Lesson 9.1 • Solving Quadratic Equations

1. Sketch the graph of a quadratic equation with
   a. One x-intercept and all nonnegative y-values.
   b. The vertex in the third quadrant and no x-intercepts.
   c. The vertex in the third quadrant and two x-intercepts.
   d. The vertex on the y-axis and two x-intercepts, opening upward.

2. Use a graph and table to approximate solutions for each equation to the nearest hundredth.
   a. \(4(x + 3)^2 + 5 = 10\)
   b. \(-4(x + 3)^2 + 5 = -8\)
   c. \((x + 5)^2 = -4\)
   d. \(3(x - 1)^2 + 1 = 15\)
   e. \(x^2 - 8x + 16 = 25\)
   f. \(x^2 - 4x + 2 = 5\)
   g. \(-3x^2 - 6x + 11 = 14\)
   h. \(2x^2 - 9x = -2\)

3. Use a symbolic method to find exact solutions for each equation. Express each answer as a rational or a radical expression.
   a. \(x^2 = 21\)
   b. \(x^2 - 50 = -1\)
   c. \((x + 1)^2 + 7 = 19\)
   d. \(3(x - 5)^2 + 2 = 17\)
   e. \(2(x + 7)^2 - 9 = -4\)
   f. \(-4(x - 3)^2 = -9\)

4. Classify each number by specifying all the number sets of which it is a member. Consider the sets: real, irrational, rational, integer, whole, and natural numbers.
   a. \(\frac{3}{8}\)
   b. \(\sqrt{5}\)
   c. 0
   d. \(-\sqrt{4}\)
   e. \(6 - \sqrt{2}\)
   f. \(-3.14\)

5. Given the two functions \(f(x) = x^2 - 4x + 5\) and \(g(x) = 3x^2 + 2x - 1\), find each answer without a calculator.
   a. \(f(2)\)
   b. \(f(3)\)
   c. \(f\left(\frac{1}{2}\right)\)
   d. \(f\left(-\frac{1}{2}\right)\)
   e. \(g(-2)\)
   f. \(g(0)\)
   g. \(g(2)\)
   h. \(g\left(\frac{1}{2}\right)\)

6. The equation \(h(t) = -4.9t^2 + 40t\) gives the height in meters at \(t\) seconds of a projectile shot vertically into the air.
   a. What is the height at 2 seconds?
   b. Use a graph or table to find the time(s) when the height is 75 meters.
      Give answers to the nearest hundredth of a second.
   c. At what time(s) is the height 0 meters? Give answers to the nearest hundredth of a second.
   d. What is a realistic domain for \(t\)?
Lesson 9.2 • Finding the Roots and the Vertex

1. Find the equation of the axis of symmetry and the coordinates of the vertex for the parabola given by each function.
   a. \( y = x^2 + 4x - 5 \), with \( x \)-intercepts \(-5\) and 1
   b. \( y = x^2 + 7x - 30 \), with \( x \)-intercepts \(-10\) and 3
   c. \( y = 2x^2 - 11x + 12 \), with \( x \)-intercepts \(\frac{3}{2}\) and 4

2. Consider the equation \((x - 2.5)^2 - 15.5 = 0\).
   a. Solve the equation symbolically. Show each step and give the exact answer.
   b. Solve the equation using a graph or table. Give the answer to the nearest thousandth.
   c. Compare your solutions in 2a and 2b.

3. Find the roots of each equation, to the nearest hundredth, by looking at a graph, zooming in on a calculator table, or both.
   a. \( y = x^2 + 6x + 5 \)
   b. \( y = x^2 + 6x + 7 \)
   c. \( y = -(x + 1)^2 + 2 \)
   d. \( y = -2x^2 + x + 3 \)
   e. \( y = x^2 - x - 12 \)
   f. \( y = 6(x - 2)^2 \)

4. Solve each equation symbolically and check your answer.
   a. \( 2(x - 1)^2 = 16 \)
   b. \( 4(x - 4)^2 = 2 \)
   c. \( \frac{1}{3}(x + 5)^2 + 4 = 12 \)
   d. \( (x + 5)^2 + 13 = 4 \)

5. The equation of a parabola is \( y = x^2 - 7x + 4 \).
   a. Use a graph or table to find the \( x \)-intercepts.
   b. Write the equation of the axis of symmetry.
   c. Find the coordinates \((h, k)\) of the vertex.
   d. Write the equation in vertex form, \( y = a(x - h)^2 + k \).
Lesson 9.3 • From Vertex to General Form

1. Is each algebraic expression a polynomial? If so, how many terms does it have? If not, give a reason why it is not a polynomial.
   a. \(x^2 + 4x - 1\)  
   b. \(12(x^5 - 6)\)  
   c. \(\frac{2}{x} - 3\)  
   d. \(9x^4\)  
   e. \(6x^3 - 4\)  
   f. \(3x^2 - 2x + 1\)  
   g. \(4x^2 - 3x + 2x^{-2}\)  
   h. \(5 + \frac{1}{2}x - \sqrt{3}x^2 + x^3\)  
   i. \(3^{-1}x^2 + 5x - 1\)

2. Expand each expression.
   a. \((x + 1)^2\)  
   b. \((x - 3)^2\)  
   c. \((x + 4)^2\)  
   d. \((x - \frac{1}{2})^2\)  
   e. \(3(x - 5)^2\)  
   f. \(\frac{1}{2}(x - 2)^2\)

3. List the first 15 perfect square whole numbers.

4. Fill in the missing values on each rectangular diagram. Then write a squared binomial and an equivalent trinomial for each diagram.
   a. 
   
   b. 
   
   c. 
   
   d. 
   
   e. 
   
   f. 

5. Convert each equation to general form. Check your answer by entering both equations into the Y= screen on your calculator and comparing their graphs.
   a. \(y = (x + 4)^2 + 1\)  
   b. \(y = (x - 5)^2 - 6\)  
   c. \(y = (x + 1)^2 - 1\)  
   d. \(y = 2(x - 4)^2 + 3\)  
   e. \(y = -4(x + 1)^2 - 2\)  
   f. \(y = (x - 3)^2 + 5\)
Lesson 9.4 • Factored Form

1. Use the zero product property to solve each equation.
   a. \((x - 3)(x - 2) = 0\)
   b. \((x + 7)(x + 1) = 0\)
   c. \(2(x - 2)(x + 2) = 0\)
   d. \(-\frac{1}{2}(x + 3)(x - 4) = 0\)
   e. \(x(x + 5) = 0\)
   f. \((x - 1)(x - 2)(x - 3) = 0\)
   g. \((4x + 3)(3x - 4) = 0\)
   h. \((3x - 6)(2x + 3) = 0\)

2. Graph each equation and then rewrite it in factored form.
   a. \(y = x^2 + 4x - 5\)
   b. \(y = x^2 + 6x + 8\)
   c. \(y = x^2 - 2x - 15\)
   d. \(y = 2x^2 - 12x + 10\)
   e. \(y = -x^2 + 3x + 4\)
   f. \(y = x^2 - 3x - 10\)

3. Name the \(x\)-intercepts of the parabola described by each quadratic equation. Check your answers by graphing the equations.
   a. \(y = (x - 7)(x + 1)\)
   b. \(y = (x + 2)(x - 6)\)
   c. \(y = (x + 8)(x - 8)\)
   d. \(y = 3(x - 5)(x - 4)\)
   e. \(y = (x - 5)^2\)
   f. \(y = (x + 0.5)(x - 3.5)\)

4. Write an equation of a quadratic function that corresponds to each pair of \(x\)-intercepts. Assume there is no vertical shrink or stretch. Write each equation in factored form and in general form.
   a. 3 and \(-1\)
   b. 1 and 5
   c. \(-\frac{1}{2}\) and \(\frac{1}{2}\)
   d. \(-4\) and \(-4\)
   e. \(\frac{1}{2}\) and \(\frac{4}{3}\)
   f. 0.2 and 0.8

5. Consider the equation \(y = 3(x - 2)(x + 2)\).
   a. How many \(x\)-intercepts does the parabola have?
   b. Find the vertex of this parabola.
   c. Write the equation in vertex form. Describe the transformations of the parent function, \(y = x^2\).

6. Reduce each rational expression by dividing out common factors from the numerator and denominator. State any restrictions on the variable.
   a. \(\frac{(x - 3)(x + 2)}{(x + 1)(x - 3)}\)
   b. \(\frac{x^2 + 6x + 8}{x^2 + 3x - 4}\)
   c. \(\frac{x^2 + 10x + 25}{x^2 - 25}\)
Lesson 9.6 • Completing the Square

1. Solve each quadratic equation.
   a. \(x^2 - 121 = 0\)  
   b. \(x^2 - 96 = 0\)
   c. \((x - 3)^2 - 1 = 0\)  
   d. \(2(x + 6)^2 - 8 = 0\)
   e. \(\frac{1}{2}(x - 5)^2 - 3 = 0\)  
   f. \(-3(x + 4)^2 - 20 = 0\)
   g. \(\frac{2}{3}(x - 6)^2 + 3 = 5\)  
   h. \(5(x + 6)^2 - 8 = 0\)
   i. \(-1.5(x + 5)^2 + 7 = 2.5\)

2. Solve each equation.
   a. \((x - 4)(x + 3) = 0\)  
   b. \((x + 9)(x - 9) = 0\)
   c. \((x + 7)(x + 1) = 0\)  
   d. \((3x + 1)(3x - 1) = 0\)
   e. \((3x + 5)(2x - 5) = 0\)  
   f. \((x - 4)(2x + 1)(3x - 2) = 0\)

3. Decide what number you must add to each expression to make a perfect-square trinomial. Then rewrite the expression as a squared binomial.
   a. \(x^2 + 6x\)  
   b. \(x^2 - 20x\)  
   c. \(x^2 - 2x\)
   d. \(x^2 + 7x\)  
   e. \(x^2 - 11x\)  
   f. \(x^2 + 10x\)
   g. \(x^2 + 24x\)  
   h. \(x^2 + \frac{5}{2}x\)  
   i. \(x^2 + (2\sqrt{7})x\)

4. Solve each quadratic equation by completing the square. Leave your answer in radical form.
   a. \(x^2 - 6x - 16 = 0\)  
   b. \(x^2 + 6x - 2 = 0\)
   c. \(x^2 - 16x + 50 = 0\)  
   d. \(x^2 - 4x = 0\)
   e. \(x^2 + 11x = 0\)  
   f. \(x^2 + 5x + 1 = 0\)
   g. \(2x^2 - 12x - 7 = 0\)  
   h. \(-x^2 + 14x - 24 = 0\)
   i. \(x^2 + 2x = -7\)

5. Rewrite each equation in vertex form. Use a graph or table to check your answers.
   a. \(y = x^2 - 8x + 6\)  
   b. \(y = x^2 + 11x\)
   c. \(y = 2x^2 - 24x + 8\)  
   d. \(y = 2(x + 1)(x - 5)\)
Lesson 9.7 • The Quadratic Formula

1. Rewrite each equation in general form. Identify the values of \(a, b, \) and \(c.\)
   a. \(x^2 + 8x = -6\)
   b. \(x^2 = 4x - 4\)
   c. \(3x = x^2\)
   d. \((x + 1)(x - 1) = 0\)
   e. \((x - 4)^2 = -3\)
   f. \((2x + 1)(2x - 3) = 4\)

2. Without using a calculator, use the discriminant, \(b^2 - 4ac,\) to determine the number of real roots for each equation in Exercise 1.

3. Use the quadratic formula to solve each equation. Give your answers in radical form and as decimals rounded to the nearest thousandth.
   a. \(x^2 + x - 6 = 0\)
   b. \(x^2 - 8x + 12 = 0\)
   c. \(2x^2 - 5x - 3 = 0\)
   d. \(x^2 + 7x - 2 = 0\)
   e. \(x^2 - 14x + 8 = 0\)
   f. \(3x^2 + 2x - 1 = 0\)
   g. \(3x^2 + 2x + 1 = 0\)
   h. \(-2x^2 + 3x + 4 = 0\)
   i. \(4x^2 + 12x + 9 = 0\)
   j. \(2x^2 - 6x + 5 = 0\)

4. Which equations from Exercise 3 could be solved by factoring? Explain how you know.

5. Solve each quadratic equation. Give your answers in radical form and as decimals rounded to the nearest hundredth.
   a. \(x^2 - 169 = 0\)
   b. \(x^2 - 82 = 0\)
   c. \((x - 5)^2 - 3 = 0\)
   d. \(2(x + 5)^2 - 9 = 0\)
   e. \(\frac{1}{2}(x - 4)^2 - 2 = 0\)
   f. \(-3(x + 5)^2 - 15 = 0\)
   g. \(\frac{2}{3}(x - 8)^2 + 8 = 3\)
   h. \(5(x + 5)^2 - 9 = 0\)

6. Consider the parabola described by the equation \(f(x) = -3x^2 + 6x + 8.\)
   a. Find the \(x\)-intercepts. Give the answers in radical form and as decimals rounded to the nearest hundredth.
   b. Find the equation of the axis of symmetry.
   c. Write the coordinates of the vertex.
   d. If \(f(x) = 5,\) find \(x.\) Give the answers in radical form and as decimals rounded to the nearest hundredth.
Lesson 9.8 • Cubic Functions

1. Write and solve an equation to find the value of x in each figure.
   a. 
      \[ \text{Volume} = x \]
      
   b. 
      \[ \text{Volume} = 35,937 \text{ cm}^3 \]
      
   c. 
      \[ \text{Volume} = 19,683 \text{ cm}^3 \]

2. Write the equation of the image of \( y = x^3 \) after the transformations.
   a. A translation right 2 units
   b. A translation up 3 units
   c. A translation right 2 units and up 3 units
   d. A vertical shrink of 0.5

3. Factor each expression by removing the largest possible common monomial factor.
   a. \( 15x^2 - 9x + 3 \)
   b. \( 4x^2 + 5x \)
   c. \( 6x^3 - 3x^2 + 12x \)
   d. \( 8x^3 + 12x^2 \)
   e. \( 2x^4 + 6x^3 - 10x^2 + 2x \)
   f. \( 5x^3 + 15x^2 - 25x \)

4. Factor each expression completely.
   a. \( x^3 + 3x^2 + 2x \)
   b. \( x^3 - 9x \)
   c. \( 3x^3 + 6x^2 + 3x \)

5. Name the x-intercepts of each function and write the equation in factored form.
   a. 
      \[ (0, 6), (1, 0), (3, 0), (-2, 0), (2, -4) \]
   b. 
      \[ (-5, 0), (-1, 0), (-4, 0), (0, 20) \]
   c. 
      \[ (0, 18), (-2, 0), (3, 0) \]

6. Use a rectangle diagram to find each missing expression.
   a. \( (5x + 2)(2x^2 + 3x) = (?) \)
   b. \( (2x - 1)(?) = 2x^3 + 5x^2 - 11x + 4 \)
   c. \( (3x - 2)(?) = 9x^3 - 6x^2 - 12x + 8 \)
Lesson 9.8 • Rational Expressions

Name ________________________________  Period ____________  Date ______________

1. Reduce each rational expression to lowest terms. State any restrictions on the variable.
   a. \( \frac{20x^4}{4x^3} \)
   b. \( \frac{(5x^3)(16x^2)}{80x^3} \)
   c. \( \frac{28(x - 5)}{7(x - 5)^2} \)
   d. \( \frac{3x^2}{15x^3} \)
   e. \( \frac{4 + 20x}{20x} \)
   f. \( \frac{15 - 5x^4}{5x} \)
   g. \( \frac{(x - 1)(x + 2)}{(x + 2)(x + 1)} \)
   h. \( \frac{x^2 + 3x + 2}{(x - 4)(x + 2)} \)
   i. \( \frac{x^2 + 2x - 15}{x^2 + 6x + 5} \)
   j. \( \frac{x^2 + 5x + 4}{x^2 - 16} \)
   k. \( \frac{x^2 - 1}{3x^2 - 3} \)
   l. \( \frac{x^2 + 12x + 36}{x^2 - 36} \)

2. Multiply or divide. State any restrictions on the variables.
   a. \( \frac{5}{n^2} \div \frac{10}{n} \)
   b. \( \frac{4x^3}{24x^5} \cdot \frac{12x^4}{15x} \)
   c. \( \frac{4xy^3}{(2x)^3} \div \frac{2y^2}{1} \)
   d. \( \frac{3(x - 6)}{18} \div \frac{4(x + 6)}{8(x - 6)} \)
   e. \( \frac{3c - 6}{8} \div \frac{5c - 10}{6} \)
   f. \( \frac{y + 4}{5y} \div \frac{20}{y^2 + 6y + 8} \)
   g. \( \frac{a^2 - 9}{a + 4} \div \frac{a - 3}{a + 4} \)
   h. \( \frac{x^2 + 3x - 10}{5x} \div \frac{x^2 - 3x}{x^2 - 5x + 6} \)
   i. \( \frac{x^2 - 5x - 6 + x^2 - 4x - 12}{x^2 + 4x + 3 + x^2 + 5x + 6} \)

3. Add or subtract. State any restrictions on the variable.
   a. \( \frac{2x + 5}{2} \)
   b. \( \frac{7}{3x} - \frac{5}{9} \)
   c. \( \frac{x - 2}{12x + 8} + \frac{3}{4} \)
   d. \( \frac{x + 3}{14x + 28} - \frac{2}{7} \)
   e. \( \frac{x + 3}{x + 7} - \frac{1}{7} \)
   f. \( \frac{x + 3}{x + 2} + \frac{x - 1}{x + 4} \)
   g. \( \frac{1}{x^2 - 16} + \frac{1}{4} \)
   h. \( \frac{1}{x} - \frac{10}{x^2(x + 3)} \)
   i. \( \frac{3}{x + 3} + \frac{1}{x^2 + 6x + 9} \)
   j. \( \frac{x - 9}{x^2 - 81} + \frac{1}{x + 9} \)