Weavers repeat steps when they make baskets and mats, creating patterns of repeating shapes. This process is not unlike recursion. In the top photo, a mat weaver in Myanmar creates a traditional design with palm fronds. The bottom photo shows bowls crafted by Native American artisans.

**OBJECTIVES**

In this chapter you will
- write recursive routines emphasizing start plus change
- study rate of change
- learn to write equations for lines using a starting value and a rate of change
- use equations and tables to graph lines
- solve linear equations
Recursive Sequences

The Empire State Building in New York City has 102 floors and is 1250 ft high. How high up are you when you reach the 80th floor? You can answer this question using a recursive sequence. In this lesson you will learn how to analyze geometric patterns, complete tables, and find missing values using numerical sequences.

A recursive sequence is an ordered list of numbers defined by a starting value and a rule. You generate the sequence by applying the rule to the starting value, then applying it to the resulting value, and repeating this process.

The table shows heights above and below ground at different floor levels in a 25-story building. Write a recursive routine that provides the sequence of heights \(-4, 9, 22, 35, \ldots, 217, \ldots\) that corresponds to the building floor numbers 0, 1, 2, \ldots. Use this routine to find each missing value in the table.

<table>
<thead>
<tr>
<th>Floor number</th>
<th>Basement (0)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>10</th>
<th>\ldots</th>
<th>\ldots</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (ft)</td>
<td>(-4)</td>
<td>9</td>
<td>22</td>
<td>35</td>
<td>\ldots</td>
<td>\ldots</td>
<td>217</td>
<td>\ldots</td>
<td></td>
</tr>
</tbody>
</table>

**Solution**

The starting value is \(-4\) because the basement is 4 ft below ground level. Each floor is 13 ft higher than the floor below it, so the rule for finding the next floor height is “add 13 to the current floor height.”

The calculator screen shows how to enter this recursive routine into your calculator. Press \(-4\) \(\text{ENTER}\) to start your number sequence. Press \(+13\) \(\text{ENTER}\). The calculator automatically displays \(\text{Ans} + 13\) and computes the next value. Simply pressing \(\text{ENTER}\) again applies the rule for finding successive floor heights. \(\text{See Calculator Note 0D.}\)

You can see that the 4th floor is at 48 ft.

How high up is the 10th floor? Count the number of times you press \(\text{ENTER}\) until you reach 10. Which floor is at a height of 217 ft? Keep counting until you see that value on your calculator screen. What’s the height of the 25th floor? Keep applying the rule by pressing \(\text{ENTER}\) and record the values in your table.

The 10th floor is at 126 ft, the 17th floor is at 217 ft, and the 25th floor is at 321 ft.
Investigation
Recursive Toothpick Patterns

In this investigation you will learn to create and apply recursive sequences by modeling them with puzzle pieces made from toothpicks.

Consider this pattern of triangles.

Step 1
Make Figures 1–3 of the pattern using as few toothpicks as possible. How many toothpicks does it take to reproduce each figure? How many toothpicks lie on the perimeter of each figure?

Step 2
Copy the table with enough rows for six figures of the pattern. Make Figures 4–6 from toothpicks by adding triangles in a row and complete the table.

<table>
<thead>
<tr>
<th>Number of toothpicks</th>
<th>Perimeter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 1</td>
<td></td>
</tr>
<tr>
<td>Figure 2</td>
<td></td>
</tr>
</tbody>
</table>

Step 3
What is the rule for finding the number of toothpicks in each figure? What is the rule for finding the perimeter? Use your calculator to create recursive routines for these rules. Check that these routines generate the numbers in your table.

Step 4
Now make Figure 10 from toothpicks. Count the number of toothpicks and find the perimeter. Does your calculator routine give the same answers? Find the number of toothpicks and the perimeter for Figure 25.

Next you'll see what sequences you can generate with a new pattern.

Step 5
Design a pattern using a row of squares, instead of triangles, with your toothpicks. Repeat Steps 1–4 and answer all the questions with the new design.

Step 6
Choose a unit of measurement and explain how to calculate the area of a square made from toothpicks. How does your choice of unit affect calculations for the areas of each figure?
Now you’ll create your own puzzle piece from toothpicks. Add identical pieces in one direction to make the succeeding figures of your design.

Step 7
Draw Figures 1–3 on your paper. Write recursive routines to generate number sequences for the number of toothpicks, perimeter, and area of each of six figures. Record these numbers in a table. Find the values for a figure made of ten puzzle pieces.

Step 8
Write three questions about your pattern that require recursive sequences to answer. For example: What is the perimeter if the area is 20? Test your questions on your classmates.

In the investigation you wrote number sequences in table columns. Remember that you can also display sequences as a list of numbers like this:

1, 3, 5, 7, . . .

Each number in the sequence is called a term. The three periods indicate that the numbers continue.

**EXAMPLE B**

Find the missing values in each sequence.

**a.** 7, 12, 17, __, 27, __, __, 42, __, 52

**b.** 5, 1, −3, __, −11, −15, __, __, −27, __

**c.** −7, __, −29, __, −51, −62, __, −84, __

**d.** 2, −4, 8, −16, 32, __, 128, −256, __, __

How many hidden numbers can you find?

**Solution**

For each sequence, identify the starting value and the operation that must be performed to get the next term.

**a.** The starting value is 7 and you add 5 each time to get the next number. The missing numbers are shown in red.

```
+ 5 + 5 + 5 + 5 + 5 + 5 + 5
7, 12, 17, 22, 27, 32, 37, 42, 47, 52
```

**b.** The starting value is 5 and you subtract 4 each time to get the next number. The missing numbers are shown in red.

```
− 4 − 4 − 4 − 4 − 4 − 4 − 4 − 4
5, 1, −3, −7, −11, −15, −19, −23, −27, −31
```
c. The starting value is \(-7\). The difference between the fifth and sixth terms shows that you subtract 11 each time.

Starting value:

\[-7, -18, -29, -40, -51, -62, -73, -84, -95\]

d. Adding or subtracting numbers does not generate this sequence. Notice that the numbers double each time. Also, they switch between positive and negative signs. So the rule is to multiply by \(2\). Multiply 32 by \(2\) to get the first missing value of 64. The last missing values are 512 and \(-1024\).

Starting value:

\[2, -4, 8, -16, 32, -64, 128, -256, 512, -1024\]

---

**EXERCISES**

You will need your graphing calculator for Exercises 2, 5, and 7.

### Practice Your Skills

**1.** Evaluate each expression without using your calculator. Then check your result with your calculator.
   
   a. \(-2(5 - 9) + 7\)
   
   b. \((-4)(-8) \div -5 + 3\)
   
   c. \(\frac{5 + (-6)(-5)}{-7}\)

**2.** Consider the sequence of figures made from a row of pentagons.

![Figures](image)

a. Copy and complete the table for five figures.

<table>
<thead>
<tr>
<th>Figure number</th>
<th>Perimeter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

b. Write a recursive routine to find the perimeter of each figure. Assume each side is 1 unit long.

c. Find the perimeter of Figure 10.

d. Which figure has a perimeter of 47?

**3.** Find the first six values generated by the recursive routine

\[-14.2, \text{ ENTER}, 3.7, \text{ ENTER}, \ldots\]

**4.** Write a recursive routine to generate each sequence. Then use your routine to find the 10th term of the sequence.

a. 3, 9, 15, 21, \ldots

b. 1.7, 1.2, 0.7, 0.2, \ldots

c. \(-3, 6, -12, 24, \ldots\)

d. 384, 192, 96, 48, \ldots
5. APPLICATION In the Empire State Building the longest elevator shaft reaches the 86th floor, 1050 ft above ground level. Another elevator takes visitors from the 86th floor to the observation area on the 102nd floor, 1224 ft above ground level. For more information about the Empire State Building, see www.keymath.com/DA.

a. Write a recursive routine that gives the height above ground level for each of the first 86 floors. Tell what the starting value and the rule mean in terms of the building.

b. Write a recursive routine that gives the heights of floors 86 through 102. Tell what the starting value and the rule mean in this routine.

c. When you are 531 ft above ground level, what floor are you on?

d. When you are on the 90th floor, how high up are you? When you are 1137 ft above ground level, what floor are you on?

6. The diagram at right shows a sequence of gray and white squares each layered under the previous one.

a. Explain how the sequence 1, 3, 5, 7, . . . is related to the areas of these squares.

b. Write a recursive routine that gives the sequence 1, 3, 5, 7, . . .

c. Use your routine to predict the number of additional unit squares you would need to enlarge this diagram by one additional row and column. Explain how you found your answers.

d. What is the 20th number in the sequence 1, 3, 5, 7, . . .?

e. The first term in the sequence is 1, and the second is 3. Which term is the number 95? Explain how you found your answer.

7. Imagine a tilted L-shaped puzzle piece made from 8 toothpicks. Its area is 3 square units. Add puzzle pieces in the corner of each “L” to form successive figures of the design. In a second figure, the two pieces “share” two toothpicks so that there are 14 toothpicks instead of 16.

a. As you did in the investigation, make a table with enough columns and rows for the number of toothpicks, perimeter, and area of each of six figures.

b. Write a recursive routine that will produce the number sequence in each column of the table.

c. Find the number of toothpicks, perimeter, and area of Figure 10.

d. Find the perimeter and area of the figure made from 152 toothpicks.
8. **APPLICATION** The table gives some floor heights in a building.

<table>
<thead>
<tr>
<th>Floor</th>
<th>...</th>
<th>−1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>...</th>
<th>...</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (m)</td>
<td>...</td>
<td>−3</td>
<td>1</td>
<td>5</td>
<td>9</td>
<td>...</td>
<td>37</td>
<td>...</td>
</tr>
</tbody>
</table>

a. How many meters are between the floors in this building?
b. Write a recursive routine that will give the sequence of floor heights if you start at the 25th floor and go to the basement (floor 0). Which term in your sequence represents the height of the 7th floor? What is the height?
c. How many terms are in the sequence in 8b?
d. Floor “−1” corresponds to the first level of the parking substructure under the building. If there are five parking levels, how far underground is level 5?

9. Consider the sequence __, −4, 8, __, 32, . . . .
a. Find two different recursive routines that could generate these numbers.
b. For each routine, what are the missing numbers? What are the next two numbers?
c. If you want to generate this number sequence with exactly one routine, what more do you need?

10. Positive multiples of 7 are generally listed as 7, 14, 21, 28, . . . .
a. If 7 is the 1st multiple of 7 and 14 is the 2nd multiple, then what is the 17th multiple?
b. How many multiples of 7 are between 100 and 200?
c. Compare the number of multiples of 7 between 100 and 200 with the number between 200 and 300. Does the answer make sense? Do all intervals of 100 have this many multiples of 7? Explain.
d. Describe two different ways to generate a list containing multiples of 7.

11. Some babies gain an average of 1.5 lb per month during the first 6 months after birth.
a. Write a recursive routine that will generate a table of monthly weights for a baby weighing 6.8 lb at birth.
b. Write a recursive routine that will generate a table of monthly weights for a baby weighing 7.2 lb at birth.
c. How are the routines in 11a and 11b the same? How are they different?
d. Copy and complete the table of data for this situation.

<table>
<thead>
<tr>
<th>Age (mo)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight of Baby A (lb)</td>
<td>6.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weight of Baby B (lb)</td>
<td>7.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
12. Write recursive routines to help you answer 12a–d.
   a. Find the 9th term of 1, 3, 9, 27, . . . .
   b. Find the 123rd term of 5, −5, 5, −5, . . . .
   c. Find the term number of the first positive term of the sequence −16.2, −14.8,
      −13.4, −12, . . . .
   d. Which term is the first to be either greater than 100 or less than −100 in the
      sequence −1, 2, −4, 8, −16, . . . ?

Review

13. The table gives the normal monthly precipitation for three cities in the
    United States.
    a. Display the data in three box plots, one for each city, and use them to
       compare the precipitation for the three cities.
    b. What information do you lose by displaying the data in a box plot? What type of
       graph might be more helpful for displaying the data?

<table>
<thead>
<tr>
<th>Precipitation for Three Cities</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Precipitation (in.)</td>
<td></td>
</tr>
<tr>
<td>Month</td>
<td>Portland, Oregon</td>
</tr>
<tr>
<td>January</td>
<td>5.4</td>
</tr>
<tr>
<td>February</td>
<td>3.9</td>
</tr>
<tr>
<td>March</td>
<td>3.6</td>
</tr>
<tr>
<td>April</td>
<td>2.4</td>
</tr>
<tr>
<td>May</td>
<td>2.1</td>
</tr>
<tr>
<td>June</td>
<td>1.5</td>
</tr>
<tr>
<td>July</td>
<td>0.7</td>
</tr>
<tr>
<td>August</td>
<td>1.1</td>
</tr>
<tr>
<td>September</td>
<td>1.8</td>
</tr>
<tr>
<td>October</td>
<td>2.7</td>
</tr>
<tr>
<td>November</td>
<td>5.3</td>
</tr>
<tr>
<td>December</td>
<td>6.1</td>
</tr>
</tbody>
</table>


It's a rainy day in Portland, Oregon.

14. Create an undo table and solve the equation listed by undoing the order of operations.

<table>
<thead>
<tr>
<th>Equation: $8 + 3(x - 5) = -14.8$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Description</strong></td>
</tr>
<tr>
<td>Pick x.</td>
</tr>
</tbody>
</table>
In this lesson you will learn that the starting value and the rule of a recursive sequence take on special meaning in certain real-world situations. When you add or subtract the same number each time in a recursive routine, consecutive terms change by a constant amount. Using your calculator, you will see how the starting value and rule let you generate data for tables quickly. You will also plot these data sets and learn that the starting value and rule relate to characteristics of the graph.

You walk into an elevator in the basement of a building. Its control panel displays “0” for the floor number. As you go up, the numbers increase one by one on the display, and the elevator rises 13 ft for each floor. The table shows the floor numbers and their heights above ground level.

a. Write recursive routines for the two number sequences in the table. Enter both routines into calculator lists.

b. Define variables and plot the data in the table for the first few floors of the building. Does it make sense to connect the points on the graph?

c. What is the highest floor with a height less than 200 ft? Is there a floor that is exactly 200 ft high?

The starting value for the floor numbers is 0, and the rule is to add 1. The starting value for the height is –4, and the rule is to add 13. You can generate both number sequences on the calculator using lists.

a. Press {0, –4} and press ENTER to input both starting values at the same time. To use the rules to get the next term in the sequence, press {Ans(1) + 1, Ans(2) + 13} ENTER. [See Calculator Note 3A.]

These commands tell the calculator to add 1 to the first term in the list and to add 13 to the second number. Press ENTER again to compute the next floor number and its corresponding height as the elevator rises.

Many elevators use Braille symbols. This alphabet for the blind was developed by Louis Braille (1809–1852). For more information about Braille, see the links at www.keymath.com/DA.
b. Let \( x \) represent the floor number and \( y \) represent the floor’s height in feet. Mark a scale from 0 to 5 on the \( x \)-axis and \(-10\) to \( 50 \) on the \( y \)-axis. Plot the data from the table. The graph starts at \((0, -4)\) on the \( y \)-axis. The points appear to be in a line. It does not make sense to connect the points because it is not possible to have a decimal or fractional floor number.

c. The recursive routine generates the points \((0, -4), (1, 9), (2, 22), \ldots, (15, 191), (16, 204), \ldots\). The height of the 15th floor is 191 ft. The height of the 16th floor is 204 ft. So the 15th floor is the highest floor with a height less than 200 ft. No floor is exactly 200 ft high.

Notice that to get to the next point on the graph from any given point, move right 1 unit on the \( x \)-axis and up 13 units on the \( y \)-axis. The points you plotted in the example showed a linear relationship between floor numbers and their heights. In what other graphs have you seen linear relationships?

### Investigation

**On the Road Again**

A green minivan starts at the Mackinac Bridge and heads south for Flint on Highway 75. At the same time, a red sports car leaves Saginaw and a blue pickup truck leaves Flint. The car and the pickup are heading for the bridge. The minivan travels 72 mi/h. The pickup travels 66 mi/h. The sports car travels 48 mi/h.

When and where will they pass each other on the highway? In this investigation you will learn how to use recursive sequences to answer questions like these.

---

**You will need**
- the worksheet On the Road Again Grid

---

**Step 1** Find each vehicle’s average speed in miles per minute (mi/min).

**Step 2** Write recursive routines to find each vehicle’s distance from Flint at each minute. What are the real-world meanings of the starting value and the rule in each routine? Use calculator lists.
Step 3 Make a table to record the highway distance from Flint for each vehicle. After you complete the first few rows of data, change your recursive routines to use 10 min intervals for up to 4 h.

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Minivan (mi)</th>
<th>Sports car (mi)</th>
<th>Pickup (mi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Step 4 Define variables and plot the information from the table onto a graph. Mark and label each axis in 10-unit intervals, with time on the horizontal axis. Using a different color for each vehicle, plot its \((time, distance)\) coordinates.

Step 5 On the graph, do the points for each vehicle seem to fall on a line? Does it make sense to connect each vehicle’s points in a line? If so, draw the line. If not, explain why not.

Use your graph and table to find the answers for Steps 6–10.

Step 6 Where does the starting value for each routine appear on the graph? How does the recursive rule for each routine affect the points plotted?

Step 7 Which line represents the minivan? How can you tell?

Step 8 Where are the vehicles when the minivan meets the first one headed north?

Step 9 How can you tell by looking at the graph whether the pickup or the sports car is traveling faster? When and where does the pickup pass the sports car?

Step 10 Which vehicle arrives at its destination first? How many minutes pass before the second and third vehicles arrive at their destinations? How can you tell by looking at the graph?

Step 11 What assumptions about the vehicles are you making when you answer the questions in the previous steps?
Consider how to model this situation more realistically. What if the vehicles are traveling at different speeds? What if one driver stops to get gas or a bite to eat? What if the vehicles’ speeds are not constant? Discuss how these questions affect the recursive routines, tables of data, and their graphs.

You will need your graphing calculator for Exercises 4–7 and 9.

### Practice Your Skills

1. Decide whether each expression is positive or negative without using your calculator. Then check your answer with your calculator.
   - a. \(-35(44) + 23\)
   - b. \((-14)(-36) - 32\)
   - c. \(25 - \frac{152}{12}\)
   - d. \(50 - 23(-12)\)
   - e. \(-12 - 38\)
   - f. \(24(15 - 76)\)

2. List the terms of each number sequence of \(y\)-coordinates for the points shown on each graph. Then write a recursive routine to generate each sequence.
   - a. \(y\) values for graph:
     - \(0, 5\)
     - \(5, 0\)
   - b. \(y\) values for graph:
     - \(0, 5\)
     - \(5, 0\)
   - c. \(y\) values for graph:
     - \(-2, 3\)
     - \(3, -2\)
   - d. \(y\) values for graph:
     - \(0, 5\)
     - \(5, 0\)

3. Make a table listing the coordinates of the points plotted in 2b and d.

4. Plot the first five points represented by each recursive routine in 4a and b on separate graphs. Then answer 4c and d.
   - a. \{0, 5\} \(\text{ENTER}\)
     - \{Ans(1) + 1, Ans(2) + 7\} \(\text{ENTER}\); \(\text{ENTER}\), ...
   - b. \{0, -3\} \(\text{ENTER}\)
     - \{Ans(1) + 1, Ans(2) - 6\} \(\text{ENTER}\); \(\text{ENTER}\), ...

[\(\text{You can use the Dynamic Algebra Exploration found at } \text{www.keymath.com/DA} \text{ to further explore the situation described in the investigation. }\)\]
c. On which axis does each starting point lie? What is the \(x\)-coordinate of each starting point?

d. As the \(x\)-value increases by 1, what happens to the \(y\)-coordinates of the points in each sequence in 4a and b? 

5. The direct variation \(y = 2.54x\) describes the relationship between two standard units of measurement where \(y\) represents centimeters and \(x\) represents inches.

a. Write a recursive routine that would produce a table of values for any whole number of inches. Use a calculator list.

b. Use your routine to complete the missing values in this table.

<table>
<thead>
<tr>
<th>Inches</th>
<th>Centimeters</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2.54</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>35.56</td>
</tr>
</tbody>
</table>

Reason and Apply

6. APPLICATION A car is moving at a speed of 68 mi/h from Dallas toward San Antonio. Dallas is about 272 mi from San Antonio.

a. Write a recursive routine to create a table of values relating time to distance from San Antonio for 0 to 5 h in 1 h intervals.

b. Graph the information in your table.

c. What is the connection between your plot and the starting value in your recursive routine?

d. What is the connection between the coordinates of any two consecutive points in your plot and the rule of your recursive routine?

e. Draw a line through the points of your plot. What is the real-world meaning of this line? What does the line represent that the points alone do not?

f. When is the car within 100 mi of San Antonio? Explain how you got your answer.

g. How long does it take the car to reach San Antonio? Explain how you got your answer.

7. APPLICATION A long-distance telephone carrier charges $1.38 for international calls of 1 minute or less and $0.36 for each additional minute.

a. Write a recursive routine using calculator lists to find the cost of a 7-minute phone call.

b. Without graphing the sequence, give a verbal description of the graph showing the costs for calls that last whole numbers of minutes. Include in your description all the important values you need in order to draw the graph.
8. These tables show the changing depths of two submarines as they come to the surface.

**USS Alabama**

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth (ft)</td>
<td>−38</td>
<td>−31</td>
<td>−24</td>
<td>−17</td>
<td>−10</td>
<td>−3</td>
<td>4</td>
</tr>
</tbody>
</table>

**USS Dallas**

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth (ft)</td>
<td>−48</td>
<td>−40</td>
<td>−32</td>
<td>−24</td>
<td>−16</td>
<td>−8</td>
<td>0</td>
</tr>
</tbody>
</table>

a. Graph the data from both tables on the same set of coordinate axes.
b. Describe what you found by graphing the data. How are the graphs the same? How are they different?
c. Does it make sense to draw a line through each set of points? Explain what these lines mean.
d. What is the real-world meaning of the point (30, 4) for the USS Alabama?

9. Each geometric design is made from tiles arranged in a row.

```
Tile Edges on the Perimeter
Number of tiles | Triangle | Rhombus | Pentagon | Hexagon
1               | 3        | 4       | 5        | 6
2               |          |         |          |
3               |          |         |          |
```

a. Make a table like the one shown. Find the number of tile edges on the perimeter of each design, and fill in ten rows of the table. Look for patterns as you add more tiles.
b. Write a recursive routine to generate the values in each table column.
c. Find the perimeter of a 50-tile design for each shape.
d. Draw four plots on the same coordinate axes using the information for designs of one to ten tiles of each shape. Use a different color for each shape. Put the number of tiles on the horizontal axis and the number of edges on the vertical axis. Label and scale each axis.

e. Compare the four scatter plots. How are they alike, and how are they different?
f. Would it make sense to draw a line through each set of points? Explain why or why not.
10. A bicyclist, 1 mi (5280 ft) away, pedals toward you at a rate of 600 ft/min for 3 min. The bicyclist then pedals at a rate of 1000 ft/min for the next 5 min.
   a. Describe what you think the plot of \((\text{time, distance from you})\) will look like. @
   b. Graph the data using 1 min intervals for your plot. @
   c. Invent a question about the situation, and use your graph to answer the question.

**Review**

11. Consider the expression 
   \[
   \frac{5.4 + 3.2(x - 2.8)}{1.2} - 2.3
   \]
   a. Use the order of operations to find the value of the expression if \(x = 7.2\).
   b. Set the expression equal to 3.8. Solve for \(x\) by undoing the sequence of operations you listed in 11a.

12. Isaac learned a way to convert from degrees Celsius to Fahrenheit. He adds 40 to the Celsius temperature, multiplies by 9, divides by 5, and then subtracts 40.
   a. Write an expression for Isaac’s conversion method. @
   b. Write the steps to convert from Fahrenheit to Celsius by undoing Isaac’s method. @
   c. Write an expression for the conversion in 12b.

13. **APPLICATION** Karen is a U.S. exchange student in Austria. She wants to make her favorite pizza recipe for her host family, but she needs to convert the quantities to the metric system. Instead of using cups for flour and sugar, her host family measures dry ingredients in grams and liquid ingredients in liters. Karen has read that 4 cups of flour weigh 1 pound.
   In her dictionary, Karen looks up conversion factors and finds that 1 ounce \(\approx 28.4\) grams, 1 pound \(\approx 454\) grams, and 1 cup \(\approx 0.236\) liter.
   a. Karen’s recipe calls for \(\frac{3}{2}\) cup water and \(1\frac{1}{2}\) cups flour. Convert these quantities to metric units.
   b. Karen’s recipe says to bake the pizza at 425°. Convert this temperature to degrees Celsius. Use your work in Exercise 12 to help you.

14. Draw and label a coordinate plane with each axis scaled from \(-10\) to 10.
   a. Represent each point named with a dot, and label it using its letter name.
   \[
   A(3, -2) \quad B(-8, 1.5) \quad C(9, 0) \quad D(-9.5, -3) \quad E(7, -4) \quad F(1, -1) \quad G(0, -6.5) \quad H(2.5, 3) \quad I(-6, 7.5) \quad J(-5, -6)
   \]
   b. List the points in Quadrant I, Quadrant II, Quadrant III, and Quadrant IV. Which points are on the \(x\)-axis? Which points are on the \(y\)-axis?
   c. Explain how to tell which quadrant a point will be in by looking at the coordinates. Explain how to tell if a point lies on one of the axes.
Time-Distance Relationships

Modeling time-distance relationships is one very useful application of algebra. You began working with this topic in Lesson 3.2. In this lesson you will explore time-distance relationships in more depth by considering various walking scenarios. You’ll learn how the starting position, speed, direction, and final position of a walker influence a graph and an equation.

The \((\text{time, distance})\) graphs below provide a lot of information about the “walks” they picture. The fact that the lines are straight and increasing means that both walkers are moving away from the motion sensor at a steady rate. The first walker starts 0.5 meter from the sensor, whereas the second walker starts 1 meter from the sensor. The first graph pictures a walker moving \(4.5 - 0.5 = 4\) meters in \(4 - 0 = 4\) seconds, or 1 meter per second. The second walker covers \(3 - 1 = 2\) meters in \(4 - 0 = 4\) seconds, or 0.5 meter per second.

In this investigation you’ll analyze time-distance graphs, and you’ll use a motion sensor to create your own graphs.

Investigation
Walk the Line

Imagine that you have a 4-meter measuring tape positioned on the floor. A motion sensor measures your distance from the tape’s 0-mark as you walk, and it graphs the information. On the calculator graphs shown here, the horizontal axis shows time from 0 to 6 seconds and the vertical axis shows distance from 0 to 4 meters.
Step 1 | Write a set of walking instructions for each graph. Tell where the walk begins, how fast the person walks, and whether the person walks toward or away from the motion sensor located at the 0-mark.

Step 2 | Graph a 6-second walk based on each set of walking instructions or data.

a. Start at the 2.5-meter mark and stand still.

b. Start at the 3-meter mark and walk toward the sensor at a constant rate of 0.4 meter per second.

c. 

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (m)</td>
<td>0.8</td>
<td>1.0</td>
<td>1.2</td>
<td>1.4</td>
<td>1.6</td>
<td>1.8</td>
<td>2.0</td>
</tr>
</tbody>
</table>

Step 3 | Write a recursive routine for the table in Step 2c.

For the next part of the investigation, you will need a graphing calculator and a motion sensor. Your group will need a space about 4 meters long and 1.5 meters wide (13 feet by 5 feet). Tape to the floor a 4-meter measuring tape or four metersticks end-to-end. Assign these tasks among your group members: walker, motion-sensor holder, coach, and timer.

Step 4 | Your group will try to create the graph shown in Step 1, graph a. Remember that you wrote walking directions for this graph. Use your motion sensor to record the walker’s motion. [See Calculator Note 3B for help using the motion sensor.] After each walk, discuss what you could have done to better replicate the graph. Repeat the walk until you have a good match for graph a.

Step 5 | Rotate jobs, and repeat Step 4 to model graphs b and c from Step 1 and the three descriptions from Step 2.

Using motion-sensor technology in the investigation, you were able to actually see how accurately you duplicated a given walk. The next examples will provide more practice with time-distance relationships.

**EXAMPLE A**

a. Graph a walk from the set of instructions “Start at the 0.5-meter mark and walk at a steady 0.25 meter per second for 6 seconds.”

b. Write a set of walking instructions based on the table data, and then sketch a graph of the walk.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (m)</td>
<td>4.0</td>
<td>3.6</td>
<td>3.2</td>
<td>2.8</td>
<td>2.4</td>
<td>2.0</td>
<td>1.6</td>
</tr>
</tbody>
</table>
Think about where the walker starts and how much distance he or she will cover in a given amount of time.

a. Walking at a steady rate of 0.25 meter per second for 6 seconds means the walker will move $0.25 \times 6 = 1.5$ m. The walker starts at 0.5 m and ends at $0.5 + 1.5 = 2$ m.

b. Walking instructions: “Start at the 4-meter mark and walk toward the sensor at 0.4 meter per second.” You can graph this walk by plotting the data points given.

**EXERCISES**

**Practice Your Skills**

1. Write a recursive routine for the table in Example A, part b.

2. Sketch a graph of a walk starting at the 1-meter mark and walking away from the sensor at a constant rate of 0.5 meter per second.

3. Write a set of walking instructions and sketch a graph of the walk described by {0, 0.8} and {Ans(1) + 1, Ans(2) + 0.2}.

4. Describe the walk shown in each graph. Include where it started and how quickly and in what direction the walker moved.

   a.

   b.
5. Describe the walk represented by the data in each table.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Distance (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td>5.8</td>
</tr>
<tr>
<td>2</td>
<td>5.6</td>
</tr>
<tr>
<td>3</td>
<td>5.4</td>
</tr>
<tr>
<td>4</td>
<td>5.2</td>
</tr>
<tr>
<td>5</td>
<td>5.0</td>
</tr>
<tr>
<td>6</td>
<td>4.8</td>
</tr>
</tbody>
</table>

b. | Time (s) | Distance (m) |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1.6</td>
</tr>
<tr>
<td>2</td>
<td>2.2</td>
</tr>
<tr>
<td>3</td>
<td>2.8</td>
</tr>
<tr>
<td>4</td>
<td>3.4</td>
</tr>
<tr>
<td>5</td>
<td>4.0</td>
</tr>
<tr>
<td>6</td>
<td>4.6</td>
</tr>
</tbody>
</table>

6. Which graph better represents a walk in which the walker starts 2 m from the motion sensor and walks away from it at a rate of 0.25 m/s for 6 s? Explain.

7. At what rate in ft/s would you walk so that you were moving at a constant speed of 1 mi/h?

8. The time-distance graph shows Carol walking at a steady rate. Her partner used a motion sensor to measure her distance from a given point.
   a. According to the graph, how much time did Carol spend walking?
   b. Was Carol walking toward or away from the motion sensor? Explain your thinking.
   c. Approximately how far away from the motion sensor was she when she started walking?
   d. If you know Carol is 2.9 m away from the motion sensor after 4 s, how fast was she walking?
   e. If the equipment will measure distances only up to 6 m, how many seconds of data can be collected if Carol continues walking at the same rate?
   f. Looking only at the graph, how do you know that Carol was neither speeding up nor slowing down during her walk?

9. Draw a scatter plot on your paper picturing (time, distance) at 1 s intervals if you start timing Carol’s walk as she walks toward her partner starting at a distance of 5.9 m and moving at a constant speed of 0.6 m/s.
10. Describe how the rate affects the graph of each situation.
   a. The graph of a person walking toward a motion sensor.
   b. The graph of a person standing still.
   c. The graph of a person walking slowly.

11. Match each calculator Answer routine to a graph.
   a. 2.5
      Ans + 0.5, ENTER, ENTER, ...

   b. 1.0
      Ans + 1.0, ENTER, ENTER, ...

   c. 2.0
      Ans + 1.0, ENTER, ENTER, ...

   d. 2.5
      Ans - 0.5, ENTER, ENTER, ...

12. Describe how you would instruct someone to walk the line \( y = x \), where \( x \) is measured in seconds and \( y \) is measured in feet. Describe how to walk the line \( y = x \), where \( x \) is measured in seconds and \( y \) is measured in meters. Which line represents a faster rate? Explain.

13. For each situation, determine if it is possible to collect such walking data and either describe how to collect it or explain why it is not possible.

   a. 
   b. 
   c. 
Review

14. Solve each proportion for $x$.
   a. $\frac{x}{3} = \frac{7}{5}$
   b. $\frac{2}{x} = \frac{9}{11}$
   c. $\frac{x}{c} = \frac{d}{e}$

15. On his Man in Motion World Tour in 1987, Canadian Rick Hansen wheeled himself 24,901.55 miles to support spinal cord injury research and rehabilitation, and wheelchair sport. He covered 4 continents and 34 countries in two years, two months, and two days. Learn more about Rick’s journey with the link at [www.keymath.com/DA](http://www.keymath.com/DA).
   a. Find Rick’s average rate of travel in miles per day. (Assume there are 365 days in a year and 30.4 days in a month.)
   b. How much farther would Rick have traveled if he had continued his journey for another $1\frac{1}{2}$ years?
   c. If Rick continued at this same rate, how many days would it take him to travel 60,000 miles? How many years is that?

16. APPLICATION Nicholai’s car burns 13.5 gallons of gasoline every 175 miles.
   a. What is the car’s fuel consumption rate?
   b. At this rate, how far will the car go on 5 gallons of gas?
   c. How many gallons does Nicholai’s car need to go 100 miles?

---

**PASCAL’S TRIANGLE**

The first five rows of Pascal’s triangle are shown.

```
1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
```

The triangle can be generated recursively. The sides of the triangle are 1’s, and each number inside the triangle is the sum of the two diagonally above it.

Complete the next five rows of Pascal’s triangle. Research its history and practical application. What is the connection between Sierpiński’s triangle and Pascal’s triangle? Can you find the sequence of triangular numbers in Pascal’s triangle? What is its connection to the Fibonacci number sequence? Present your findings in a paper or a poster.

What became known as Pascal’s triangle was first published in *Siyuan yujian xicao* by Zhu Shijie in 1303. This ancient version actually has one error. Can you find it?
Linear Equations and the Intercept Form

So far in this chapter you have used recursive routines, graphs, and tables to model linear relationships. In this lesson you will learn to write linear equations from recursive routines. You’ll begin to see some common characteristics of linear equations and their graphs, starting with the relationship between exercise and calorie consumption.

Different physical activities cause people to burn calories at different rates depending on many factors such as body type, height, age, and metabolism. Coaches and trainers consider these factors when suggesting workouts for their athletes.

Investigation
Working Out with Equations

Manisha starts her exercise routine by jogging to the gym. Her trainer says this activity burns 215 calories. Her workout at the gym is to pedal a stationary bike. This activity burns 3.8 calories per minute.

First you’ll model this scenario with your calculator.

<table>
<thead>
<tr>
<th>Pedaling time (min)</th>
<th>Total calories burned</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>( y' )</td>
</tr>
<tr>
<td>0</td>
<td>215</td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td></td>
</tr>
<tr>
<td>45</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td></td>
</tr>
</tbody>
</table>

Step 1 Use calculator lists to write a recursive routine to find the total number of calories Manisha has burned after each minute she pedals the bike. Include the 215 calories she burned on her jog to the gym.

Step 2 Copy and complete the table using your recursive routine.

Step 3 After 20 minutes of pedaling, how many calories has Manisha burned? How long did it take her to burn 443 total calories?
Next you'll learn to write an equation that gives the same values as the calculator routines.

**Step 4**
Write an expression to find the total calories Manisha has burned after 20 minutes of pedaling. Check that your expression equals the value in the table.

**Step 5**
Write and evaluate an expression to find the total calories Manisha has burned after pedaling 38 minutes. What are the advantages of this expression over a recursive routine?

**Step 6**
Let \( x \) represent the pedaling time in minutes, and let \( y \) represent the total number of calories Manisha burns. Write an equation relating time to total calories burned.

**Step 7**
Check that your equation produces the corresponding values in the table.

Now you'll explore the connections between the linear equation and its graph.

**Step 8**
Plot the points from your table on your calculator. Then enter your equation into the \( Y = \) menu. Graph your equation to check that it passes through the points. Give two reasons why drawing a line through the points realistically models this situation. [► See Calculator Note 1J to review how to plot points and graph an equation. ]

**Step 9**
Substitute 538 for \( y \) in your equation to find the elapsed time required for Manisha to burn a total of 538 calories. Explain your solution process. Check your result.

**Step 10**
How do the starting value and the rule of your recursive routine show up in your equation? How do the starting value and the rule of your recursive routine show up in your graph? When is the starting value of the recursive routine also the value where the graph crosses the \( y \)-axis?

The equation for Manisha's workout shows a linear relationship between the total calories burned and the number of minutes pedaling on the bike. You probably wrote this linear equation as

\[
y = 215 + 3.8x \quad \text{or} \quad y = 3.8x + 215
\]

The form \( y = a + bx \) is the intercept form. The value of \( a \) is the \textbf{y-intercept}, which is the value of \( y \) when \( x \) is zero. The intercept gives the location where the graph crosses the \( y \)-axis. The number multiplied by \( x \) is \( b \), which is called the \textbf{coefficient} of \( x \).
In the equation \( y = 215 + 3.8x \), 215 is the value of \( a \). It represents the 215 calories Manisha burned while jogging before her workout. The value of \( b \) is 3.8. It represents the rate her body burned calories while she was pedaling. What would happen if Manisha chose a different physical activity before pedaling on the stationary bike?

You can also think of direct variations in the form \( y = kx \) as equations in intercept form. For instance, Sam's trainer tells him that swimming will burn 7.8 calories per minute. When the time spent swimming is 0, the number of calories burned is 0, so \( a \) is 0 and drops out of the equation. The number of calories burned is proportional to the time spent swimming, so you can write the equation \( y = 7.8x \).

The constant of variation \( k \) is 7.8, the rate at which Sam's body burns calories while he is swimming. It plays the same role as \( b \) in \( y = a + bx \).

**EXAMPLE A**

Suppose Sam has already burned 325 calories before he begins to swim for his workout. His swim will burn 7.8 calories per minute.

a. Create a table of values for the calories Sam will burn by swimming 60 minutes and the total calories he will burn after each minute of swimming.

b. Define variables and write an equation in intercept form to describe this relationship.

c. On the same set of axes, graph the equation for total calories burned and the direct variation equation for calories burned by swimming.

d. How are the graphs similar? How are they different?

**Solution**

a. The total numbers of calories burned appear in the third column of the table. Each entry is 325 plus the corresponding entry in the middle column.

<table>
<thead>
<tr>
<th>Swimming time (min)</th>
<th>Calories burned by swimming</th>
<th>Total calories burned</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>325</td>
</tr>
<tr>
<td>1</td>
<td>7.8</td>
<td>332.8</td>
</tr>
<tr>
<td>2</td>
<td>15.6</td>
<td>340.6</td>
</tr>
<tr>
<td>20</td>
<td>156</td>
<td>481</td>
</tr>
<tr>
<td>30</td>
<td>234</td>
<td>559</td>
</tr>
<tr>
<td>45</td>
<td>351</td>
<td>676</td>
</tr>
<tr>
<td>60</td>
<td>468</td>
<td>793</td>
</tr>
</tbody>
</table>

b. Let \( y \) represent the total number of calories burned, and let \( x \) represent the number of minutes Sam spends swimming.

\[
y = 325 + 7.8x
\]
c. The direct variation equation is \( y = 7.8x \). Enter it into \( Y_1 \) on your calculator. Enter the equation \( y = 325 + 7.8x \) into \( Y_2 \). Check to see that these equations give the same values as the table by looking at the calculator table.

d. The lower line shows the calories burned by swimming and is a direct variation. The upper line shows the total calories burned. It is 325 units above the first line because, at any particular time, Sam has burned 325 more calories. Both graphs have the same value of \( b \), which is 7.8 calories per minute. The graphs are similar because both are lines with the same steepness. They are different because they have different \( y \)-intercepts.

What will different values of \( a \) in the equation \( y = a + bx \) do to the graph?

**EXAMPLE B**

A minivan is 220 mi from its destination, Flint. It begins traveling toward Flint at 72 mi/h.

a. Define variables and write an equation in intercept form for this relationship.

b. Use your equation to calculate the location of the minivan after 2.5 h.

c. Use your equation to calculate when the minivan will be 130 mi from Flint.

d. Graph the relationship and locate the points that are the solutions to parts b and c.

e. What is the real-world meaning of the rate of change in this relationship? What does the sign of the rate of change indicate?

► Solution

a. Let the independent variable, \( x \), represent the time in hours since the beginning of the trip. Let \( y \) represent the distance in miles between the minivan and Flint. The equation for the relationship is \( y = 220 - 72x \).

b. Substitute the time, 2.5 h, for \( x \).

\[
y = 220 - 72 \cdot 2.5 = 40
\]

So the minivan is 40 mi from Flint.

c. Substitute 130 mi for \( y \) and solve the equation \( 220 - 72x = 130 \).

\[
220 + -72x = 130 \quad \text{Original equation. The subtraction of 72x is written as addition of } -72x.
\]

\[
-72x = -90 \quad \text{Subtract 220 to undo the addition.}
\]

\[
x = 1.25 \quad \text{Divide by -72 to undo the multiplication.}
\]

The minivan will be 130 mi from Flint after 1.25 h. You can change 0.25 h to minutes using dimensional analysis. \( 0.25 \text{ h} \cdot \frac{60 \text{ min}}{1 \text{ h}} = 15 \text{ min} \), so you can also write the answer as 1 h 15 min.
d. Set your calculator window to 
\[0, 3.5, 1, 0, 250, 50,\]
graph the equation, and press TRACE and the arrow keys to find the points where \( x = 1.25 \) and \( x = 2.5 \).
e. The rate of change indicates the speed of the car. If it is negative, the minivan is getting closer to Flint. That is, as time increases the distance decreases. A positive rate of change would mean that the vehicle was moving away from Flint.

In linear equations it is sometimes helpful to say which variable is the input variable and which is the output variable. The horizontal axis represents the input variable, and the vertical axis represents the output variable. In Example B, the input variable, \( x \), represents time so the \( x \)-axis is labeled time, and the output variable, \( y \), represents distance so the \( y \)-axis is labeled distance. What are the input and output variables in the investigation and in Example A?

**EXERCISES**

1. Match the recursive routine in the first column with the equation in the second column.

   a. \( 3 \) \( \text{ENTER} \); \( \text{ENTER} \); \( \text{ENTER} \); \( \text{ENTER} \), \( \ldots \) (i) \( y = 4 - 3x \)
   
   Ans + 4 \( \text{ENTER} \); \( \text{ENTER} \); \( \text{ENTER} \); \( \text{ENTER} \), \( \ldots \)

   b. \( 3 \) \( \text{ENTER} \) \( \text{ENTER} \) \( \text{ENTER} \) \( \text{ENTER} \) \( \ldots \) (ii) \( y = 3 + 4x \)
   
   Ans + 3 \( \text{ENTER} \); \( \text{ENTER} \); \( \text{ENTER} \); \( \text{ENTER} \), \( \ldots \)

   c. \( -3 \) \( \text{ENTER} \); \( \text{ENTER} \); \( \text{ENTER} \); \( \text{ENTER} \), \( \ldots \) (iii) \( y = -3 - 4x \)
   
   Ans - 4 \( \text{ENTER} \); \( \text{ENTER} \); \( \text{ENTER} \); \( \text{ENTER} \), \( \ldots \)

   d. \( 4 \) \( \text{ENTER} \) \( \text{ENTER} \) \( \text{ENTER} \) \( \text{ENTER} \) \( \ldots \) (iv) \( y = 4 + 3x \)
   
   Ans - 3 \( \text{ENTER} \); \( \text{ENTER} \); \( \text{ENTER} \); \( \text{ENTER} \), \( \ldots \)

2. You can use the equation \( d = 24 - 45t \) to model the distance from a destination for someone driving down the highway, where distance \( d \) is measured in miles and time \( t \) is measured in hours. Graph the equation and use the trace function to find the approximate time for each distance given in \( 2a \) and \( b \).

   a. \( d = 16 \text{ mi} \) (i)
   
   b. \( d = 3 \text{ mi} \)
   
   c. What is the real-world meaning of 24? (ii)
   
   d. What is the real-world meaning of 45?
   
   e. Solve the equation \( 24 - 45t = 16 \).
3. You can use the equation \( d = 4.7 + 2.8t \) to model a walk in which the distance from a motion sensor \( d \) is measured in feet and the time \( t \) is measured in seconds. Graph the equation and use the trace function to find the approximate distance from a motion sensor for each time value given in 3a and b.
   a. \( t = 12 \) s
   b. \( t = 7.4 \) s
   c. What is the real-world meaning of 4.7?
   d. What is the real-world meaning of 2.8?

4. Undo the order of operations to find the \( x \)-value in each equation.
   a. \( 3(x - 5.2) + 7.8 = 14 \)
   b. \( 3.5\left(\frac{x - 8}{4}\right) = 2.8 \)

5. The equation \( y = 35 + 0.8x \) gives the distance a sports car is from Flint after \( x \) minutes.
   a. How far is the sports car from Flint after 25 minutes?
   b. How long will it take until the sports car is 75 miles from Flint? Show how to find the solution using two different methods.

### Reason and Apply

6. **APPLICATION** Louis is beginning a new exercise workout. His trainer shows him the calculator table with \( x \)-values showing his workout time in minutes. The \( Y_1 \)-values are the total calories Louis burned while running, and the \( Y_2 \)-values are the number of calories he wants to burn.
   a. Find how many calories Louis has burned before beginning to run, how many he burns per minute running, and the total calories he wants to burn.
   b. Write a recursive routine that will generate the values listed in \( Y_1 \).
   c. Use your recursive routine to write a linear equation in intercept form. Check that your equation generates the table values listed in \( Y_1 \).
   d. Write a recursive routine that will generate the values listed in \( Y_2 \).
   e. Write an equation that generates the table values listed in \( Y_2 \).
   f. Graph the equations in \( Y_1 \) and \( Y_2 \) on your calculator. Your window should show a time of up to 30 minutes. What is the real-world meaning of the \( y \)-intercept in \( Y_1 \)?
   g. Use the trace function to find the approximate coordinates of the point where the lines meet. What is the real-world meaning of this point?

7. Jo mows lawns after school. She finds that she can use the equation \( P = -300 + 15N \) to calculate her profit.
   a. Give some possible real-world meanings for the numbers \(-300\) and \(15\) and the variable \( N \).
   b. Invent two questions related to this situation and then answer them.
   c. Solve the equation \( P = -300 + 15N \) for the variable \( N \).
   d. What does the equation in 7c tell you?
8. As part of a physics experiment, June threw an object off a cliff and measured how fast it was traveling downward. When the object left June's hand, it was traveling 5 m/s, and it sped up as it fell. The table shows a partial list of the data she collected as the object fell.
   a. Write an equation to represent the speed of the object.
   b. What was the object's speed after 3 s?
   c. If it were possible for the object to fall long enough, how many seconds would pass before it reached a speed of 83.4 m/s?
   d. What limitations do you think this equation has in modeling this situation?

9. APPLICATION Manny has a part-time job as a waiter. He makes $45 per day plus tips. He has calculated that his average tip is 12% of the total amount his customers spend on food and beverages.
   a. Define variables and write an equation in intercept form to represent Manny's daily income in terms of the amount his customers spend on food and beverages.
   b. Graph this relationship for food and beverage amounts between $0 and $900.
   c. Write and evaluate an expression to find how much Manny makes in one day if his customers spend $312 on food and beverages.
   d. What amounts spent on food and beverages will give him a daily income between $105 and $120?

10. APPLICATION Paula is cross-training for a triathlon in which she cycles, swims, and runs. Before designing an exercise program for Paula, her coach consults a table listing rates for calories burned in various activities.
   a. On Monday, Paula starts her workout by biking for 30 minutes and then swimming. Write an equation for the calories she burns on Monday in terms of the number of minutes she swims.
   b. On Wednesday, Paula starts her workout by swimming for 30 minutes and then jogging. Write an equation for the number of calories she burns on Wednesday in terms of the number of minutes she jogs.
   c. On Friday, Paula starts her workout by swimming 15 minutes, then biking for 15 minutes, then running. Write an equation for the number of calories she burns on Friday in terms of the number of minutes she spends running.
   d. How many total calories does Paula burn on each day described in 10a–c if she does a 60-minute workout?
Review

11. At a family picnic, your cousin tells you that he always has a hard time remembering how to compute percents. Write him a note explaining what percent means. Use these problems as examples of how to solve the different types of percent problems, with an answer for each.
   a. 8 is 15% of what number?
   b. 15% of 18.95 is what number?
   c. What percent of 64 is 326?
   d. 10% of what number is 40?

12. APPLICATION Carl has been keeping a record of his gas purchases for his new car. Each time he buys gas, he fills the tank completely. Then he records the number of gallons he bought and the miles since the last fill-up. Here is his record:
   a. Copy and complete the table by calculating the ratio of miles per gallon for each purchase.
   b. What is the average rate of miles per gallon so far?
   c. The car’s tank holds 17.1 gallons. To the nearest mile, how far should Carl be able to go without running out of gas?
   d. Carl is planning a trip across the United States. He estimates that the trip will be 4230 miles. How many gallons of gas can Carl expect to buy?

<table>
<thead>
<tr>
<th>Miles traveled</th>
<th>Gallons</th>
<th>miles per gallon</th>
</tr>
</thead>
<tbody>
<tr>
<td>363</td>
<td>16.2</td>
<td></td>
</tr>
<tr>
<td>342</td>
<td>15.1</td>
<td></td>
</tr>
<tr>
<td>285</td>
<td>12.9</td>
<td></td>
</tr>
</tbody>
</table>

Consumer CONNECTION

Many factors influence the rate at which cars use gas, including size, age, and driving conditions. Advertisements for new cars often give the average mpg for city traffic (slow, congested) and highway traffic (fast, free flowing). These rates help consumers make an informed purchase. For more information about fuel economy, see the links at www.keymath.com/DA.

13. Match each recursive routine to a graph below. Explain how you made your decision and tell what assumptions you made.
   a. 2.5 \(\text{ENTER}\)
      Ans + 0.5 \(\text{ENTER} ; \text{ENTER} , \ldots\)
   b. 1.0 \(\text{ENTER}\)
      Ans + 1.0 \(\text{ENTER} ; \text{ENTER} , \ldots\)
   c. 2.0 \(\text{ENTER}\)
      Ans + 1.0 \(\text{ENTER} ; \text{ENTER} , \ldots\)
   d. 2.5 \(\text{ENTER}\)
      Ans − 0.5 \(\text{ENTER} ; \text{ENTER} , \ldots\)

   ![Graphs i, ii, iii, iv]
14. Bjarne is training for a bicycle race by riding on a stationary bicycle with a time-distance readout. He is riding at a constant speed. The graph shows his accumulated distance and time as he rides.
   a. How fast is Bjarne bicycling?
   b. Copy and complete the table. 🟡
   c. Write a recursive routine for Bjarne’s ride.
   d. Looking at the graph, how do you know that Bjarne is neither slowing down nor speeding up during his ride?
   e. If Bjarne keeps up the same pace, how far will he ride in one hour?

Bicyclists race through San Luis Obispo, California.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Distance (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

15. Consider the expression \(\frac{4(y - 8)}{3}\).
   a. Find the value of the expression if \(y = 5\). Make a table to show the order of operations. 🟡
   b. Solve the equation \(\frac{4(y - 8)}{3} = 8\) by undoing the sequence of operations. 🟡

**IMPROVING YOUR REASONING SKILLS**

You have two containers of the same size; one contains juice and the other contains water. Remove one tablespoon of juice and put it into the water and stir. Then remove one tablespoon of the water and juice mixture and put it into the juice. Is there more water in the juice or more juice in the water?
In this lesson you will continue to develop your skills with equations, graphs, and tables of data by exploring the role that the value of $b$ plays in the equation $y = a + bx$.

You have already studied the intercept form of a linear equation in several real-world situations. You have used the intercept form to relate calories to minutes spent exercising, floor heights to floor numbers, and distances to time. So, defining variables is an important part of writing equations. Depending on the context of an equation, its numbers take on different real-world meanings. Can you recall how these equations modeled each scenario?

In most linear equations, there are different output values for different input values. This happens when the coefficient of $x$ is not zero. You’ll explore how this coefficient relates input and output values in the examples and the investigation.

In addition to giving the actual temperature, weather reports often indicate the temperature you feel as a result of the wind chill factor. The wind makes it feel colder than it actually is. In the next example you will use recursive routines to answer some questions about wind chill.
The table relates the approximate wind chills for different actual temperatures when the wind speed is 15 mi/h. Assume the wind chill is a linear relationship for temperatures between $-5^\circ$ and $35^\circ$.

<table>
<thead>
<tr>
<th>Temperature ($^\circ$F)</th>
<th>-5</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wind chill ($^\circ$F)</td>
<td>25.8</td>
<td>-19.4</td>
<td>-13</td>
<td>6.2</td>
<td>19</td>
<td>25.4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. What are the input and output variables?

b. What is the change in temperature from one table entry to the next? What is the corresponding change in the wind chill?

c. Use calculator lists to write a recursive routine that generates the table values. What are the missing entries?

**Solution**

a. The input variable is the actual air temperature in $^\circ$F. The output variable is the temperature you feel as a result of the wind chill factor.

b. For every $5^\circ$ increase in temperature, the wind chill increases $6.4^\circ$.

c. The recursive routine to complete the missing table values is $\{-5, -25.8\}$ and $\{\text{Ans}(1) + 5, \text{Ans}(2) + 6.4\}$. The calculator screen displays the missing entries.

In Example A, the rate at which the wind chill drops can be calculated from the ratio $\frac{6.4}{5}$, or $\frac{1.28}{1}$. In other words, it feels $1.28^\circ$ colder for every $1^\circ$ drop in air temperature. This number is the rate of change for a wind speed of 15 mi/h. The rate of change is equal to the ratio of the change in output values divided by the corresponding change in input values.

Do you think the rate of change differs with various wind speeds?

**Investigation**

**Wind Chill**

In this investigation you’ll use the relationship between temperature and wind chill to explore the concept of rate of change and its connections to tables, scatter plots, recursive routines, equations, and graphs.

The data in the table represent the approximate wind chill temperatures in degrees Fahrenheit for a wind speed of 20 mi/h. Use this data set to complete each task.

**Step 1** Define the input and output variables for this relationship.

<table>
<thead>
<tr>
<th>Temperature ($^\circ$F)</th>
<th>Wind chill ($^\circ$F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5</td>
<td>-28.540</td>
</tr>
<tr>
<td>0</td>
<td>-21.980</td>
</tr>
<tr>
<td>1</td>
<td>-20.668</td>
</tr>
<tr>
<td>2</td>
<td>-19.356</td>
</tr>
<tr>
<td>5</td>
<td>-15.420</td>
</tr>
<tr>
<td>15</td>
<td>-2.300</td>
</tr>
<tr>
<td>35</td>
<td>23.940</td>
</tr>
</tbody>
</table>

[Data sets: TMPWS, WINDCH]
Step 2: Plot the points and describe the viewing window you used.

Step 3: Write a recursive routine that gives the pairs of values listed in the table.

Step 4: Copy the table. Complete the third and fourth columns of the table by recording the changes between consecutive input and output values. Then find the rate of change.

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
<th>Change in input values</th>
<th>Change in output values</th>
<th>Rate of change</th>
</tr>
</thead>
<tbody>
<tr>
<td>−5</td>
<td>−28.540</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>−21.980</td>
<td>5</td>
<td>6.56</td>
<td>$\frac{+6.56}{+5}$ =</td>
</tr>
<tr>
<td>1</td>
<td>−20.668</td>
<td>1</td>
<td>1.312</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>−19.356</td>
<td>1</td>
<td>1.312</td>
<td>$\frac{+1.312}{+1}$ =</td>
</tr>
<tr>
<td>5</td>
<td>−15.420</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>−2.300</td>
<td>13.12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>23.940</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Step 5: Use your routine to write a linear equation in intercept form that relates wind chill to temperature. Note that the starting value, −28.540, is not the $y$-intercept. How does the rule of the routine appear in your equation?

Step 6: Graph the equation on the same set of axes as your scatter plot. Use the calculator table to check that your equation is correct. Does it make sense to draw a line through the points? Where does the $y$-intercept show up in your equation?

Step 7: What do you notice about the values for rate of change listed in your table? How does the rate of change show up in your equation? In your graph?

Step 8: Explain how to use the rate of change to find the actual temperature if the weather report indicates a wind chill of 9.5° with 20 mi/h winds.

High wind speeds in Saskatchewan, Canada, drop temperatures below freezing.
This table shows the temperature of the air outside an airplane at different altitudes.

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Altitude (m)</td>
<td>Temperature (°C)</td>
</tr>
<tr>
<td>1000</td>
<td>7.7</td>
</tr>
<tr>
<td>1500</td>
<td>4.2</td>
</tr>
<tr>
<td>2200</td>
<td>−0.7</td>
</tr>
<tr>
<td>3000</td>
<td>−6.3</td>
</tr>
<tr>
<td>4700</td>
<td>−18.2</td>
</tr>
<tr>
<td>6000</td>
<td>−27.3</td>
</tr>
</tbody>
</table>

a. Add three columns to the table, and record the change in input values, the change in output values, and the corresponding rate of change.

b. Use the table and a recursive routine to write a linear equation in intercept form $y = a + bx$.

c. What are the real-world meanings of the values for $a$ and $b$ in your equation?

### Solution

a. Record the change in input values, change in output values, and rate of change in a table. Note the units of each value.

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
<th>Change in input values (m)</th>
<th>Change in output values (°C)</th>
<th>Rate of change (°C/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Altitude (m)</td>
<td>Temperature (°C)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td>7.7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1500</td>
<td>4.2</td>
<td>500</td>
<td>−3.5</td>
<td>$\frac{3.5}{500} = 0.007$</td>
</tr>
<tr>
<td>2200</td>
<td>−0.7</td>
<td>700</td>
<td>−4.9</td>
<td>$\frac{4.9}{700} = 0.007$</td>
</tr>
<tr>
<td>3000</td>
<td>−6.3</td>
<td>800</td>
<td>−5.6</td>
<td>$\frac{5.6}{800} = 0.007$</td>
</tr>
<tr>
<td>4700</td>
<td>−18.2</td>
<td>1700</td>
<td>−11.9</td>
<td>$\frac{11.9}{1700} = 0.007$</td>
</tr>
<tr>
<td>6000</td>
<td>−27.3</td>
<td>1300</td>
<td>−9.1</td>
<td>$\frac{9.1}{1300} = 0.007$</td>
</tr>
</tbody>
</table>

b. Note that the rate of change, or slope, is always $−0.007$, or $\frac{−7}{1000}$. You can also write the rate of change as $\frac{−0.7}{100}$, so this recursive routine models the relationship:

$$
\{1000, 7.7\} \text{ ENTER }
\{\text{Ans}(1) + 100, \text{Ans}(2) - 0.7\} \text{ ENTER }
$$

Working this routine backward, $\{\text{Ans}(1) - 100, \text{Ans}(2) + 0.7\}$, will eventually give the result $\{0, 14.7\}$. So the intercept form of the equation is $y = 14.7 − 0.007x$, where $x$ represents the altitude in meters and $y$ represents the air temperature in °C.
Note that the starting value of the recursive routine is not the same as the value of the y-intercept in the equation.

c. The value of \( a \), 14.7, is the temperature (in °C) of the air at sea level. The value of \( b \) indicates that the temperature drops 0.007°C for each meter that a plane climbs.

**EXERCISES**

You will need your graphing calculator for Exercises 4, 5, and 10.

**Practice Your Skills**

1. Copy and complete the table of output values for each equation.
   
   a. \( y = 50 + 2.5x \)
   
<table>
<thead>
<tr>
<th>Input ( x )</th>
<th>Output ( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td></td>
</tr>
<tr>
<td>-30</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
</tr>
<tr>
<td>-12.5</td>
<td></td>
</tr>
</tbody>
</table>

   b. \( L_2 = -5.2 - 10 \cdot L_1 \)

<table>
<thead>
<tr>
<th>( L_1 ) ( x )</th>
<th>( L_2 ) ( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>-8</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td></td>
</tr>
<tr>
<td>-35</td>
<td></td>
</tr>
<tr>
<td>-5.2</td>
<td></td>
</tr>
</tbody>
</table>

2. Use the equation \( w = -29 + 1.4t \), where \( t \) is temperature and \( w \) is wind chill, both in °F, to approximate the wind chill temperatures for a wind speed of 40 mi/h.
   
   a. Find \( w \) for \( t = 32° \).
   b. Find \( t \) for a wind chill of \( w = -8 \).
   c. What is the real-world meaning of 1.4?
   d. What is the real-world meaning of -29?

3. Describe what the rate of change looks like in each graph.
   
   a. the graph of a person walking at a steady rate toward a motion sensor
   b. the graph of a person standing still
   c. the graph of a person walking at a steady rate away from a motion sensor
   d. the graph of one person walking at a steady rate faster than another person

4. Use the “Easy” setting of the INOUT game on your calculator to produce four data tables. Copy each table and write the equation you used to match the data values in the table. See Calculator Note 3C to learn how to run the program.
5. Each table below shows a different input-output relationship.

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-6</td>
</tr>
<tr>
<td>1</td>
<td>-2.5</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>4.5</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-7</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>25</td>
</tr>
<tr>
<td>5</td>
<td>41</td>
</tr>
<tr>
<td>7</td>
<td>57</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>-8</td>
<td>34.2</td>
</tr>
<tr>
<td>-7</td>
<td>32.8</td>
</tr>
<tr>
<td>-3</td>
<td>27.2</td>
</tr>
<tr>
<td>2</td>
<td>20.2</td>
</tr>
<tr>
<td>8</td>
<td>11.8</td>
</tr>
</tbody>
</table>

a. Find the rate of change in each table. Explain how you found this value.

b. For each table, find the output value that corresponds to an input value of 0. What is this value called?

c. Use your results from 5a and b to write an equation in intercept form for each table.

d. Use a calculator list of input values to check that each equation actually produces the output values shown in the table.

6. The wind chill temperatures for a wind speed of 35 mi/h are given in the table.

<table>
<thead>
<tr>
<th>Temperature (°F)</th>
<th>-5</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>35</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wind chill (°F)</td>
<td>-35</td>
<td>-21</td>
<td>-14</td>
<td>0</td>
<td>21</td>
</tr>
</tbody>
</table>

a. Define input and output variables.

b. Find the rate of change. Explain how you got your answer.

c. Write an equation in intercept form.

d. Plot the points and graph the equation on the same set of axes. How are the graphs for the points and the equation similar? How are they different?

7. Samantha's walk was recorded by a motion sensor. A graph of her walk and a few data points are shown here.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Distance (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3.5</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
</tr>
</tbody>
</table>

a. Write an equation in the form

\[ \text{Distance from sensor = start distance + change} \]

b. If she continues to walk at a constant rate, at what time would she pass the sensor?
8. You can use the equation $7.3x = 200$ to describe a rectangle with an area of 200 square units like the one shown. What are the real-world meanings of the numbers and the variable in the equation? Solve the equation for $x$ and explain the meaning of your solution. Is the rectangle drawn to scale? How can you tell?

<table>
<thead>
<tr>
<th>200 square units</th>
<th>7.3 units</th>
</tr>
</thead>
</table>

9. The total area of the figure at right is 1584 square units. You can use the equation $1584 - 33x = 594$ to represent an area of 1584 square units minus the area of 33$x$ square units. The area remaining is 594 square units.

a. What is the area of the shaded rectangle? ☐

b. Write the equation you would use to find the height of the shaded rectangle. ☐

c. Solve the equation you wrote in 9b to find the height of the shaded rectangle. ☐

10. Use the “Medium” setting of the INOUT game on your calculator to produce four data tables. Copy each table and write the equation you used to match the data values in the table. [See Calculator Note 3C.]

Review

11. Show how you can solve these equations by using an undoing process. Check your results by substituting the solutions into the original equations.

a. $-15 = -52 + 1.6x$  

b. $7 - 3x = 52$

12. **APPLICATION** To plan a trip downtown, you compare the costs of three different parking lots. ABC Parking charges $5 for the first hour and $2 for each additional hour or fraction of an hour. Cozy Car charges $3 per hour or fraction of an hour, and The Corner Lot charges a $15 flat rate for a whole day.

a. Make a table similar to the one shown. Write recursive routines to calculate the cost of parking up to 10 hours at each of the three lots.

<table>
<thead>
<tr>
<th>Hours parked</th>
<th>ABC Parking</th>
<th>Cozy Car</th>
<th>The Corner Lot</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. Make three different scatter plots on the same pair of axes showing the parking rates at the three different lots. Use a different color for each parking lot. Put the hours on the horizontal axis and the cost on the vertical axis.

c. Compare the three scatter plots. Under what conditions is each parking lot the best deal for your trip? Use the graph to explain.

d. Would it make sense to draw a line through each set of points? Explain why or why not.
13. Today while Don was swimming, he started wondering how many lengths he would have to swim in order to swim different distances. At one end of the pool, he stopped, gasping for breath, and asked the lifeguard. She told him that 1 length of the pool is 25 yards and that 72 lengths is 1 mile. As he continued swimming, he wondered:
   a. Is 72 lengths really a mile? Exactly how many lengths would it take to swim a mile? 
   b. If it took him a total of 40 minutes to swim a mile, what was his average speed in feet per second?
   c. How many lengths would it take to swim a kilometer?
   d. Last summer Don got to swim in a pool that was 25 meters long. How many lengths would it take to swim a kilometer there? How many for a mile?

14. **APPLICATION** Holly has joined a video rental club. After paying $6 a year to join, she then has to pay only $1.25 for each new release she rents.
   a. Write an equation in intercept form to represent Holly’s cost for movie rentals.
   b. Graph this situation for up to 60 movie rentals.
   c. Video Unlimited charges $60 for a year of unlimited movie rentals. How many movies would Holly have to rent for this to be a better deal?

---

**LEGAL LIMITS**

To make a highway accessible to more vehicles, engineers reduce its steepness, also called its **gradient** or grade. This highway was designed with switchbacks so the gradient would be small.

A gradient is the inclination of a roadway to the horizontal surface. Research the federal, state, and local standards for the allowable gradients of highways, streets, and railway routes.

Find out how gradients are expressed in engineering terms. Give the standards for roadway types designed for vehicles of various weights, speeds, and engine power in terms of rate of change. Describe the alternatives available to engineers to reduce the gradient of a route in hilly or mountainous terrain. What safety measures do they incorporate to minimize risk on steep grades? Bring pictures to illustrate a presentation about your research, showing how engineers have applied standards to roads and routes in your home area.
Solving Equations Using the Balancing Method

In the previous two lessons, you learned about rate of change and the intercept form of a linear equation. In this lesson you’ll learn symbolic methods to solve these equations. You’ve already seen the calculator methods of tracing on a graph and zooming in on a table. These methods usually give approximate solutions. Working backward to undo operations is a symbolic method that gives exact solutions. Another symbolic method that you can apply to solve equations is the balancing method. In this lesson you’ll investigate how to use the balancing method to solve linear equations. You’ll discover that it’s closely related to the undoing method.

Investigation
Balancing Pennies

Here is a visual model of the equation $2x + 3 = 7$. A cup represents the variable $x$ and pennies represent numbers. Assume that each cup has the same number of pennies in it and that the containers themselves are weightless.

Step 1 | How many pennies must be in each cup if the left side of the scale balances with the right side? Explain how you got your answer.

Your answer to Step 1 is the solution to the equation $2x + 3 = 7$. It’s the number that can replace $x$ to make the statement true. In Steps 2 and 3, you’ll use pictures and equations to show stages that lead to the solution.

Step 2 | Redraw the picture above, but with three pennies removed from each side of the scale. Write the equation that your picture represents.
Step 3 | Redraw the picture, this one showing half of what was on each side of the scale in Step 2. There should be just one cup on the left side of the scale and the correct number of pennies on the right side needed to balance it. Write the equation that this picture represents. This is the solution to the original equation.

Now your group will create a pennies-and-cups equation for another group to solve.

Step 4 | Divide the pennies into two equal piles. If you have one left over, put it aside. Draw a large equal sign (or form one with two pencils) and place the penny stacks on opposite sides of it.

Step 5 | From the pile on one side of your equal sign, make three identical stacks, leaving at least a few pennies out of the stacks. Hide each stack under a paper cup. You should now have three cups and some pennies on one side of your equal sign.

Step 6 | On the other side you should have a pile of pennies. On both sides of the equal sign you have the same number of pennies, but on one side some of the pennies are hidden under cups. You can think of the two sides of the equal sign as being the two sides of a balance scale. Write an equation for this setup, using $x$ to represent the number of pennies hidden under one cup.

Step 7 | Move to another group's setup. Look at their arrangement of pennies and cups, and write an equation for it. Solve the equation; that is, find how many pennies are under one cup without looking. When you're sure you know how many pennies are under each cup, you can look to check your answer.

Step 8 | Write a brief description of how you solved the equation.

You can do problems like those in the investigation using a balance scale as long as the weight of the cup is very small. But an actual balance scale can only model equations in which all the numbers involved are positive. Still, the idea of balancing equations can apply to equations involving negative numbers. Just remember, when you add any number to its opposite, you get 0. For this reason, the opposite of a number is called the **additive inverse**. Think of negative and positive numbers as having opposite effects on a balance scale. You can remove 0 from either side of a balance-scale picture without affecting the balance. These three figures all represent 0:

- $1 + (-1) = 0$
- $1 + (-1) = 0$
- $-3 + 3 = 0$
- $2x + (-2x) = 0$
EXAMPLE A

The goal is to end up with a single \( x \)-cup on one side of the balance scale. One way to get rid of something on one side is to add its opposite to both sides.

Here is the equation \( 6 = -2 + 4x \) solved by the balancing method:

<table>
<thead>
<tr>
<th>Picture</th>
<th>Action taken</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Image" /></td>
<td>Original equation.</td>
<td>( 6 = -2 + 4x )</td>
</tr>
<tr>
<td><img src="image2.png" alt="Image" /></td>
<td>Add 2 to both sides.</td>
<td>( 6 + 2 = -2 + 2 + 4x )</td>
</tr>
<tr>
<td><img src="image3.png" alt="Image" /></td>
<td>Remove the 0.</td>
<td>( 8 = 4x )</td>
</tr>
<tr>
<td><img src="image4.png" alt="Image" /></td>
<td>Divide both sides by 4.</td>
<td>( \frac{8}{4} = \frac{4x}{4} )</td>
</tr>
<tr>
<td><img src="image5.png" alt="Image" /></td>
<td>Reduce.</td>
<td>( 2 = x ) or ( x = 2 )</td>
</tr>
</tbody>
</table>

In the second and third equations, you saw \( 6 + 2 \) combine to 8, and \( -2 + 2 \) combine to 0. You can combine numbers because they are like terms. However, in the first equation you could not combine \( -2 \) and \( 4x \), because they are not like terms. Like terms are terms in which the variable component is the same, and they may differ only by a coefficient.
Balance-scale pictures can help you see what to do to solve an equation by the balancing method. But you won’t need the pictures once you get the idea of doing the same thing to both sides of an equation. And pictures are less useful if the numbers in the equation aren’t “nice.”

**EXAMPLE B**

Solve the equation $-31 = -50.25 + 1.55x$ using each method.

a. undoing operations

b. the balancing method
c. tracing on a calculator graph
d. zooming in on a calculator table

**Solution**

Each of these methods will give the same answer, but notice the differences among the methods. When might you prefer to use a particular method?

a. undoing operations

Start with $-31$.

b. the balancing method

\[
-31 = -50.25 + 1.55x \\
-31 + 50.25 = -50.25 + 50.25 + 1.55x \\
19.25 = 1.55x \\
\frac{19.25}{1.55} = \frac{1.55x}{1.55} \\
12.42 = x, \text{ or } x = 12.42
\]

This chart shows how balancing equations is related to the undoing method that you’ve been using. In the last column, as you work up from the bottom, you can see how the equation changes as you apply the “undo” operation to both sides of the equation.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Equation:</strong> $-31 = -50.25 + 1.55x$</td>
<td><strong>Description</strong></td>
<td><strong>Undo</strong></td>
<td><strong>Result</strong></td>
</tr>
<tr>
<td>Pick $x$.</td>
<td>$\approx 12.42$</td>
<td>$12.42 \approx x$</td>
<td></td>
</tr>
<tr>
<td>Multiply by 1.55.</td>
<td>$/(1.55)$</td>
<td>$19.25$</td>
<td>$19.25 = 1.55x$</td>
</tr>
<tr>
<td>Subtract 50.25.</td>
<td>$+(50.25)$</td>
<td>$-31$</td>
<td>$-31 = -50.25 + 1.55x$</td>
</tr>
</tbody>
</table>

In parts a and b, if you convert the answer to a fraction, you get an exact solution of $\frac{385}{31}$. 

---

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50 | DISCOVERING ALGEBRA COURSE SAMPLER
c. tracing on a calculator graph

Enter the equation into Y1. Adjust your window settings and graph. Press TRACE and use the arrow keys to find the x-value for a y-value of –31. (See Example B in Lesson 3.4 to review this procedure.) You can see that for a y-value of approximately –31.6 the x-value is 12.02.

d. zooming in on a calculator table

To find a starting value for the table, use guess-and-check or a calculator graph to find an approximate answer. Then use the calculator table to find the answer to the desired accuracy.

Once you have determined a reasonable starting value, zoom in on a calculator table to find the answer using smaller and smaller values for the table increment. [See Calculator Note 2A to review zooming in on a table.]

You can also check your answer by using substitution.

The calculator result isn’t exactly –31 because 12.42 is a rounded answer. If you substitute an exact solution such as $\frac{19.25}{1.55}$ or $\frac{385}{31}$, you’ll get exactly –31.

From Example B, you can see that each method has its advantages. The methods of balancing and undoing use the same process of working backward to get an exact solution. The two calculator methods are easy to use but usually give approximate solutions to the equation. You may prefer one method over others, depending on the equation you need to solve. If you are able to solve an equation using two or more different methods, you can check to see that each method gives the same result. With practice, you may develop symbolic solving methods of your own. Knowing a variety of methods, such as the balancing and undoing methods, as well as the calculator methods, will improve your equation-solving skills, regardless of which method you prefer.

In Exercise 12, you’ll see how to use the balancing method to solve an equation that has the variable on both sides.


**EXERCISES**

You will need your graphing calculator for Exercises 8 and 14.

## Practice Your Skills

1. Give the equation that each picture models.
   
   a. 
   
   ![Equation 1]
   
   b. 
   
   ![Equation 2]
   
   c. 
   
   ![Equation 3]
   
   d. 
   
   ![Equation 4]

2. Copy and fill in the table to solve the equation as in Example A.

<table>
<thead>
<tr>
<th>Picture</th>
<th>Action taken</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Equation 5]</td>
<td>Add 2 to both sides.</td>
<td></td>
</tr>
<tr>
<td>![Equation 6]</td>
<td>Divide both sides by 2.</td>
<td>( \frac{2x}{2} = \frac{6}{2} )</td>
</tr>
<tr>
<td>![Equation 7]</td>
<td>Reduce.</td>
<td></td>
</tr>
</tbody>
</table>
3. Give the next stages of the equation, matching the action taken, to reach the solution.

a. \( 0.1x + 12 = 2.2 \)  
   Original equation.  
   Subtract 12 from both sides.  
   Remove the 0 and subtract.  
   Divide both sides by 0.1.

b. \( \frac{12}{3} + 3.12x = -100 \)  
   Original equation.  
   Multiply both sides by 3.  
   Subtract 12 from both sides.  
   Remove the 0.  
   Divide both sides by 3.12

4. Complete the tables to solve the equations.

<table>
<thead>
<tr>
<th>Description</th>
<th>Undo</th>
<th>Result</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equation: ( \frac{3(x - 8)}{5} + 7 = 34 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pick x.</td>
<td></td>
<td>53</td>
<td>( x = 53 )</td>
</tr>
<tr>
<td>Subtract 8.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multiply by 3.</td>
<td>/ (3)</td>
<td>( 3(x - 8) = 135 )</td>
<td></td>
</tr>
<tr>
<td>Multiply by 5.</td>
<td>( * (5) )</td>
<td>27</td>
<td></td>
</tr>
<tr>
<td>( 34 ) ( \frac{3(x - 8)}{5} + 7 = 34 )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Description</th>
<th>Undo</th>
<th>Result</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>b.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equation: ( 7 \left( \frac{2 + x}{4} \right) - 5 = 16 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pick x.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( x = )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5. Give the additive inverse of each number.

a. \( \frac{1}{5} \)  
   b. 17  
   c. \(-2.3\)  
   d. \(-x\)

**Reason and Apply**

6. A **multiplicative inverse** is a number or expression that you can multiply by something to get a value of 1. The multiplicative inverse of 4 is \( \frac{1}{4} \) because \( 4 \cdot \frac{1}{4} = 1 \).

   Give the multiplicative inverse of each number.

a. \( 12 \)  
   b. \( \frac{1}{6} \)  
   c. 0.02  
   d. \( -\frac{1}{2} \)

7. Solve these equations. Tell what action you take at each stage.

a. \( 144x = 12 \)  
   b. \( \frac{1}{6}x + 2 = 8 \)

8. **Mini-Investigation** A solution to the equation \(-10 + 3x = 5\) is shown below.

   \(-10 + 3x = 5\)  
   \(3x = 15\)  
   \(x = 5\)

   a. Describe the steps that transform the original equation into the second equation and the second equation into the third (the solution).

   b. Graph \( Y_1 = -10 + 3x \) and \( Y_2 = 5 \), and trace to the lines’ intersection. Write the coordinates of this point.
c. Graph \( Y_1 = 3x \) and \( Y_2 = 15 \), and trace to the lines’ intersection. Write the coordinates of this point.

d. Graph \( Y_1 = x \) and \( Y_2 = 5 \), and trace to the lines’ intersection. Write the coordinates of this point.

e. What do you notice about your answers to 8b–d? Explain what this illustrates.

9. Solve the equation \( 4 + 1.2x = 12.4 \) by using each method.
   a. balancing  
   b. undoing 
   c. tracing on a graph 
   d. zooming in on a table

10. Solve each equation symbolically using the balancing method.
    a. \( 3 + 2x = 17 \) 
    b. \( 0.5x + 2.2 = 101.0 \) 
    c. \( x + 307.2 = 2.1 \) 
    d. \( 2(2x+2) = 7 \) 
    e. \( \frac{4 + 0.01x}{6.2} = 6.2 \) 

11. You can solve familiar formulas for a specific variable. For example, solving \( A = lw \) for \( l \) you get
    \[
    \frac{A}{w} = \frac{lw}{w} \quad \text{Original equation.} \\
    \frac{A}{w} = l \quad \text{Divide both sides by} \ w. \\
    \frac{A}{w} = l \quad \text{Reduce.}
    \]
    You can also write \( l = \frac{A}{w} \). Now try solving these formulas for the given variable.
    a. \( C = 2\pi r \) for \( r \) 
    b. \( A = \frac{1}{2}(hb) \) for \( h \) 
    c. \( P = 2(l + w) \) for \( l \) 
    d. \( P = 4s \) for \( s \) 
    e. \( d = rt \) for \( t \) 
    f. \( A = \frac{1}{2}h(a + b) \) for \( h \)

12. An equation can have the variable on both sides. In these cases you can maintain the balance by eliminating the \( x \)'s from one of the sides before you begin undoing.
    a. Copy and complete this table to solve the equation.

<table>
<thead>
<tr>
<th>Picture</th>
<th>Action taken</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( +1 ) ( x ) ( x ) ( +1 ) ( +1 ) ( +1 ) ( \Delta )</td>
<td>Original equation. ( 2 + 4x = x + 8 )</td>
<td></td>
</tr>
<tr>
<td>( +1 ) ( x ) ( +1 ) ( +1 ) ( +1 ) ( \Delta )</td>
<td>( 3x = 6 )</td>
<td></td>
</tr>
<tr>
<td>( +1 ) ( x ) ( +1 ) ( +1 ) ( \Delta )</td>
<td>Divide both sides by 3.</td>
<td></td>
</tr>
</tbody>
</table>

b. Show the steps used to solve \( 5x - 4 = 2x + 5 \) using the balancing method. Substitute your solution into the original equation to check your answer.
Review

13. **APPLICATION** Economy drapes for a certain size window cost $90. They have shallow pleats, and the width of the fabric is $\frac{2}{3}$ times the window width. Luxury drapes of the same fabric for the same size window have deeper pleats. The width of the fabric is 3 times the window width. What price should the store manager ask for the luxury drapes?

14. Run the easy level of the LINES program on your calculator. See Calculator Note 3D to learn how to use the LINES program. Sketch a graph of the randomly generated line on your paper. Use the trace function to locate the y-intercept and to determine the rate of change. When the calculator says you have the correct equation, write it under the graph. Repeat this program until you get three correct equations in a row.

15. The local bagel store sells a baker’s dozen of bagels for $6.49, while the grocery store down the street sells a bag of 6 bagels for $2.50.
   a. Copy and complete the tables showing the cost of bagels at the two stores.

<table>
<thead>
<tr>
<th>Bagels</th>
<th>13</th>
<th>26</th>
<th>39</th>
<th>52</th>
<th>65</th>
<th>78</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bagels</th>
<th>6</th>
<th>12</th>
<th>18</th>
<th>24</th>
<th>30</th>
<th>36</th>
<th>42</th>
<th>48</th>
<th>54</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   b. Graph the information for each market on the same coordinate axes. Put bagels on the horizontal axis and cost on the vertical axis.
   c. Find equations to describe the cost of bagels at each store.
   d. How much does one bagel cost at each store? How do these cost values relate to the equations you wrote in 15c?
   e. Looking at the graphs, how can you tell which store is the cheaper place to buy bagels?
   f. Bernie and Buffy decided to use a recursive routine to complete the tables. Bernie used this routine for the bagel store:

   6.49 ENTER
   Ans • 2 ENTER

   Buffy says that this routine isn’t correct, even though it gives the correct answer for 13 and 26 bagels. Explain to Bernie what is wrong with his recursive routine. What routine should he use?
CHAPTER 3 Linear Equations

LESSON 3.7

**Modeling Data**

Whenever measuring is involved in collecting data, you can expect some variation in the pattern of data points. Usually, you can’t construct a mathematical model that fits the data exactly. But in general, the better a model fits, the more useful it is for making predictions or drawing conclusions from the data.

**Activity**

**Tying Knots**

In this activity, you’ll explore the relationship between the number of knots in a rope and the length of the rope and write an equation to model the data.

<table>
<thead>
<tr>
<th>Number of knots</th>
<th>Length of knotted rope (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

**You will need**

- two pieces of rope of different lengths (around 1 m) and thickness
- a meterstick or a tape measure

**Step 1**
Choose one piece of rope and record its length in a table like the one shown. Tie 6 or 7 knots, remeasuring the rope after you tie each knot. As you measure, add data to complete a table like the one above.

**Step 2**
Graph your data, plotting the number of knots on the x-axis and the length of the knotted rope on the y-axis. What pattern does the data seem to form?

**Step 3**
What is the approximate rate of change for this data set? What is the real-world meaning of the rate of change? What factors have an effect on it?

**Step 4**
What is the y-intercept for the line that best models the data? What is its real-world meaning?
Step 5 | Write an equation in intercept form for the line that you think best models the data. Graph your equation to check that it’s a good fit.

Now you’ll make predictions and draw some conclusions from your data using the line model as a summary of the data.

Step 6 | Use your equation to predict the length of your rope with 7 knots. What is the difference between the actual measurement of your rope with 7 knots and the length you predicted using your equation?

Step 7 | Use your equation to predict the length of a rope with 17 knots. Explain the problems you might have in making or believing your prediction.

Step 8 | What is the maximum number of knots that you can tie with your piece of rope? Explain your answer.

Step 9 | Does your graph cross the $x$-axis? Explain the real-world meaning, if any, of the $x$-value of the intersection point.

Step 10 | Substitute a value for $y$ into the equation. What question does the equation ask? What is the answer?

Step 11 | Repeat Steps 1–5 using a different piece of rope. Graph the data on the same pair of axes.

Step 12 | Compare the graphs of the lines of fit for both ropes. Give reasons for the differences in their $y$-intercepts, in their $x$-intercepts, and in their rates of change.

---

**IMPROVING YOUR REASONING SKILLS**

There are 100 students and 100 lockers in a school hallway. All of the lockers are closed. The first student walks down the hallway and opens every locker. A second student closes every even-numbered locker. The third student goes to every third locker and opens it if it is closed or closes it if it is open. This pattern repeats so that the $n$th student leaves every $n$th locker the opposite of how it was before. After all 100 students have opened or closed the lockers, how many lockers are left open?
You started this chapter by investigating **recursive sequences** by using their starting values and **rates of change** to write **recursive routines**. You saw how rates of change and starting values appear in plots.

In a walking investigation you observed, interpreted, and analyzed graphical representations of relationships between time and distance. What does the graph look like when you stand still? When you move away from or move toward the motion sensor? If you speed up or slow down? You identified real-world meanings of the **slope-intercept** and the rate of change of a **linear relationship**, and used them to write a **linear equation** in the **intercept form**, \[ y = a + bx. \] You learned the role of \( b \), the coefficient of \( x \). You explored relationships among verbal descriptions, tables, recursive rules, equations, and graphs.

Throughout the chapter you developed your equation-solving skills. You found solutions to equations by continuing to practice an undoing process and by using a **balancing** process. You found approximate solutions by tracing calculator graphs and by zooming in on calculator tables. Finally, you learned how to model data that don't lie exactly on a line, and you used your model to predict inputs and outputs.

### Exercises

You will need your graphing calculator for Exercises 4, 6, and 7.

1. Solve these equations. Give reasons for each step.
   a. \(-x = 7\)
   b. \(4.2 = -2x - 42.6\)

2. These tables represent linear relationships. For each relationship, give the rate of change, the \(y\)-intercept, the recursive rule, and the equation in intercept form.
   a. \[
   \begin{array}{c|c}
   x & y \\
   \hline
   0 & 3 \\
   1 & 4 \\
   2 & 5 \\
   \end{array}
   \]
   b. \[
   \begin{array}{c|c}
   x & y \\
   \hline
   1 & 0.01 \\
   2 & 0.02 \\
   3 & 0.03 \\
   \end{array}
   \]
   c. \[
   \begin{array}{c|c}
   x & y \\
   \hline
   -2 & 1 \\
   0 & 5 \\
   3 & 11 \\
   \end{array}
   \]
   d. \[
   \begin{array}{c|c}
   x & y \\
   \hline
   -4 & 5 \\
   12 & -3 \\
   2 & 2 \\
   \end{array}
   \]
3. Match these walking instructions with their graph sketches.

- i. $d$ over $t$
- ii. $d$ over $t$
- iii. $d$ over $t$

a. The walker stands still.
b. The walker takes a few steps toward the 0-mark, then walks away.
c. The walker steps away from the 0-mark, stops, then continues more slowly in the same direction.

4. Graph each equation on your calculator, and trace to find the approximate $y$-value for the given $x$-value.
   a. $y = 1.21 - x$ when $x = 70.2$
   b. $y = 6.02 + 44.3x$ when $x = 96.7$
   c. $y = -0.06 + 0.313x$ when $x = 0.64$
   d. $y = 1183 - 2140x$ when $x = -111$

5. Write the equations for linear relationships that have these characteristics.
   a. The output value is equal to the input value.
   b. The output value is 3 less than the input value.
   c. The rate of change is 2.3 and the $y$-intercept is $-4.3$.
   d. The graph contains the points (1, 1), (2, 1), and (3, 1).

6. The profit for a small company depends on the number of bookcases it sells. One way to determine the profit is to use a recursive routine such as

```
{0, -850} ENTER
{Ans(1) + 1, Ans(2) + 70} ENTER; ENTER, ...
```

a. Explain what the numbers and expressions $0$, $-850$, $\text{Ans}(1)$, $\text{Ans}(1) + 1$, $\text{Ans}(2)$, and $\text{Ans}(2) + 70$ represent.
b. Make a plot of this situation.
c. When will the company begin to make a profit? Explain.
d. Explain the relationship between the values $-850$ and $70$ and your graph.
e. Does it make sense to connect the points in the graph with a line? Explain.

7. A single section and a double section of a log fence are shown.

a. How many additional logs are required each time the fence is increased by a single section?
b. Copy and fill in the missing values in the table below.

<table>
<thead>
<tr>
<th>Number of sections</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>...</th>
<th>...</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of logs</td>
<td>4</td>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>91</td>
</tr>
</tbody>
</table>

c. Describe a recursive routine that relates the number of logs required to the number of sections.

d. If each section is 3 meters long, what is the longest fence you can build with 217 logs?

8. Suppose a new small-business computer system costs $5,400. Every year its value drops by $525.
   a. Define variables and write an equation modeling the value of the computer in any given year.
   b. What is the rate of change, and what does it mean in the context of the problem?
   c. What is the y-intercept, and what does it mean in the context of the problem?
   d. What is the x-intercept, and what does it mean in the context of the problem?

9. Andrei and his younger brother are having a race. Because the younger brother can’t run as fast, Andrei lets him start out 5 m ahead. Andrei runs at a speed of 7.7 m/s. His younger brother runs at 6.5 m/s. The total length of the race is 50 m.
   a. Write an equation to find how long it will take Andrei to finish the race. Solve the equation to find the time.
   b. Write an equation to find how long it will take Andrei’s younger brother to finish the race. Solve the equation to find the time.
   c. Who wins the race? How far ahead was the winner at the time he crossed the finish line?

10. Solve each equation using the method of your choice. Then use a different method to verify your solution.
   a. \(14x = 63\)  
   b. \(-4.5x = 18.6\)  
   c. \(8 = 6 + 3x\)  
   d. \(5(x - 7) = 29\)  
   e. \(3(x - 5) + 8 = 12\)
11. For each table, write a formula for list L₂ in terms of list L₁.

a. | L₁ | L₂ |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-5.7</td>
</tr>
<tr>
<td>1</td>
<td>-3.4</td>
</tr>
<tr>
<td>2</td>
<td>-1.1</td>
</tr>
<tr>
<td>3</td>
<td>1.2</td>
</tr>
<tr>
<td>4</td>
<td>3.5</td>
</tr>
<tr>
<td>5</td>
<td>5.8</td>
</tr>
</tbody>
</table>

b. | L₁ | L₂ |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>19</td>
</tr>
<tr>
<td>-1</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>-5</td>
</tr>
<tr>
<td>2</td>
<td>-21</td>
</tr>
<tr>
<td>5</td>
<td>-45</td>
</tr>
<tr>
<td>6</td>
<td>-53</td>
</tr>
</tbody>
</table>

c. | L₁ | L₂ |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>13.5</td>
</tr>
<tr>
<td>-2</td>
<td>11</td>
</tr>
<tr>
<td>-9</td>
<td>7.5</td>
</tr>
<tr>
<td>0</td>
<td>12</td>
</tr>
<tr>
<td>6</td>
<td>15</td>
</tr>
<tr>
<td>-5</td>
<td>9.5</td>
</tr>
</tbody>
</table>

12. You can represent linear relationships with a graph, a table of values, an equation, or a rule stated in words. Here are two linear relationships. Give all the other ways to show each relationship.

a. [Graph of a linear relationship]

b. | x | y |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>2</td>
</tr>
<tr>
<td>-1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>-3</td>
</tr>
</tbody>
</table>

---

**MIXED REVIEW**

13. **APPLICATION** Sonja bought a pair of 210 cm cross-country skis. Will they fit in her ski bag, which is $6\frac{1}{2}$ ft long? Why or why not?

14. Fifteen students counted the number of letters in their first and last names. Here is the data set: [Data set: NMLET]:

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6</td>
<td>15</td>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>13</td>
<td>15</td>
<td>14</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>16</td>
<td>15</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

a. Make a histogram of the data with a bin width of 2.

b. What is the mean number of letters?

15. Evaluate these expressions.

a. $-3 \cdot 8 - 5 \cdot 6$

b. $[-2 - (-4)] \cdot 8 - 11$

c. $7 \cdot 8 + 4 \cdot (-12)$

d. $11 - 3 \cdot 9 - 2$

16. On a recent trip to Detroit, Tom started from home, which is 12 miles from Traverse City. After 4 hours he had traveled 220 miles.

a. Write a recursive routine to model Tom's distance from Traverse City during this trip. State at least two assumptions you're making.

b. Use your recursive routine to determine his distance from Traverse City for each hour during the first 5 hours of the trip.

c. What is the rate of change, and what does it mean in the context of this situation?
17. California has many popular national parks. This table shows the number of visitors in thousands to national parks in 2003.
   a. Find the mean number of visitors.
   b. What is the five-number summary for the data?
   c. Create a box plot for the data.
   d. Identify any parks in California that are outliers in the numbers of visitors they had. Explain why they are outliers.

<table>
<thead>
<tr>
<th>National park</th>
<th>Visitors (thousands)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channel Islands</td>
<td>586</td>
</tr>
<tr>
<td>Death Valley</td>
<td>890</td>
</tr>
<tr>
<td>Joshua Tree</td>
<td>1283</td>
</tr>
<tr>
<td>Kings Canyon</td>
<td>556</td>
</tr>
<tr>
<td>Lassen Volcanic</td>
<td>404</td>
</tr>
<tr>
<td>Redwood</td>
<td>408</td>
</tr>
<tr>
<td>Sequoia</td>
<td>979</td>
</tr>
<tr>
<td>Yosemite</td>
<td>3379</td>
</tr>
</tbody>
</table>

(U.S. National Park Service) [Data set: CAPRK]

18. Ohm’s law states that electrical current is inversely proportional to the resistance. A current of 18 amperes is flowing through a conductor whose resistance is 4 ohms.
   a. What is the current that flows through the system if the resistance is 8 ohms?
   b. What is the resistance of the conductor if a current of 12 amperes is flowing?

Every knob or lever of this sound recording console regulates electric resistance in a current. The resistance varies directly with voltage and inversely with current.
19. Consider the equation \(2(x - 6) = -5\).
   a. Solve the equation.
   b. Show how you can check your result by substituting it into the original equation.

20. **APPLICATION** Amber makes $6 an hour at a sandwich shop. She wants to know how many hours she needs to work to save $500 in her bank account. On her first paycheck, she notices that her net pay is about 75% of her gross pay.
   a. How many hours must she work to earn $500 in gross pay?
   b. How many hours must she work to earn $500 in net pay?

---

**TAKE ANOTHER LOOK**

1. The picture at right is a **contour map**. This type of map reveals the character of the terrain. All points on an **isometric line** are the same height in feet above sea level. The graph below shows how the hiker’s walking speed changes as she covers the terrain on the dotted-line trail shown on the map.

![Graph](image)

   a. What quantities are changing in the graph and in the map?
   b. How does each display reveal rate of change?
   c. How could you measure distance on each display?
   d. What would the graph sketch of this hike look like if distance were plotted on the vertical axis instead of speed?
   e. What do these two displays tell you when you study them together?

2. You’ve learned that a rational number is a number that can be written as a ratio of two integers. Every rational number can also be written in an equivalent decimal form. In Lesson 2.1, you learned how to convert fractions into decimal form. In some cases the result was a **terminating decimal**, and in other cases the result was a **repeating decimal**, in which a digit or group of digits repeated.
a. Rewrite each of these fractions in decimal form. If the digits appear to repeat, indicate this by placing a bar over those digits that repeat.

\[
\frac{1}{2}, \frac{7}{16}, \frac{11}{125}, \frac{9}{15}, \frac{11}{22}, \frac{7}{30}, \frac{20}{20}
\]

b. Describe how you can predict whether a fraction will convert to a terminating decimal or a repeating decimal.

**Reversing the process—converting decimals to fractions**

c. Write the decimals 0.25, 0.8, 0.13, and 0.412 as fractions.

You can use what you’ve learned in this chapter about solving equations to help you write an infinite repeating decimal, like \(0.\overline{1}\), as a fraction. For example, to find a fraction equal to \(0.\overline{1}\), you are looking for a fraction \(F\) such that \(F = 0.11111\ldots\). Follow the steps shown.

\[
F = 0.11111\ldots \\
10F = 1.11111\ldots
\]

So, \(10F - F = 1.11111\ldots - 0.11111\ldots\)

\[
9F = 1 \\
F = \frac{1}{9}
\]

Here, the trick was to multiply by 10 so that \(10F\) and \(F\) had the same decimal part. Then, when you subtract \(10F - F\), the decimal portion is eliminated.

d. Write the repeating decimal 0.1\overline{8} as a fraction. (Hint: What can you multiply \(F = 0.\overline{18}\) by so that you can subtract off the same decimal part?)

e. Write these repeating decimals as fractions.

i. \(0.\overline{32}\)  
ii. \(0.\overline{325}\)  
iii. \(0.2\overline{325}\)

### IMPROVING YOUR **REASONING** SKILLS

Did these plants grow at the same rate? If not, which plant was tallest on Day 4? Which plant took the most time to reach 8 cm? Redraw the graphs so that you can compare their growth rates more easily.
Assessing What You’ve Learned

GIVING A PRESENTATION

Making presentations is an important career skill. Most jobs require workers to share information, to help orient new coworkers, or to represent the employer to clients. Making a presentation to the class is a good way to develop your skill at organizing and delivering your ideas clearly and in an interesting way. Most teachers will tell you that they have learned more by trying to teach something than they did simply by studying it in school.

Here are some suggestions to make your presentation go well:

► Work with a group. Acting as a panel member might make you less nervous than giving a talk on your own. Be sure the role of each panel member is clear so that the work and the credit are equally shared.

► Choose the topic carefully. You can summarize the results of an investigation, do research for a project and present what you’ve learned and how it connects to the chapter, or give your own thinking on Take Another Look or Improving Your Reasoning Skills.

► Prepare thoroughly. Outline your presentation and think about what you have to say on each point. Decide how much detail to give, but don't try to memorize whole sentences. Illustrate your presentation with models, a poster, a handout, or overhead transparencies. Prepare these visual aids ahead of time and decide when to introduce them.

► Speak clearly. Practice talking loudly and clearly. Show your interest in the subject. Don’t hide behind a poster or the projector. Look at the listeners when you talk.

Here are other ways to assess what you’ve learned:

UPDATE YOUR PORTFOLIO  Choose a piece of work you did in this chapter to add to your portfolio—your graph from the investigation On the Road Again (Lesson 3.2), the most complicated equation you’ve solved, or your research on a project.

WRITE IN YOUR JOURNAL  What method for solving equations do you like best? Do you always remember to define variables before you graph or write an equation? How are you doing in algebra generally? What things don’t you understand?

ORGANIZE YOUR NOTEBOOK  You might need to update your notebook with examples of balancing to solve an equation, or with notes about how to trace a line or search a table to approximate the coordinates of the solution. Be sure you understand the meanings of important words like linear equation, rate of change, and intercept form.