example B, students moved the cursor to trace a graph and find specific points on the graph. In this exercise, they trace to find the intersection of a horizontal line \( y = 700 \) with the line \( y = 400 + 20.7x \).

6a. Louis has burned 400 calories before beginning to run. His calorie-burning rate is 20.7 calories per minute, and he wants to burn 700 total calories.

6f. The \( y \)-intercept of \( Y_1 \), which is 400, is the number of calories burned after 0 min of running (before Louis begins to run).

6g. The approximate coordinates of the point where the lines meet are (14.5, 700). This means that after 14.5 min of running, Louis will have burned off his desired total of 700 calories.

**Exercise 7** Students may be confused by the variable names. Suggest that they write the appropriate dimensions.

7a. One possible scenario: Jo has an initial start-up cost of $300 for equipment and expenses. She makes $15 for every lawn she mows, \( N \).

7b. Sample questions: How many lawns will Jo have to mow to break even? [Solve the equation \( -300 + 15N = 0 \); Jo must mow 20 lawns.] How much profit will Jo earn if she mows 40 lawns? [Substitute 40 for \( N \); $300.]
8. As part of a physics experiment, June threw an object off a cliff and measured how fast it was traveling downward. When the object left June’s hand, it was traveling 5 m/s, and it sped up as it fell. The table shows a partial list of the data she collected as the object fell.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Speed (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5.0</td>
</tr>
<tr>
<td>0.5</td>
<td>9.9</td>
</tr>
<tr>
<td>1.0</td>
<td>14.8</td>
</tr>
<tr>
<td>1.5</td>
<td>19.7</td>
</tr>
</tbody>
</table>

a. Write an equation to represent the speed of the object. \( s = 5 + 9.8t \), where \( t \) is time in seconds.

b. What was the object’s speed after 3 s? \( s = 9.8t + 5 \), so after 3 s, the speed is 34.4 m/s.

c. If it were possible for the object to fall long enough, how many seconds would pass before it reached a speed of 83.4 m/s? \( 8s = 34.4 \), so \( s = 4.3 \) seconds.

d. What limitations do you think this equation has in modeling this situation? It doesn’t account for air resistance and terminal speed.

9. APPLICATION Manny has a part-time job as a waiter. He makes $45 per day plus tips. He has calculated that his average tip is 12% of the total amount his customers spend on food and beverages.

a. Define variables and write an equation in intercept form to represent Manny’s daily income in terms of the amount his customers spend on food and beverages. Let \( x \) represent dollar amounts customers spend and \( y \) represents Manny’s daily income in dollars.

\[ y = 45 + 0.12x \]

b. Graph this relationship for food and beverage amounts between $0 and $900.

c. Write and evaluate an expression to find how much Manny makes in one day if his customers spend $312 on food and beverages.

\[ 45 + 0.12 \cdot 312 = 82.44 \] dollars.

d. What amounts spent on food and beverages will give him a daily income between $105 and $120? Between $500 and $625.

10. APPLICATION Paula is cross-training for a triathlon in which she cycles, swims, and runs. Before designing an exercise program for Paula, her coach consults a table listing rates for calories burned in various activities.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Calories burned (per min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Walking</td>
<td>3.2</td>
</tr>
<tr>
<td>Bicycling</td>
<td>3.8</td>
</tr>
<tr>
<td>Swimming</td>
<td>6.9</td>
</tr>
<tr>
<td>Jogging</td>
<td>7.3</td>
</tr>
<tr>
<td>Running</td>
<td>11.3</td>
</tr>
</tbody>
</table>

a. On Monday, Paula starts her workout by biking for 30 minutes and then swimming. Write an equation for the calories she burns on Monday in terms of the number of minutes she bikes and swims.

\[ y = 114 + 6.9x + 30 \]

b. On Wednesday, Paula starts her workout by swimming for 30 minutes and then jogging. Write an equation for the number of calories she burns on Wednesday in terms of the number of minutes she swims and jogs.

\[ y = 207 + 7.3x \]

c. On Friday, Paula starts her workout by swimming 15 minutes, then biking for 15 minutes, then running. Write an equation for the number of calories she burns on Friday in terms of the number of minutes she spends swimming, biking, and running.

\[ y = 160.5 + 11.3x \]

d. How many total calories does Paula burn on each day described in 10a–c if she does a 60-minute workout? Monday: 321 calories; Wednesday: 426 calories; Friday: 499.5 calories.
Review

2.2 11. At a family picnic, your cousin tells you that he always has a hard time remembering how to compute percents. Write him a note explaining what percent means. Use these problems as examples of how to solve the different types of percent problems, with an answer for each.
   a. 8 is 15% of what number? \[ \frac{8}{n} = \frac{15}{100} \Rightarrow n \approx 53.3 \]
   b. 15% of 18.95 is what number? \[ \frac{0.15 \times 18.95}{100} \approx 2.8 \]
   c. What percent of 64 is 32? \[ \frac{32}{64} = \frac{n}{100} \Rightarrow n = 50 \]
   d. 10% of what number is 40? \[ \frac{0.1 \times n}{100} = 40 \Rightarrow n = 4000 \]

2.3 12. APPLICATION
Carl has been keeping a record of his gas purchases for his new car. Each time he buys gas, he fills the tank completely. Then he records the number of gallons he bought and the miles since the last fill-up. Here is his record:

<table>
<thead>
<tr>
<th>Miles traveled</th>
<th>Gallons</th>
<th>Miles/gallon</th>
</tr>
</thead>
<tbody>
<tr>
<td>363</td>
<td>16.2</td>
<td>22.4</td>
</tr>
<tr>
<td>342</td>
<td>15.1</td>
<td>22.6</td>
</tr>
<tr>
<td>285</td>
<td>12.9</td>
<td>22.1</td>
</tr>
</tbody>
</table>

Carl’s Purchases

a. Copy and complete the table by calculating the ratio of miles per gallon for each purchase.
b. What is the average rate of miles per gallon so far? 22.4 mi/gal
c. The car’s tank holds 17.1 gallons. To the nearest mile, how far should Carl be able to go without running out of gas? 383 mi
d. Carl is planning a trip across the United States. He estimates that the trip will be 4230 miles. How many gallons of gas can Carl expect to buy? approximately 189 gal

Consumer Connection

Many factors influence the rate at which cars use gas, including size, age, and driving conditions. Advertisements for new cars often give the average mpg for city traffic (slow, congested) and highway traffic (fast, free flowing). These rates help consumers make an informed purchase. For more information about fuel economy, see the links at www.keymath.com/DA.

3.2 13. Match each recursive routine to a graph below. Explain how you made your decision and tell what assumptions you made.

ii a. 2.5 (Enter)
   Ans + 0.5 (Enter); (Enter), . . .

ii b. 1.0 (Enter)
   Ans + 1.0 (Enter); (Enter), . . .

iii c. 2.0 (Enter)
   Ans + 1.0 (Enter); (Enter), . . .

iii d. 2.5 (Enter)
   Ans − 0.5 (Enter); (Enter), . . .

Exercise 11 Students can complete this exercise using proportions as they learned in Chapter 2, but allow other correct methods.

11. Partial answer: Write the percent as one ratio of a proportion. Put the part over the whole in the other ratio.

13. Sample explanation: I matched the rate of change to each graph. I assumed the starting value was the y-intercept.
3.3 14. Bjarne is training for a bicycle race by riding on a stationary bicycle with a time-distance readout. He is riding at a constant speed. The graph shows his accumulated distance and time as he rides.

a. How fast is Bjarne bicycling? 14 m/s
b. Copy and complete the table.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Distance (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14</td>
</tr>
<tr>
<td>2</td>
<td>28</td>
</tr>
<tr>
<td>3</td>
<td>42</td>
</tr>
<tr>
<td>4</td>
<td>56</td>
</tr>
<tr>
<td>5</td>
<td>70</td>
</tr>
<tr>
<td>6</td>
<td>84</td>
</tr>
<tr>
<td>7</td>
<td>98</td>
</tr>
<tr>
<td>8</td>
<td>112</td>
</tr>
<tr>
<td>9</td>
<td>126</td>
</tr>
<tr>
<td>10</td>
<td>140</td>
</tr>
</tbody>
</table>

d. Looking at the graph, how do you know that Bjarne is neither slowing down nor speeding up during his ride?
ej. If Bjarne keeps up the same pace, how far will he ride in one hour? 50,400 m, or 50.4 km

IMPROVING YOUR REASONING SKILLS

You have two containers of the same size; one contains juice and the other contains water. Remove one tablespoon of juice and put it into the water and stir. Then remove one tablespoon of the water and juice mixture and put it into the juice. Is there more water in the juice or more juice in the water?

IMPROVING REASONING SKILLS

If students are having difficulty figuring out that the percentage of juice in the water is the same as the percentage of the water in the juice, you might suggest that they think about particular amounts of liquid, such as 10 oz of each with 1 oz being transferred. Or produce some decks of playing cards. Give each pair of students a pile of ten red cards and a pile of ten black cards. Have one student pull any number of red cards out and mix them among the black cards. The other student pulls out the same number of cards from the other pile and puts them into the red pile. Keeping track of how many of each color are moving and trying it with extreme cases might deepen students’ understanding of the ratios involved.
How can it be that mathematics, being after all a product of human thought independent of experience, is so admirably adapted to the objects of reality?

ALBERT EINSTEIN

In this lesson you will continue to develop your skills with equations, graphs, and tables of data by exploring the role that the value of $b$ plays in the equation $y = a + bx$.

You have already studied the intercept form of a linear equation in several real-world situations. You have used the intercept form to relate calories to minutes spent exercising, floor heights to floor numbers, and distances to time. So, defining variables is an important part of writing equations. Depending on the context of an equation, its numbers take on different real-world meanings. Can you recall how these equations modeled each scenario?

In most linear equations, there are different output values for different input values. This happens when the coefficient of $x$ is not zero. You’ll explore how this coefficient relates input and output values in the examples and the investigation.

In addition to giving the actual temperature, weather reports often indicate the temperature you feel as a result of the wind chill factor. The wind makes it feel colder than it actually is. In the next example you will use recursive routines to answer some questions about wind chill.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Situation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = 215 + 3.8x$</td>
<td>calories burned in a workout</td>
</tr>
<tr>
<td>$y = 321 - 13x$</td>
<td>floor heights in a building</td>
</tr>
<tr>
<td>$y = -300 + 15x$</td>
<td>earnings from mowing lawns</td>
</tr>
<tr>
<td>$y = 45 + 0.12x$</td>
<td>income from restaurant tabs</td>
</tr>
<tr>
<td>$y = 220 - 1.2x$</td>
<td>distance a car is from Flint</td>
</tr>
</tbody>
</table>

Winds of 40 mi/h blow on North Michigan Ave. in 1955 Chicago.

In [ELL] Input variables are those that are put in. Output variables are those that come out.

Students get closer to the notion of slope by studying rates of change as they construct linear equations and their graphs from input-output tables. The concept of input and output variables is a precursor to the study of functions. (The input value is the independent variable and the output value is the dependent variable, although those terms aren’t used until Lesson 7.3.)
The table relates the approximate wind chills for different actual temperatures when the wind speed is 15 mi/h. Assume the wind chill is a linear relationship for temperatures between $-5^\circ$ and $35^\circ$.

<table>
<thead>
<tr>
<th>Temperature ($^\circ$F)</th>
<th>-5</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wind chill ($^\circ$F)</td>
<td>-25.8</td>
<td>-19.4</td>
<td>-13</td>
<td>6.2</td>
<td>19</td>
<td>25.4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. What are the input and output variables?

b. What is the change in temperature from one table entry to the next? What is the corresponding change in the wind chill?

c. Use calculator lists to write a recursive routine that generates the table values. What are the missing entries?

a. The input variable is the actual air temperature in °F. The output variable is the temperature you feel as a result of the wind chill factor.

b. For every 5° increase in temperature, the wind chill increases 6.4°.

c. The recursive routine to complete the missing table values is \{Ans(1), Ans(2) - 6.4\} and \{Ans(1) + 5, Ans(2) + 6.4\}.

The calculator screen displays the missing entries.

In Example A, the rate at which the wind chill drops can be calculated from the ratio $\frac{6.4}{5}$, or $1.28$°. In other words, it feels 1.28° colder for every 1° drop in air temperature. This number is the rate of change for a wind speed of 15 mi/h.

The rate of change is equal to the ratio of the change in output values divided by the corresponding change in input values.

Do you think the rate of change differs with various wind speeds?
Step 3 Starting value: \((-5, -28.540)\)  
Rule for 1° increment: \(\text{Ans}(1) + 1, \text{Ans}(2) + 1.312\)

Plot the points and describe the viewing window you used.

Step 4 Write a recursive routine that gives the pairs of values listed in the table.

Copy the table. Complete the third and fourth columns of the table by recording the changes between consecutive input and output values. Then find the rate of change.

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
<th>Change in input values</th>
<th>Change in output values</th>
<th>Rate of change</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5</td>
<td>-28.540</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>-21.980</td>
<td>5</td>
<td>6.56</td>
<td>1.312</td>
</tr>
<tr>
<td>1</td>
<td>-20.668</td>
<td>1</td>
<td>1.312</td>
<td>1.312</td>
</tr>
<tr>
<td>2</td>
<td>-19.356</td>
<td>1</td>
<td>1.312</td>
<td>1.312</td>
</tr>
<tr>
<td>5</td>
<td>-15.420</td>
<td>3</td>
<td>3.936</td>
<td>1.312</td>
</tr>
<tr>
<td>15</td>
<td>-2.300</td>
<td>10</td>
<td>13.12</td>
<td>1.312</td>
</tr>
<tr>
<td>35</td>
<td>23.940</td>
<td>20</td>
<td>26.24</td>
<td>1.312</td>
</tr>
</tbody>
</table>

Step 5 If students are having difficulty, remind them that they can find the rate of change easily if they know output values for input values that are 1 unit apart. Also, elicit the idea that an input value of 0 gives the y-intercept as an output value.

Step 6 Yes, a line represents every possible temperature. The y-intercept shows up as the value of \(a\) in the equation. It is not the starting value of the routine.

Step 7 The values for rate of change are all equivalent. The rate of change appears as \(b\), the coefficient of \(x\). In the graph, to go from one point to the next you move right 1 unit and up 1.312 units, which is the rate of change.

Step 8 Encourage a variety of approaches: tracing, calculator tables, or working backward to solve the equation 

\[9.5 = -21.98 + 1.312x\]  

Step 8 Explanations will vary. Students can add 1.312° nine times to –2.3 and add 9° to 15°. They can also subtract 1.312° eleven times from 23.940° and subtract 11° from 35°. The answer is 24°F.

**SHARING IDEAS**  
Have students present several approaches to Step 8, describing their equations in the process. Mention that the constant rate of change is sometimes called the wind chill factor at this wind speed. Draw out the fact that a rate of change is a rate as defined in Chapter 2—that is, a ratio with denominator 1. The rate of change is the output change with each additional unit of input.

**[Ask]** “What do the equations have in common?”  
[Elicit the idea that the output variable is usually isolated on the left side, making it easy to enter functions in the calculator for graphing. The right side is like the recursive routine: A constant corresponds to the starting value, and the rate of change is multiplied by the input variable.]
**EXAMPLE B**

This table shows the temperature of the air outside an airplane at different altitudes.

<table>
<thead>
<tr>
<th>Altitude (m)</th>
<th>Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>7.7</td>
</tr>
<tr>
<td>1500</td>
<td>4.2</td>
</tr>
<tr>
<td>2200</td>
<td>-0.7</td>
</tr>
<tr>
<td>3000</td>
<td>-6.3</td>
</tr>
<tr>
<td>4700</td>
<td>-18.2</td>
</tr>
<tr>
<td>6000</td>
<td>-27.3</td>
</tr>
</tbody>
</table>

**a.** Add three columns to the table, and record the change in input values, the change in output values, and the corresponding rate of change.

**b.** Use the table and a recursive routine to write a linear equation in intercept form $y = a + bx$.

**c.** What are the real-world meanings of the values for $a$ and $b$ in your equation?

**Solution**

**a.** Record the change in input values, change in output values, and rate of change in a table. Note the units of each value.

<table>
<thead>
<tr>
<th>Altitude (m)</th>
<th>Temperature (°C)</th>
<th>Change in input values (m)</th>
<th>Change in output values (°C)</th>
<th>Rate of change (°C/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>7.7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1500</td>
<td>4.2</td>
<td>500</td>
<td>-3.5</td>
<td>$-3.3/500 = -0.007$</td>
</tr>
<tr>
<td>2200</td>
<td>-0.7</td>
<td>700</td>
<td>-4.9</td>
<td>$-4.9/700 = -0.007$</td>
</tr>
<tr>
<td>3000</td>
<td>-6.3</td>
<td>800</td>
<td>-5.6</td>
<td>$-5.6/800 = -0.007$</td>
</tr>
<tr>
<td>4700</td>
<td>-18.2</td>
<td>1700</td>
<td>-11.9</td>
<td>$-11.9/1700 = -0.007$</td>
</tr>
<tr>
<td>6000</td>
<td>-27.3</td>
<td>1300</td>
<td>-9.1</td>
<td>$-9.1/1300 = -0.007$</td>
</tr>
</tbody>
</table>

**b.** Note that the rate of change, or slope, is always $-0.007$, or $-\frac{7}{1000}$. You can also write the rate of change as $-0.7/100$, so this recursive routine models the relationship:

```
[1000, 7.7] ENTER
[Ans(1) + 100, Ans(2) - 0.7] ENTER
```

Working this routine backward, $[\text{Ans}(1) - 100, \text{Ans}(2) + 0.7]$, will eventually give the result $[0, 14.7]$. So the intercept form of the equation is $y = 14.7 - 0.007x$, where $x$ represents the altitude in meters and $y$ represents the air temperature in °C.
Note that the starting value of the recursive routine is not the same as the value of the \( y \)-intercept in the equation.

c. The value of \( a \), 14.7, is the temperature (in °C) of the air at sea level. The value of \( b \) indicates that the temperature drops 0.007°C for each meter that a plane climbs.

### EXERCISES

You will need your graphing calculator for Exercises 4, 5, and 10.

#### Practice Your Skills

1. a. \( y = 50 + 2.5x \)
   
<table>
<thead>
<tr>
<th>Input ( x )</th>
<th>Output ( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>100</td>
</tr>
<tr>
<td>-30</td>
<td>-25</td>
</tr>
<tr>
<td>16</td>
<td>90</td>
</tr>
<tr>
<td>15</td>
<td>87.5</td>
</tr>
<tr>
<td>-12.5</td>
<td>18.75</td>
</tr>
</tbody>
</table>

2. b. \( L_2 = -5.2 - 10 \cdot L_1 \)
   
<table>
<thead>
<tr>
<th>( L_1 )</th>
<th>( L_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-5.2</td>
</tr>
<tr>
<td>-8</td>
<td>74.8</td>
</tr>
<tr>
<td>24</td>
<td>-245.2</td>
</tr>
<tr>
<td>-35</td>
<td>344.8</td>
</tr>
<tr>
<td>-5.2</td>
<td>46.8</td>
</tr>
</tbody>
</table>

2. Use the equation \( w = -29 + 1.4t \), where \( t \) is temperature and \( w \) is wind chill, both in °F; to approximate the wind chill temperatures for a wind speed of 40 mi/h.

   a. Find \( w \) for \( t = 32° \) on the left.
   
   b. Find \( t \) for a wind chill of \( w = 15°F \) on the right.
   
   c. What is the real-world meaning of 1.4? (See Calculator Note 3C to learn how to run the program.)
   
   d. What is the real-world meaning of -29? (See Calculator Note 3C to learn how to run the program.)

3. Describe what the rate of change looks like in each graph.

   a. The graph of a person walking at a steady rate toward a motion sensor.
   
   b. The graph of a person standing still.
   
   c. The graph of a person walking at a steady rate away from a motion sensor.
   
   d. The graph of one person walking at a steady rate faster than another person.

4. Use the "Easy" setting of the INOUT game on your calculator to produce four data tables. Copy each table and write the equation you used to match the data values in the table. [See Calculator Note 3C to learn how to run the program.]

3c. The rate is positive, so the line goes from the lower left to the upper right.

3d. The rate for the speedier walker will be greater than the rate for the person walking more slowly, so the graph for the speedier walker will be steeper than the graph for the slower walker.

4. A sample:

   \[
   \begin{bmatrix}
   0 & -5 \\
   2 & -1 \\
   3 & 4 \\
   5 & 6
   \end{bmatrix}
   \]

   GUESS: -5 + 2L1

   The rate is positive, so the line goes from the lower left to the upper right.

   The rate for the speedier walker will be greater than the rate for the person walking more slowly, so the graph for the speedier walker will be steeper than the graph for the slower walker.

   A sample:
Reason and Apply

5. Each table below shows a different input-output relationship.

<table>
<thead>
<tr>
<th>i. Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-6</td>
</tr>
<tr>
<td>1</td>
<td>-2.5</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>4.5</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ii. Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-7</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>25</td>
</tr>
<tr>
<td>5</td>
<td>41</td>
</tr>
<tr>
<td>7</td>
<td>57</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>iii. Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>-8</td>
<td>34.2</td>
</tr>
<tr>
<td>-7</td>
<td>32.8</td>
</tr>
<tr>
<td>-3</td>
<td>27.2</td>
</tr>
<tr>
<td>2</td>
<td>20.2</td>
</tr>
<tr>
<td>8</td>
<td>11.8</td>
</tr>
</tbody>
</table>

a. Find the rate of change in each table. Explain how you found this value. i. 3.5; ii. 8; iii. -1.4
b. For each table, find the output value that corresponds to an input value of 0.
   What is this value called? i. -6; ii. 1; iii. 23; the y-intercept
c. Use your results from 5a and b to write an equation in intercept form for each table. i. y = -6 + 3.5x; ii. y = 1 + 8x; iii. y = 23 - 1.4x
d. Use a calculator list of input values to check that each equation actually produces the output values shown in the table.

6. The wind chill temperatures for a wind speed of 35 mi/h are given in the table.

<table>
<thead>
<tr>
<th>Temperature (°F)</th>
<th>Wind chill (°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5</td>
<td>-35</td>
</tr>
<tr>
<td>5</td>
<td>-21</td>
</tr>
<tr>
<td>10</td>
<td>-14</td>
</tr>
<tr>
<td>20</td>
<td>0</td>
</tr>
<tr>
<td>35</td>
<td>21</td>
</tr>
</tbody>
</table>

a. Define input and output variables. [Temperature, Wind chill]
b. Find the rate of change. Explain how you got your answer. [1.4°]
c. Write an equation in intercept form. \( y = -28 + 1.4x \)
d. Plot the points and graph the equation on the same set of axes. How are the graphs for the points and the equation similar? How are they different?

7. Samantha’s walk was recorded by a motion sensor. A graph of her walk and a few data points are shown here.

a. Write an equation in the form  
   \[ \text{Distance from sensor} = \text{start distance} + \text{change} \]  
   to model this walk. \[ 6 = 2 \]
b. If she continues to walk at a constant rate, at what time would she pass the sensor?
8. You can use the equation $7.3x = 200$ to describe a rectangle with an area of 200 square units like the one shown. What are the real-world meanings of the numbers and the variable in the equation? Solve the equation for $x$ and explain the meaning of your solution. Is the rectangle drawn to scale? How can you tell?

9. The total area of the figure at right is 1584 square units. You can use the equation $1584 - 33x = 594$ to represent an area of 1584 square units minus the area of 33x square units. The area remaining is 594 square units.
   a. What is the area of the shaded rectangle? $\text{Ans} = 990$ square units
   b. Write the equation you would use to find the height of the shaded rectangle. possible answers: $33x = 990; x = \frac{990}{33}$
   c. Solve the equation you wrote in 9b to find the height of the shaded rectangle. $\text{Ans} = 30$ units

10. Use the “Medium” setting of the INOUT game on your calculator to produce four data tables. Copy each table and write the equation you used to match the data values in the table. [See Calculator Note 3C.]

> Review

2.8

11. Show how you can solve these equations by using an undoing process. Check your results by substituting the solutions into the original equations.
   a. $-15 = -52 + 1.6x$
   b. $7 - 3x = 52$

3.2

12. **APPLICATION** To plan a trip downtown, you compare the costs of three different parking lots. ABC Parking charges $5 for the first hour and $2 for each additional hour or fraction of an hour. Cozy Car charges $3 per hour or fraction of an hour, and The Corner Lot charges a $15 flat rate for a whole day.
   a. Make a table similar to the one shown. Write recursive routines to calculate the cost of parking up to 10 hours at each of the three lots.
   b. Make three different scatter plots on the same pair of axes showing the parking rates at the three different lots. Use a different color for each parking lot. Put the hours on the horizontal axis and the cost on the vertical axis.
   c. Compare the three scatter plots. Under what conditions is each parking lot the best deal for your trip? Use the graph to explain.
   d. Would it make sense to draw a line through each set of points? Explain why or why not. No; because you have to pay for a whole hour for any fraction of the hour, the price of parking does not increase continuously.

<table>
<thead>
<tr>
<th>Hours</th>
<th>ABC Parking</th>
<th>Cozy Car</th>
<th>The Corner Lot</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>3</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>6</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>9</td>
<td>15</td>
</tr>
</tbody>
</table>

11a. $-15$
   Ans + 52
   Ans/1.6
   $-52 + 1.6(23.125) = -15$ Check.

11b. 52
   Ans − 7
   Ans/3
   $7 - 3(-15) = 52$ Check.

12b. Downtown Parking

Exercise 8

This exercise gives an opportunity to emphasize that a direct variation is a special kind of linear equation.

8. Because height times width gives area, 7.3 and $x$ represent the height and width, respectively. The number 200 represents the area of the rectangle in square units. The solution is about 27.4 units. The rectangle is not drawn to scale. The length should be about 3.8 times the width.

10. A sample:

   ![INOUT Game](image)

   **Exercise 12b**

   On calculators without a color display, students can use different graph marks.

<table>
<thead>
<tr>
<th>Hours</th>
<th>ABC</th>
<th>Cozy</th>
<th>Corner</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>11</td>
<td>12</td>
<td>15</td>
</tr>
<tr>
<td>5</td>
<td>13</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>6</td>
<td>15</td>
<td>18</td>
<td>15</td>
</tr>
<tr>
<td>7</td>
<td>17</td>
<td>21</td>
<td>15</td>
</tr>
<tr>
<td>8</td>
<td>19</td>
<td>24</td>
<td>15</td>
</tr>
<tr>
<td>9</td>
<td>21</td>
<td>27</td>
<td>15</td>
</tr>
<tr>
<td>10</td>
<td>23</td>
<td>30</td>
<td>15</td>
</tr>
</tbody>
</table>

   ABC: $\{1, 5\}$, $\{\text{Ans}(1) + 1, \text{Ans}(2) + 2\}$; Cozy: $\{1, 3\}$, $\{\text{Ans}(1) + 1, \text{Ans}(2) + 3\}$; Corner: $\{1, 15\}$, $\{\text{Ans}(1) + 1, \text{Ans}(2) + 0\}$

12c. For less than 3 h, Cozy Car is the least expensive option because on the graph its points are lower than the points of the others. For exactly 3 h, ABC and Cozy Car cost the same. For 3 to 5 h, ABC Parking has the best price, because its graph has a lower cost in that time frame. For exactly 6 h, ABC and The Corner Lot cost the same. For more than 6 h, The Corner Lot is the least expensive option.
To make a highway accessible to more vehicles, engineers reduce its steepness, also called its gradient or grade. This highway was designed with switchbacks so the gradient would be small.

A gradient is the inclination of a roadway to the horizontal surface. Research the federal, state, and local standards for the allowable gradients of highways, streets, and railway routes.

Find out how gradients are expressed in engineering terms. Give the standards for roadway types designed for vehicles of various weights, speeds, and engine power in terms of rate of change. Describe the alternatives available to engineers to reduce the gradient of a route in hilly or mountainous terrain. What safety measures do they incorporate to minimize risk on steep grades? Bring pictures to illustrate a presentation about your research, showing how engineers have applied standards to roads and routes in your home area.

2.3 13. Today while Don was swimming, he started wondering how many lengths he would have to swim in order to swim different distances. At one end of the pool, he stopped, gasping for breath, and asked the lifeguard. She told him that 1 length of the pool is 25 yards and that 72 lengths is 1 mile. As he continued swimming, he wondered:

70.4 lengths

a. Is 72 lengths really a mile? Exactly how many lengths would it take to swim a mile?

b. If it took him a total of 40 minutes to swim a mile, what was his average speed in feet per second? 2.2 ft/s

c. How many lengths would it take to swim a kilometer? about 44 lengths

d. Last summer Don got to swim in a pool that was 25 meters long. How many lengths would it take to swim a kilometer there? How many for a mile?

2.2 ft/s

3.4 14. APPLICATION Holly has joined a video rental club. After paying $6 a year to join, she then has to pay only $1.25 for each new release she rents.

a. Write an equation in intercept form to represent Holly's cost for movie rentals. $y = 6 + 1.25x$

b. Graph this situation for up to 60 movie rentals.

c. Video Unlimited charges $60 for a year of unlimited movie rentals. How many movies would Holly have to rent for this to be a better deal? 44 movies

**LEGAL LIMITS**

To make a highway accessible to more vehicles, engineers reduce its steepness, also called its gradient or grade. This highway was designed with switchbacks so the gradient would be small.

A gradient is the inclination of a roadway to the horizontal surface. Research the federal, state, and local standards for the allowable gradients of highways, streets, and railway routes.

Find out how gradients are expressed in engineering terms. Give the standards for roadway types designed for vehicles of various weights, speeds, and engine power in terms of rate of change. Describe the alternatives available to engineers to reduce the gradient of a route in hilly or mountainous terrain. What safety measures do they incorporate to minimize risk on steep grades? Bring pictures to illustrate a presentation about your research, showing how engineers have applied standards to roads and routes in your home area.

**OUTCOMES**

- Gradient is defined. Stated as a percent, 20% means a rise of 20 ft every 100 ft. Stated as a ratio, 1 to 5 is the same as 20%.
- The report summarizes standards. A 20% gradient, an angle of 11.5°, is considered steep. A maximum of 15% sustained gradient is recommended. Where there is heavy snowfall, the maximum is 10%.
- The report includes techniques for reducing gradients and safety measures that can minimize risk on steep grades.
- The report includes relevant pictures of nearby roads.
- Gradients for roads are compared with gradients for railroads.
- The slope of a line is carefully defined.

**Supporting the Project**

**MOTIVATION**

Steep roads are harder to travel and harder to maintain. Engineers must consider steepness as they design roads. What guidelines and regulations do they follow?
LESSON 3.6 Solving Equations Using the Balancing Method

In the previous two lessons, you learned about rate of change and the intercept form of a linear equation. In this lesson you’ll learn symbolic methods to solve these equations. You’ve already seen the calculator methods of tracing on a graph and zooming in on a table. These methods usually give approximate solutions. Working backward to undo operations is a symbolic method that gives exact solutions.

Another symbolic method that you can apply to solve equations is the balancing method. In this lesson you’ll investigate how to use the balancing method to solve linear equations. You’ll discover that it’s closely related to the undoing method.

Investigation
Balancing Pennies

Here is a visual model of the equation $2x + 3 = 7$. A cup represents the variable $x$ and pennies represent numbers. Assume that each cup has the same number of pennies in it and that the containers themselves are weightless.

**Step 1**
How many pennies must be in each cup if the left side of the scale balances with the right side? Explain how you got your answer.

Your answer to Step 1 is the solution to the equation $2x + 3 = 7$. It’s the number that can replace $x$ to make the statement true. In Steps 2 and 3, you’ll use pictures and equations to show stages that lead to the solution.

**Step 2**
Redraw the picture above, but with three pennies removed from each side of the scale. Write the equation that your picture represents.

**NCTM STANDARDS**

<table>
<thead>
<tr>
<th>CONTENT</th>
<th>PROCESS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>Problem Solving</td>
</tr>
<tr>
<td>Algebra</td>
<td>Reasoning</td>
</tr>
<tr>
<td>Geometry</td>
<td>Communication</td>
</tr>
<tr>
<td>Measurement</td>
<td>Connections</td>
</tr>
<tr>
<td>Data/Probability</td>
<td>Representation</td>
</tr>
</tbody>
</table>

**LESSON OBJECTIVE**

- Learn the balancing method to solve equations by doing the same thing to both sides

**Guiding the Investigation**

**One Step**
Show the Pan Balance transparency. Pose this problem: “How could you use pennies and cups on a pan balance to represent and solve the equation $2x + 3 = 7$?” Be open to a variety of approaches. As needed, point out...
that an unbalanced scale doesn't tell as much as a balanced one. Remind students that to keep a balance they must do the same thing to both sides. If there's time, have each group set up an equation for other groups to solve.

**Step 1** If students ask, point out that the balance pictured is a pan balance, with all masses on each side concentrated on one point, as opposed to the beam balance of Chapter 2, in which the distribution of masses was important.

**Steps 7 and 8** Students may use undoing to solve the equation. Encourage them to see how working backward is used on one side of the balance to isolate the unknown.

**SHARING IDEAS**
At one station for groups to visit during Steps 4 through 8, you might place five pennies under each of two cups, lay out four pennies and seven washers on the same side, and put ten pennies and three washers on the other side. Explain to visiting students that each washer represents $-1$. Then, when the class is together, ask how they solved the equation $2x + 4 - 7 = 10 - 3$, or $2x - 3 = 7$. Elicit the idea that, just as they combined the four pennies with four washers and removed them, they could add three pennies to each side to “cancel out” the three washers remaining on the left. In other words, when you add a number to its opposite, you get 0, and you can remove 0’s from anywhere on the balance without effect. (This wouldn't work on an actual pan balance, because the washers would have positive weights.)

If you don’t have washers, ask during Sharing how you might model the equation $2x - 3 = 7$ on a pan balance. Students may suggest that you could add three pennies to the seven on the right. Point out that doing so models the equation $2x = 7 + 3$. [Ask] “Do the equations $2x - 3 = 7$ and $2x = 7 + 3$ have the same solution?” [yes]

For more practice with balancing, use the Sketchpad demonstration The Balancing Method.

---

**Step 3** $x = 2$

**Redraw the picture,** this one showing half of what was on each side of the scale in Step 2. There should be just one cup on the left side of the scale and the correct number of pennies on the right side needed to balance it. Write the equation that this picture represents. This is the solution to the original equation.

**Step 4** Divide the pennies into two equal piles. If you have one left over, put it aside. Draw a large equal sign (or form one with two pencils) and place the penny stacks on opposite sides of it.

**Step 5** From the pile on one side of your equal sign, make three identical stacks, leaving at least a few pennies out of the stacks. Hide each stack under a paper cup. You should now have three cups and some pennies on one side of your equal sign.

**Step 6** On the other side you should have a pile of pennies. On both sides of the equal sign you have the same number of pennies, but on one side some of the pennies are hidden under cups. You can think of the two sides of the equal sign as being the two sides of a balance scale. Write an equation for this setup, using $x$ to represent the number of pennies hidden under one cup.

**Step 7** Move to another group’s setup. Look at their arrangement of pennies and cups, and write an equation for it. Solve the equation; that is, find how many pennies are under one cup without looking. When you’re sure you know how many pennies are under each cup, you can look to check your answer.

**Step 8** Write a brief description of how you solved the equation.

You can do problems like those in the investigation using a balance scale as long as the weight of the cup is very small. But an actual balance scale can only model equations in which all the numbers involved are positive. Still, the idea of balancing equations can apply to equations involving negative numbers. Just remember, when you add any number to its opposite, you get 0. For this reason, the opposite of a number is called the additive inverse. Think of negative and positive numbers as having opposite effects on a balance scale. You can remove 0 from either side of a balance-scale picture without affecting the balance. These three figures all represent 0:
**EXAMPLE A**  
Draw a series of balance-scale pictures to solve the equation \(6 = -2 + 4x\).

**Solution**  
The goal is to end up with a single \(x\)-cup on one side of the balance scale. One way to get rid of something on one side is to add its opposite to both sides.

Here is the equation \(6 = -2 + 4x\) solved by the balancing method:

<table>
<thead>
<tr>
<th>Picture</th>
<th>Action taken</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Image" /></td>
<td>Original equation.</td>
<td>(6 = -2 + 4x)</td>
</tr>
<tr>
<td><img src="image2.png" alt="Image" /></td>
<td>Add 2 to both sides.</td>
<td>(6 + 2 = -2 + 2 + 4x)</td>
</tr>
<tr>
<td><img src="image3.png" alt="Image" /></td>
<td>Remove the 0.</td>
<td>(8 = 4x)</td>
</tr>
<tr>
<td><img src="image4.png" alt="Image" /></td>
<td>Divide both sides by 4.</td>
<td>(\frac{8}{4} = \frac{4x}{4})</td>
</tr>
<tr>
<td><img src="image5.png" alt="Image" /></td>
<td>Reduce.</td>
<td>(2 = x) or (x = 2)</td>
</tr>
</tbody>
</table>

In the second and third equations, you saw \(6 + 2\) combine to 8, and \(-2 + 2\) combine to 0. You can combine numbers because they are **like terms**. However, in the first equation you could not combine \(-2\) and \(4x\), because they are **not like terms**. **Like terms** are terms in which the variable component is the same, and they may differ only by a coefficient.

**Assessing Progress**  
Observe how well students follow directions, work in groups, and understand the idea of balance.

**EXAMPLE A**  
This example continues the idea of working with negative numbers on a picture of a pan balance (not an actual balance). You might want to reinforce the new vocabulary **additive inverse** as you discuss this example. Exercise 6 defines **multiplicative inverse**.

If you decide to show students an example of an equation in which \(x\) has a negative coefficient, be aware of the pitfalls of the balancing model. You may use undoing and divide or multiply by the multiplicative inverse.

Another option is to move the term to the other side of the equation to give it a positive coefficient.
Balance-scale pictures can help you see what to do to solve an equation by the balancing method. But you won’t need the pictures once you get the idea of doing the same thing to both sides of an equation. And pictures are less useful if the numbers in the equation aren’t “nice.”

**EXAMPLE B**

Solve the equation $-31 = -50.25 + 1.55x$ using each method.

a. undoing operations

b. the balancing method
c. tracing on a calculator graph
d. zooming in on a calculator table

**Solution**

Each of these methods will give the same answer, but notice the differences among the methods. When might you prefer to use a particular method?

a. undoing operations

Start with $-31$.

b. the balancing method

\[
-31 = -50.25 + 1.55x \\
-31 + 50.25 = -50.25 + 50.25 + 1.55x \\
19.25 = 1.55x \\
19.25 \div 1.55 = 12.42 \\
12.42 \approx x, \text{ or } x \approx 12.42
\]

This chart shows how balancing equations is related to the undoing method that you’ve been using. In the last column, as you work up from the bottom, you can see how the equation changes as you apply the “undo” operation to both sides of the equation.

<table>
<thead>
<tr>
<th>Description</th>
<th>Undo</th>
<th>Result</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pick $x$</td>
<td></td>
<td>$12.42$</td>
<td>$12.42 = x$</td>
</tr>
<tr>
<td>Multiply by $1.55$</td>
<td>$(1.55)$</td>
<td>$19.25$</td>
<td>$19.25 = 1.55x$</td>
</tr>
<tr>
<td>Subtract $50.25$</td>
<td>$(50.25)$</td>
<td>$-31$</td>
<td>$-31 = -50.25 + 1.55x$</td>
</tr>
</tbody>
</table>

In parts a and b, if you convert the answer to a fraction, you get an exact solution of $\frac{35}{31}$. 
c. tracing on a calculator graph

Enter the equation into Y1. Adjust your window settings and graph. Press TRACE and use the arrow keys to find the x-value for a y-value of –31. (See Example B in Lesson 3.4 to review this procedure.) You can see that for a y-value of approximately –31.6 the x-value is 12.02.

d. zooming in on a calculator table

To find a starting value for the table, use guess-and-check or a calculator graph to find an approximate answer. Then use the calculator table to find the answer to the desired accuracy.

Once you have determined a reasonable starting value, zoom in on a calculator table to find the answer using smaller and smaller values for the table increment. See Calculator Note 2A to review zooming in on a table.

You can also check your answer by using substitution.

The calculator result isn’t exactly –31 because 12.42 is a rounded answer. If you substitute an exact solution such as \( \frac{15}{2} \) or \( \frac{30}{2} \), you’ll get exactly –31.

From Example B, you can see that each method has its advantages. The methods of balancing and undoing use the same process of working backward to get an exact solution. The two calculator methods are easy to use but usually give approximate solutions to the equation. You may prefer one method over others, depending on the equation you need to solve. If you are able to solve an equation using two or more different methods, you can check to see that each method gives the same result. With practice, you may develop symbolic solving methods of your own.

Knowing a variety of methods, such as the balancing and undoing methods, as well as the calculator methods, will improve your equation-solving skills, regardless of which method you prefer.

In Exercise 12, you’ll see how to use the balancing method to solve an equation that has the variable on both sides.

Closing the Lesson

Of the two exact methods for solving linear equations, the balancing method is useful when the variable appears more than once in the equation. The undoing method helps students focus on the order of operations.

BUILDING UNDERSTANDING

Students practice solving equations by a variety of methods.

ASSIGNING HOMEWORK

| Essential | 1–6, 9, 12 |
| Performance assessment | 9 |
| Portfolio | 9 |
| Journal | 6 |
| Group | 8, 10–12 |
| Review | 13–15 |
EXERCISES

Practice Your Skills

1. Give the equation that each picture models.
   a. \[ \frac{2x}{3} = 6 \]
   b. \[ x + 2 = 5 \]
   c. \[ 2x - 1 = 3 \]
   d. \[ 2 = 2x - 3 \]

2. Copy and fill in the table to solve the equation as in Example A.

<table>
<thead>
<tr>
<th>Picture</th>
<th>Action taken</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Image](original equation: 2x - 2 = 4)</td>
<td>Add 2 to both sides.</td>
<td>[ 2x - 2 = 4 + 4 ]</td>
</tr>
<tr>
<td>![Image](add 2 to both sides: 2 - 2 + 4 + 2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>![Image](remove 0 from left side: 2x = 6)</td>
<td>Divide both sides by 2.</td>
<td>[ \frac{2x}{2} = \frac{6}{2} ]</td>
</tr>
<tr>
<td>![Image](divide both sides by 2: x = 3)</td>
<td>Reduce.</td>
<td>[ x = 3 ]</td>
</tr>
</tbody>
</table>
3. Give the next stages of the equation, matching the action taken, to reach the solution.
   a. 0.1x + 12 = 2.2 \(\circ\) Original equation.
   b. \(\frac{12 + 3.12x}{3} = -100\) Original equation.
   0.1x + 12 = 2.2 \(\longrightarrow\) Subtract 12 from both sides.
   \(\frac{12 + 3.12x}{3} = -300\) Multiply both sides by 3.
   0.1x = -9.8 \(\longrightarrow\) Remove the 0 and subtract.
   12 = -312 \(\longrightarrow\) Subtract 12 from both sides.
   x = -98 \(\longrightarrow\) Divide both sides by 0.1.
   x = -100 \(\longrightarrow\) Divide both sides by 3.12

4. Complete the tables to solve the equations.

   a.  
      | Description | Undo | Result | Equation |
      |-------------|------|--------|----------|
      | Pick x.     | 53   | x = 53 | 3(x - 8) + 7 = 34 |
      | Subtract 8. | + (8) | 45     | x - 8 = 45 |
      | Multiply by 3. | / (3) | 135    | 3(x - 8) = 135 |
      | Divide by 5. | x (5) | 27     | \(\frac{3(x - 8)}{5}\) = 27 |
      | Add 7.      | - (7) | 34     | \(\frac{3(x - 8)}{5}\) + 7 = 34 |

   b.  
      | Description | Undo | Result | Equation |
      |-------------|------|--------|----------|
      | Pick x.     | 10   | x = 10 | \(\frac{2 + x}{4}\) - 5 = 16 |
      | Add 2.      | - (2) | 12     | 2 + x = 12 |
      | Divide by 4. | x (4) | 3      | \(\frac{2 + x}{4}\) = 3 |
      | Multiply by 7. | / (7) | 21    | \(\frac{2 + x}{4}\) = 3 |
      | Subtract 5. | + (5) | 16     | \(\frac{2 + x}{4}\) = 3 |

5. Give the additive inverse of each number.
   a. \(-\frac{1}{2}\) \(\circ\) \(-\frac{1}{5}\) b. 17 \(-17\) c. -2.3 \(\circ\) 2.3 d. \(-x\) \(-x\)

6. A **multiplicative inverse** is a number or expression that you can multiply by something to get a value of 1. The multiplicative inverse of 4 is \(\frac{1}{4}\) because \(4 \cdot \frac{1}{4} = 1\).
   Give the multiplicative inverse of each number.
   a. 12 \(\circ\) \(\frac{1}{12}\) b. \(\frac{1}{6}\) \(\circ\) 6 c. 0.02 \(\circ\) 50 d. \(-\frac{1}{2}\) \(-2\)

7. Solve these equations. Tell what action you take at each stage.
   a. 144x = 12 \(\longrightarrow\) x = \(\frac{1}{12}\)
   b. \(\frac{1}{6}\)x + 2 = 8 \(\longrightarrow\) x = 36

8. **Mini-Investigation** A solution to the equation \(-10 + 3x = 5\) is shown below.
   
   \(-10 + 3x = 5\)
   
   \(3x = 15\)
   
   \(x = 5\)
   
   a. Describe the steps that transform the original equation into the second equation and the second equation into the third (the solution). **Add 10 to both sides, divide both sides by 3.**
   b. Graph \(Y_1 = -10 + 3x\) and \(Y_2 = 5\), and trace to the lines' intersection. Write the coordinates of this point. (5, 5)
c. Graph \( Y_1 = 3x \) and \( Y_2 = 15 \), and trace to the lines’ intersection. Write the coordinates of this point: \((5, 15)\).

d. Graph \( Y_1 = x \) and \( Y_2 = 5 \), and trace to the lines’ intersection. Write the coordinates of this point: \((5, 5)\).

e. What do you notice about your answers to 8b–d? Explain what this illustrates.

9. Solve the equation \( 4 + 1.2x = 12.4 \) by using each method.

a. balancing 

b. undoing 

c. tracing on a graph 

d. zooming in on a table 

10. Solve each equation symbolically using the balancing method.

a. \( 3 + 2x = 17 \)  

b. \( 0.5x + 2.2 = 101.0 \)  

c. \( x + 307.2 = 2.1 \)  

d. \( 2(2x + 2) = 7 \)  

e. \( \frac{4 + 0.01x}{6.2} - 6.2 = 0 \)  

11. You can solve familiar formulas for a specific variable. For example, solving \( A = hw \) for \( l \) you get

\[ \frac{A}{w} = l \]

Divide both sides by \( w \).

You can also write \( l = \frac{A}{w} \). Now try solving these formulas for the given variable.

a. \( C = 2\pi r \) for \( r \)  

b. \( A = \frac{1}{2}(bh) \) for \( h \)  

c. \( P = 2(l + w) \) for \( l \)  

d. \( P = 4s \) for \( s \)  

e. \( d = rt \) for \( t \)  

f. \( A = \frac{1}{2}(a + b) \) for \( h \)

12. An equation can have the variable on both sides. In these cases you can maintain the balance by eliminating the x’s from one of the sides before you begin undoing.

a. Copy and complete this table to solve the equation: \( 4 + 1.2x = 12.4 \).

<table>
<thead>
<tr>
<th>Picture</th>
<th>Action taken</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Picture" /></td>
<td>Original equation.</td>
<td>( 2 + 4x = x + 8 )</td>
</tr>
<tr>
<td><img src="image2" alt="Picture" /></td>
<td>Subtract 1x from both sides.</td>
<td>( 2 + 3x = 8 )</td>
</tr>
<tr>
<td><img src="image3" alt="Picture" /></td>
<td>Subtracted 2 from both sides.</td>
<td>( 3x = 6 )</td>
</tr>
<tr>
<td><img src="image4" alt="Picture" /></td>
<td>Divided both sides by 3.</td>
<td>( x = 2 )</td>
</tr>
</tbody>
</table>

b. Show the steps used to solve \( 5x - 4 = 2x + 5 \) using the balancing method. Substitute your solution into the original equation to check your answer.
2.1 13. APPLICATION Economy drapes for a certain size window cost $90. They have shallow pleats, and the width of the fabric is $\frac{2}{3}$ times the window width. Luxury drapes of the same fabric for the same size window have deeper pleats. The width of the fabric is 3 times the window width. What price should the store manager ask for the luxury drapes?

$\frac{90}{2.25} = \frac{3}{x} \Rightarrow x = 120$

3.4 14. Run the easy level of the LINES program on your calculator. See Calculator Note 3D to learn how to use the LINES program. Sketch a graph of the randomly generated line on your paper. Use the trace function to locate the $y$-intercept and to determine the rate of change. When the calculator says you have the correct equation, write it under the graph. Repeat this program until you get three correct equations in a row.

3.2 15. The local bagel store sells a baker’s dozen of bagels for $6.49, while the grocery store down the street sells a bag of 6 bagels for $2.50.

a. Copy and complete the tables showing the cost of bagels at the two stores.

<table>
<thead>
<tr>
<th>Bagel Store</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bagels</td>
</tr>
<tr>
<td>13</td>
</tr>
<tr>
<td>26</td>
</tr>
<tr>
<td>39</td>
</tr>
<tr>
<td>52</td>
</tr>
<tr>
<td>65</td>
</tr>
<tr>
<td>78</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Grocery Store</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bagels</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>12</td>
</tr>
<tr>
<td>18</td>
</tr>
<tr>
<td>24</td>
</tr>
<tr>
<td>30</td>
</tr>
<tr>
<td>36</td>
</tr>
<tr>
<td>42</td>
</tr>
<tr>
<td>48</td>
</tr>
<tr>
<td>54</td>
</tr>
<tr>
<td>60</td>
</tr>
</tbody>
</table>

b. Graph the information for each market on the same coordinate axes. Put bagels on the horizontal axis and cost on the vertical axis.

c. Find equations to describe the cost of bagels at each store.

d. How much does one bagel cost at each store? How do these cost values relate to the equations you wrote in 15c?

e. Looking at the graphs, how can you tell which store is the cheaper place to buy bagels?

f. Bernie and Buffy decided to use a recursive routine to complete the tables. Bernie used this routine for the bagel store:

Bernie’s routine calculates each price by doubling the last. It works the first time, because if you buy twice as many bagels, you pay twice as much. But using Bernie’s routine, if you buy 36 bagels at the bagel store, you pay $25.96 instead of $19.47, which amounts to paying four times as much as a single dozen instead of three times the price of a dozen. The routine should be:

6.49 ENTER

Ans: 2.50 ENTER

Buffy says that this routine isn’t correct, even though it gives the correct answer for 13 and 26 bagels. Explain to Bernie what is wrong with his recursive routine. What routine should he use?

15b. $y$ represents cost; $x$ represents number of bagels.

bagel store: $y = \frac{6.49}{13} x$ (or $y \approx 0.50x$)

grocery store: $y = \frac{2.50}{6} x$ (or $y \approx 0.42x$)

15d. Bagel store: about 50¢ per bagel; grocery store: about 42¢ per bagel; these are the coefficients of $x$ or constants of variation in the equations.
In this activity you’ll explore the relationship between the number of knots in a rope and the length of the rope and write an equation to model the data.

**Step 1**
Choose one piece of rope and record its length in a table like the one shown. Tie 6 or 7 knots, remeasuring the rope after you tie each knot. As you measure, add data to complete a table like the one above.

**Step 2**
The data should show a linear relationship.

**Step 3**
Answers will vary depending on the thickness of rope and type of knot. The rate of change represents the reduction of rope length for each knot tied. It is a negative number.

**Step 4**
Choose one piece of rope and record its length in a table like the one shown. Tie 6 or 7 knots, remeasuring the rope after you tie each knot. As you measure, add data to complete a table like the one above.

Graph your data, plotting the number of knots on the x-axis and the length of the knotted rope on the y-axis. What pattern does the data seem to form?

What is the approximate rate of change for this data set? What is the real-world meaning of the rate of change? What factors have an effect on it?

What is the intercept for the line that best models the data? What is its real-world meaning? The y-intercept is the length of rope without any knots.
Step 5 Write an equation in intercept form for the line that you think best models the data. Graph your equation to check that it’s a good fit. Graphs will vary. The line should go down from a positive y-intercept. It should pass through, or near, most of the points.

Step 6 Use your equation to predict the length of your rope with 7 knots. What is the difference between the actual measurement of your rope with 7 knots and the length you predicted using your equation? Answers will vary.

Step 7 The equation says that you will eventually have a rope of length 0 if you tie enough knots.

Step 8 What is the maximum number of knots that you can tie with your piece of rope? Explain your answer.

Step 9 Does your graph cross the x-axis? Explain the real-world meaning, if any, of the x-value of the intersection point.

Step 10 Substitute a value for y into the equation. What question does the equation ask? What is the answer? The question it asks is “How many knots are tied to produce a rope of length y?”

Step 11 Repeat Steps 1–5 using a different piece of rope. Graph the data on the same pair of axes.

Step 12 Different lengths of ropes will account for different y-intercepts. Different thicknesses will account for different rates of change. Both affect the value of the x-intercepts.

IMPROVING YOUR REASONING SKILLS

There are 100 students and 100 lockers in a school hallway. All of the lockers are closed. The first student walks down the hallway and opens every locker. A second student closes every even-numbered locker. The third student goes to every third locker and changes its state to the opposite of how it was before. After all 100 students have opened or closed the lockers, how many lockers are left open?

IMPROVING REASONING SKILLS

After accumulating data, many students will see the pattern that the lockers left open at the end correspond to perfect squares—1, 4, 9, 16, 25, 36, 49, 64, 81, and 100. If needed, explain that left open means open after all 100 students pass. Be sure they actually answer the question of how many lockers are left open. [10] Ask students to explain.

[A locker changes once for each factor of its number. For example, locker number 24 is changed by students 1, 2, 3, 4, 6, 8, 12, and 24. So if a locker’s number has an even number of factors, it is left closed. If a locker’s number has an odd number of factors, it is left open. The numbers with an odd number of factors are the perfect squares.]

[Ask] “Why do perfect squares have an odd number of factors?” [Factors of numbers come in pairs: (1, 24), (2, 12), (3, 8), (4, 6). The square root of a perfect square is paired with itself; factors of 36: (1, 36), (2, 18), (3, 12), (4, 9), (6, 6). So the number of distinct factors is an odd number.]
You started this chapter by investigating recursive sequences by using their starting values and rates of change to write recursive routines. You saw how rates of change and starting values appear in plots.

In a walking investigation you observed, interpreted, and analyzed graphical representations of relationships between time and distance. What does the graph look like when you stand still? When you move away from or move toward the motion sensor? If you speed up or slow down? You identified real-world meanings of the $y$-intercept and the rate of change of a linear relationship, and used them to write a linear equation in the intercept form, $y = a + bx$. You learned the role of $b$, the coefficient of $x$. You explored relationships among verbal descriptions, tables, recursive rules, equations, and graphs.

Throughout the chapter you developed your equation-solving skills. You found solutions to equations by continuing to practice an undoing process and by using a balancing process. You found approximate solutions by tracing calculator graphs and by zooming in on calculator tables. Finally, you learned how to model data that don’t lie exactly on a line, and you used your model to predict inputs and outputs.

### Exercises

You will need your graphing calculator for Exercises 4, 6, and 7.

**Answers are provided for all exercises in this set.**

1. Solve these equations. Give reasons for each step.
   a. $-x = 7$  
      $x = -7$
   b. $4.2 = -2x - 42.6$  
      $x = -23.4$

2. These tables represent linear relationships. For each relationship, give the rate of change, the $y$-intercept, the recursive rule, and the equation in intercept form.

   **a.**
   
<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
</tbody>
</table>

   1; 3; add 1; $y = 3 + x$

   **b.**
   
<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.01</td>
</tr>
<tr>
<td>2</td>
<td>0.02</td>
</tr>
<tr>
<td>3</td>
<td>0.03</td>
</tr>
</tbody>
</table>

   0.01; 0; add 0.01; $y = 0.01x$

   **c.**
   
<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
</tr>
</tbody>
</table>

   2; 5; add 2; $y = 5 + 2x$

   **d.**
   
<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>5</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>12</td>
<td>-3</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

   $\frac{1}{2}$; 3; subtract $\frac{1}{2}$; $y = 3 - \frac{1}{2}x$

**Helping with the Exercises**

Exercise 1 Reasons students give for each step will depend on their method of solving the equation.
3. Match these walking instructions with their graph sketches.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>d</td>
<td>t</td>
</tr>
<tr>
<td>ii</td>
<td>d</td>
<td>t</td>
</tr>
<tr>
<td>iii</td>
<td>d</td>
<td>t</td>
</tr>
</tbody>
</table>

a. The walker stands still.  

b. The walker takes a few steps toward the 0-mark, then walks away.

c. The walker steps away from the 0-mark, stops, then continues more slowly in the same direction.

4. Graph each equation on your calculator, and trace to find the approximate y-value for the given x-value.

a. \( y = 1.21 - x \) when \( x = 70.2 \)  
   \( y = -68.99 \)

b. \( y = 6.02 + 44.3x \) when \( x = 96.7 \)  
   \( y = 4289.83 \)

c. \( y = -0.06 + 0.313x \) when \( x = 0.64 \)  
   \( y = 0.14032 \)

d. \( y = 1183 - 2140x \) when \( x = -111 \)  
   \( y = 238,723 \)

5. Write the equations for linear relationships that have these characteristics.

a. The output value is equal to the input value.  
   \( y = x \)

b. The output value is 3 less than the input value.  
   \( y = -3 + x \)

c. The rate of change is 2.3 and the y-intercept is \(-4.3\).  
   \( y = -4.3 + 2.3x \)

d. The graph contains the points (1, 1), (2, 1), and (3, 1).  
   \( y = 1 \)

6. The profit for a small company depends on the number of bookcases it sells. One way to determine the profit is to use a recursive routine such as

```
0, -850 ENTER
{Ans(1) + 1, Ans(2) + 70} ENTER ; ENTER ; 
```

a. Explain what the numbers and expressions 0, \(-850\), Ans(1), Ans(1) + 1, Ans(2), and Ans(2) + 70 represent.

b. Make a plot of this situation.

c. When will the company begin to make a profit? Explain.

d. Explain the relationship between the values \(-850\) and 70 and your graph.

e. Does it make sense to connect the points in the graph with a line? Explain.

No; partial bookcases cannot be sold.

7. A single section and a double section of a log fence are shown.

a. How many additional logs are required each time the fence is increased by a single section?  3
b. Copy and fill in the missing values in the table below.

<table>
<thead>
<tr>
<th>Number of sections</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>...</th>
<th>30</th>
<th>...</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of logs</td>
<td>4</td>
<td>7</td>
<td>10</td>
<td>13</td>
<td>...</td>
<td>91</td>
<td>...</td>
<td>151</td>
</tr>
</tbody>
</table>

c. Describe a recursive routine that relates the number of logs required to the number of sections.

d. If each section is 3 meters long, what is the longest fence you can build with 217 logs? 216 m

8. Suppose a new small-business computer system costs $5,400. Every year its value drops by $525.

a. Define variables and write an equation modeling the value of the computer in any given year.

b. What is the rate of change, and what does it mean in the context of the problem?

c. What is the y-intercept, and what does it mean in the context of the problem?

d. What is the x-intercept, and what does it mean in the context of the problem?

9. Andrei and his younger brother are having a race. Because the younger brother can’t run as fast, Andrei lets him start out 5 m ahead. Andrei runs at a speed of 7.7 m/s. His younger brother runs at 6.5 m/s. The total length of the race is 50 m.

a. Write an equation to find how long it will take Andrei to finish the race. Solve the equation to find the time.

b. Write an equation to find how long it will take Andrei’s younger brother to finish the race. Solve the equation to find the time.

c. Who wins the race? How far ahead was the winner at the time he crossed the finish line?

10. Solve each equation using the method of your choice. Then use a different method to verify your solution.

a. $14x = 63 \quad x = 4.5$

b. $-4.5x = 18.6 \quad x = -4.13$

c. $8 = 6 + 3x \quad x = 0.6$

d. $5(x - 7) = 29 \quad x = 12.8$

e. $3(x - 5) + 8 = 12 \quad x = 6.3$
11. For each table, write a formula for list $L_2$ in terms of list $L_1$.

<table>
<thead>
<tr>
<th></th>
<th>$L_1$</th>
<th>$L_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>0</td>
<td>5.7</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>3.4</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.1</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>3.5</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>5.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$L_1$</th>
<th>$L_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>b.</td>
<td>3</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>53</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$L_1$</th>
<th>$L_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>c.</td>
<td>3</td>
<td>13.5</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>7.5</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>9.5</td>
</tr>
</tbody>
</table>

12. You can represent linear relationships with a graph, a table of values, an equation, or a rule stated in words. Here are two linear relationships. Give all the other ways to show each relationship.

a. $y = 1 + \frac{1}{2}x$; the output value is half the input value plus 1.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1.5</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>2.5</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

b. $y = -x$; the output value is the additive inverse (or opposite) of the input value, or the sum of the input value and the output value is 0.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>-3</td>
</tr>
<tr>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

13. **Application** Sonja bought a pair of 210 cm cross-country skis. Will they fit in her ski bag, which is $6\frac{1}{2}$ ft long? Why or why not? No, they won’t fit; 210 cm is 6.89 ft.

14. Fifteen students counted the number of letters in their first and last names. Here is the data set: [Data set: NMLET]:

| Number of students |
|---|---|---|---|---|---|
| 6 | 15 | 8 | 12 | 8 | 17 | 9 | 7 |
| 13 | 15 | 14 | 9 | 16 | 15 | 10 |

a. Make a histogram of the data with a bin width of 2.

b. What is the mean number of letters? 11.6 letters

15. Evaluate these expressions.

a. $-3 \cdot 8 - 5 \cdot 6 = -54$

b. $[-2 - (4)] \cdot 8 - 11 = 5$

c. $7 \cdot 8 + 4 \cdot (-12) = 8$

d. $11 - 3 \cdot 9 - 2 = -18$

16. On a recent trip to Detroit, Tom started from home, which is 12 miles from Traverse City. After 4 hours he had traveled 220 miles.

a. Write a recursive routine to model Tom’s distance from Traverse City during this trip. State at least two assumptions you’re making.

b. Use your recursive routine to determine his distance from Traverse City for each hour during the first 5 hours of the trip.

c. What is the rate of change, and what does it mean in the context of this situation?

16a. The starting value is 12; Ans + 55. Possible assumptions: Tom’s home is 12 mi closer to Detroit than to Traverse City. He travels at a constant speed. We are measuring highway distance.

16b. Hours | Distance (mi)
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>12</td>
</tr>
<tr>
<td>1</td>
<td>67</td>
</tr>
<tr>
<td>2</td>
<td>122</td>
</tr>
<tr>
<td>3</td>
<td>177</td>
</tr>
<tr>
<td>4</td>
<td>232</td>
</tr>
<tr>
<td>5</td>
<td>287</td>
</tr>
</tbody>
</table>

16c. Tom traveled 55 mi each additional hour. The rate of change is 55 mi/h.
17a. approximately 1061 thousand (or 1,016,000) visitors

17b. 404, 482, 738, 1131, 3379

17c. [0, 3500, 500, 0, 2, 1]

17d. Yosemite; the number of visitors exceeds 1131 by more than 1.5(1131 − 482).

1.3 17. California has many popular national parks. This table shows the number of visitors in thousands to national parks in 2003.

   a. Find the mean number of visitors.
   b. What is the five-number summary for the data?
   c. Create a box plot for the data.
   d. Identify any parks in California that are outliers in the numbers of visitors they had. Explain why they are outliers.

<table>
<thead>
<tr>
<th>National park</th>
<th>Visitors (thousands)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channel Islands</td>
<td>586</td>
</tr>
<tr>
<td>Death Valley</td>
<td>890</td>
</tr>
<tr>
<td>Joshua Tree</td>
<td>1283</td>
</tr>
<tr>
<td>Kings Canyon</td>
<td>556</td>
</tr>
<tr>
<td>Lassen Volcanic</td>
<td>404</td>
</tr>
<tr>
<td>Redwood</td>
<td>408</td>
</tr>
<tr>
<td>Sequoia</td>
<td>979</td>
</tr>
<tr>
<td>Yosemite</td>
<td>3379</td>
</tr>
</tbody>
</table>

(U.S. National Park Service) [Data set CAPRK]

2.5 18. Ohm’s law states that electrical current is inversely proportional to the resistance. A current of 18 amperes is flowing through a conductor whose resistance is 4 ohms.

   a. What is the current that flows through the system if the resistance is 8 ohms?
   b. What is the resistance of the conductor if a current of 12 amperes is flowing?

   Every knob or lever of this sound recording console regulates electric resistance in a current. The resistance varies directly with voltage and inversely with current.
2.8 19. Consider the equation \(2(x - 6) = -5\).
   a. Solve the equation. Solution methods will vary; \(x = 3.5\).
   b. Show how you can check your result by substituting it into the original equation.
   \[ \frac{2(3.5 - 6)}{2} = \frac{-5}{2} = -2.5 \]

2.3 20. APPLICATION Amber makes \$6 an hour at a sandwich shop. She wants to know how many hours she needs to work to save \$500 in her bank account. On her first paycheck, she notices that her net pay is about 75% of her gross pay.
   a. How many hours must she work to earn \$500 in gross pay?
   \[ \frac{500}{0.75} = 66.67 \text{ h} \]
   b. How many hours must she work to earn \$500 in net pay?
   \[ \frac{500}{0.75 	imes 6} = 111.11 \text{ h} \]

TAKE ANOTHER LOOK

1a. The graph shows the change in the hiker’s speed over time. The contour map shows the change in the hiker’s elevation as she follows the dotted line. It also shows the horizontal distance she has traveled from her starting point.
   b. The graph shows rate of change in speed as the steepness (slope) of a line. At first the speed is steadily decreasing, then it is increasing, then it remains constant. The contour map shows rate of change in elevation by the distance between the contour lines. When the lines are very close together, the elevation is changing quickly.
   c. If a scale were provided, you could infer distance from the graph by estimating the average speed up to a certain point and multiplying that by the time at that point. Students might refer to the formula \( \text{distance} = \text{speed} \times \text{time} \) \((d = rt)\). You could measure distance on the contour map using the map scale.
   d. Student sketches of the graph of \((\text{distance, time})\) should show three sections. In each, the graph is increasing. In the first section, the graph is a curve that is concave down, showing a steadily decreasing speed. The second section of the graph is concave up because the speed is increasing, and the third section is a steep straight line, showing a constant, fast speed.

1e. The graph and the contour map together show that the speed of the hiker was decreasing at the beginning because she was climbing a hill. As she began to go down the hill, she kept moving faster and faster until she was running. She stayed at a constant run after she reached level ground.
a. Rewrite each of these fractions in decimal form. If the digits appear to repeat, indicate this by placing a bar over those digits that repeat.

\[
\frac{1}{7}, \frac{11}{7}, \frac{1}{11}, \frac{2}{7}, \frac{13}{7}, \frac{9}{11}, \frac{7}{21}, \frac{15}{22}, \frac{30}{20}.
\]

b. Describe how you can predict whether a fraction will convert to a terminating decimal or a repeating decimal.

Reversing the process—converting decimals to fractions

c. Write the decimals 0.25, 0.8, 0.13, and 0.412 as fractions.

You can use what you’ve learned in this chapter about solving equations to help you write an infinite repeating decimal, like \(0.\overline{1}\), as a fraction. For example, to find a fraction equal to \(0.\overline{1}\), you are looking for a fraction \(F\) such that \(10F = 1.\overline{1}\). Follow the steps shown.

\[
F = 0.\overline{1} \quad \quad \quad \quad \quad \quad 10F = 1.\overline{1} = 1 + 0.\overline{1}
\]

\[
9F = 1
\]

So, \(F = \frac{1}{9}\).

Here, the trick was to multiply by 10 so that 10\(F\) and \(F\) had the same decimal part. Then, when you subtract 10\(F\) \(-\) \(F\), the decimal portion is eliminated.

d. Write the repeating decimal \(0.1\overline{8}\) as a fraction. (Hint: What can you multiply \(F = 0.1\overline{8}\) by so that you can subtract off the same decimal part?)

e. Write these repeating decimals as fractions.

i. \(0.3\overline{2}\)  
   ii. \(0.3\overline{25}\)  
   iii. \(0.2\overline{325}\)

IMPROVING YOUR REASONING SKILLS

Did these plants grow at the same rate? If not, which plant was tallest on Day 4? Which plant took the most time to reach 8 cm? Redraw the graphs so that you can compare their growth rates more easily.

The differences in the vertical scale (height) indicate that the plants are growing at different rates. Plant 3 is the fastest growing; it is about 20 cm tall in 4 days. Plant 2 is slowest growing; it takes almost 6 days to reach 7 cm. This should suggest to students that the steepness of a line is relative to the scales on the axes. They have seen this many times on their calculators.
Assessing What You’ve Learned

CHAPTER 3 REVIEW

Making presentations is an important career skill. Most jobs require workers to share information, to help orient new coworkers, or to represent the employer to clients. Making a presentation to the class is a good way to develop your skill at organizing and delivering your ideas clearly and in an interesting way. Most teachers will tell you that they have learned more by trying to teach something than they did simply by studying it in school.

Here are some suggestions to make your presentation go well:

► Work with a group. Acting as a panel member might make you less nervous than giving a talk on your own. Be sure the role of each panel member is clear so that the work and the credit are equally shared.

► Choose the topic carefully. You can summarize the results of an investigation, do research for a project and present what you’ve learned and how it connects to the chapter, or give your own thinking on Take Another Look or Improving Your Reasoning Skills.

► Prepare thoroughly. Outline your presentation and think about what you have to say on each point. Decide how much detail to give, but don’t try to memorize whole sentences. Illustrate your presentation with models, a poster, a handout, or overhead transparencies. Prepare these visual aids ahead of time and decide when to introduce them.

► Speak clearly. Practice talking loudly and clearly. Show your interest in the subject. Don’t hide behind a poster or the projector. Look at the listeners when you talk.

Here are other ways to assess what you’ve learned:

UPDATE YOUR PORTFOLIO Choose a piece of work you did in this chapter to add to your portfolio—your graph from the investigation On the Road Again (Lesson 3.2), the most complicated equation you’ve solved, or your research on a project.

WRITE IN YOUR JOURNAL What method for solving equations do you like best? Do you always remember to define variables before you graph or write an equation? How are you doing in algebra generally? What things don’t you understand?

ORGANIZE YOUR NOTEBOOK You might need to update your notebook with examples of balancing to solve an equation, or with notes about how to trace a line or search a table to approximate the coordinates of the solution. Be sure you understand the meanings of important words like linear equation, rate of change, and intercept form.