CONDENSED LESSON

10.1 Solving Quadratic Equations

In this lesson you will

- look at **quadratic functions** that model **projectile motion**
- use tables and graphs to approximate solutions to **quadratic equations**
- **solve quadratic equations** by undoing the order of operations

When an object is projected straight up into the air, its position at any time depends on its starting height, its initial velocity, and the force of gravity. If you plot the object's height at each instant of time, the resulting graph is a parabola. Read Example A in your book and look at the graph of the height of a popped-up baseball over time.

The motion of an object projected into the air can be modeled by a **quadratic function.** A quadratic function is any transformation of the parent function, $f(x) = x^2$.

Investigation: Rocket Science

A model rocket blasts off from a position 2.5 meters above the ground with a starting velocity of 49 m/sec. If the rocket travels straight up, and if the only force acting on it is gravity, then the **projectile motion** of the rocket can be described by the function

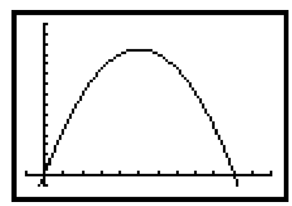
$$h(t) = \frac{1}{2}(-9.8)t^2 + 49t + 2.5$$

where t is the number of seconds after blastoff, and $h(t)$ is the height at time t.

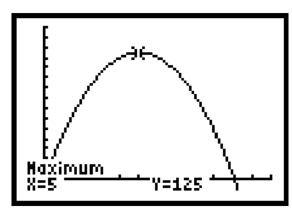
The fact that $h(0) = 2.5$ means that the initial height of the rocket is 2.5 meters.

In metric units, the acceleration due to gravity is 9.8 m/sec^2. This value appears in the t^2 term of the equation $\frac{1}{2}(-9.8)t^2$. The negative symbol shows that the force is downward.

Graph the function on your calculator. Be sure to use a window that shows all the important features of the parabola. Here is the graph in the window $[-1, 12, 1, -10, 150, 10]$.



You can trace the graph to find the coordinates of the highest point (the vertex).



The coordinates are (5, 125), indicating that the rocket reaches a maximum height of 125 meters 5 seconds after blastoff.

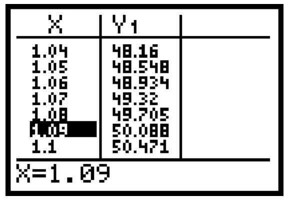
To find the amount of time the rocket is in flight, find the coordinates of the point where the graph intersects the positive x-axis. The coordinates are (10.05, 0), indicating that the rocket hits the ground (that is, the height is 0) after 10.05 seconds. So, the flight of the rocket lasts just over 10 seconds.

To find the value of t when $h(t) = 50$, you would need to solve

$$\frac{1}{2}(-9.8)t^2 + 49t + 2.5 = 50$$

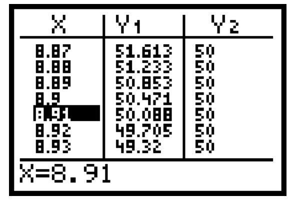
(continued)

A calculator table shows that the approximate solutions to this equation are 1.1 and 8.9, indicating that the rocket is at height 50 meters after about 1.1 seconds (when it is on its way up) and after about 8.9 seconds (when it is on its way down).



X	Y1
1.04	48.16
1.05	48.548
1.06	48.934
1.07	49.32
1.08	49.705
1.09	50.088
1.1	50.471

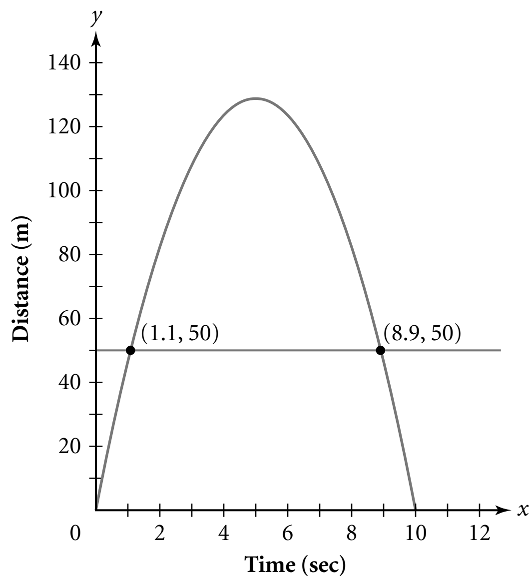
X=1.09



X	Y1	Y2
8.87	51.613	50
8.88	51.233	50
8.89	50.853	50
8.9	50.471	50
8.91	50.088	50
8.92	49.705	50
8.93	49.32	50

X=8.91

On a graph, the solutions are the x-coordinates of the points where the graphs of $h(t) = \frac{1}{2}(-9.8)t^2 + 49t + 2.5$ and $h(t) = 50$ intersect.



In the investigation, you approximated solutions to a quadratic equation using tables and graphs. To solve a quadratic equation using the symbolic methods you are familiar with, you must put the equation in a particular form. Example B in your book solves a quadratic equation by "undoing" the order of operations. Below is another example. (Note: Later, you will learn new methods that let you solve any quadratic equation.)

EXAMPLE Solve $-2(x-1)^2 + 9 = 4$ symbolically.

▶ ***Solution*** Undo each operation as you would when solving a linear equation. Keep the equation balanced by doing the same thing to both sides. To undo squaring, take the square root of both sides.

$-2(x-1)^2 + 9 = 4$ The original equation.

$-2(x-1)^2 = -5$ Subtract 9 from both sides.

$(x-1)^2 = 2.5$ Divide both sides by -2.

$\sqrt{(x-1)^2} = \sqrt{2.5}$ Take the square root of both sides.

$x - 1 = \pm\sqrt{2.5}$ The $\pm$ symbol shows the two numbers $+\sqrt{2.5}$ and $-\sqrt{2.5}$ whose square is 2.5.

$x = 1 \pm \sqrt{2.5}$ Add 1 to both sides.

The two solutions are $1 + \sqrt{2.5}$ and $1 - \sqrt{2.5}$, or approximately 2.58 and -0.58.

Read the rest of the investigation in your book.

CONDENSED LESSON

10.2 Finding the Roots and the Vertex

In this lesson you will

- model a real-word situation with a quadratic function
- identify the ***x*-intercepts, vertex,** and **line of symmetry** of a parabola
- rewrite a quadratic function in **vertex form**

You have looked at quadratic functions that model projectile motion. In this lesson you'll explore another situation that can be modeled with a quadratic function.

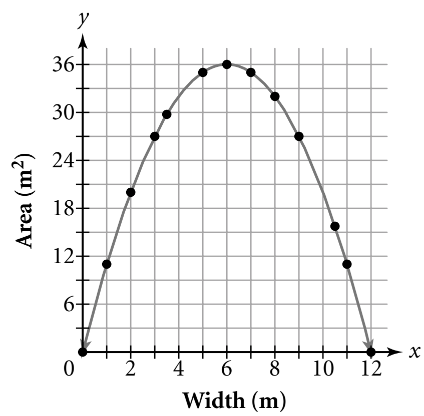
Investigation: Making the Most of It

Suppose you have 24 meters of fencing to use to enclose a rectangular space for a garden. This table shows the width, length, and area of some of the fences you could build.

Width (m)	0	1	3.5	5	6	8	10.5	12
Length (m)	12	11	8.5	7	6	4	1.5	0
Area (sq. m)	0	11	29.75	35	36	32	15.75	0

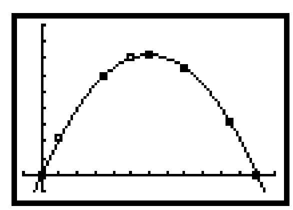
Notice that width values of 0 and 12 give areas of 0. Enter the width values into list L1 and enter the area values into list L2.

Make a graph of the points (x, y), where x is the width of the rectangle and y is the area. Plot additional points to help you determine the shape of the graph. The points appear to fall in the shape of a parabola. Any real-number value between 0 and 12 makes sense for the width, so you can connect the points with a smooth curve.



The graph appears to reach its highest point at (6, 36), indicating that a rectangle with width 6 meters has the greatest area, 36 square meters. The length of this rectangle is also 6 meters, so the rectangle is a square.

A garden with width 2 meters has length 10 meters. A garden with width 4.3 meters has length 7.7 meters. In general, if x is the width of the garden, the length is $12 - x$ and the equation for the area y is $y = x(12 - x)$. In the same window, graph this equation and plot the (L1, L2) values.



$[-1, 13, 1, -1, 45, 5]$

By tracing the graph of $y = x(12 - x)$ to find the coordinates of the vertex, you can verify that the square with side length 6 has the maximum area.

(continued)

The points where a graph crosses the x-axis are called the ***x*-intercepts.** The x-intercepts of the graph above are (0, 0) and (12, 0). This indicates that the rectangle has no area if the width is 0 meter or 12 meters. (Note: When naming an x-intercept, sometimes just the x-coordinate, rather than the ordered pair, is given. So, we might say that the x-intercepts of the graph above are 0 and 12.)

If you repeat this process for different perimeters (total fence lengths), you will find that the rectangle with the greatest area is always a square.

The x-coordinates of the x-intercepts of a graph are the solutions of the equation $f(x) = 0$. These solutions are called the **roots** of the function $y = f(x)$. For the equation in the investigation, the roots are 0 and 12, the width values that make the area equal zero.

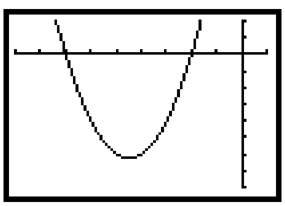
Read the rest of the lesson in your book. Example A shows you how to use your calculator to estimate the roots of a quadratic function. Example B shows how to find the vertex and the **line of symmetry** of a parabola based on its equation and then how to use this information to rewrite the equation in **vertex form.** Here is another example.

EXAMPLE

Find the line of symmetry and the vertex of the parabola $y = x^2 + 9x + 14$. Then, write the equation in vertex form $y = a(x - h)^2 + k$.

▶ Solution

You can use a calculator graph to find that the roots are −7 and −2.



[−9, 1, 1, −8, 2, 1]

The line of symmetry is $x = -4.5$, the vertical line halfway between the x-intercepts. The vertex lies on the line of symmetry, so it has an x-coordinate of −4.5. To find its y-coordinate, substitute −4.5 for x in the equation.

$$y = (-4.5)^2 + 9(-4.5) + 14$$

$$= 20.25 - 40.5 + 14$$

$$= -6.25$$

So, the vertex is (−4.5, −6.25).

The graph is a transformation of the parent function, $f(x) = x^2$. Because the vertex is (−4.5, −6.25), there is a translation left 4.5 units and down 6.25 units, so the equation is of the form $y = a(x + 4.5)^2 - 6.25$. If you graph $y = (x + 4.5) - 6.25$ in the same window as the original equation, you will find that the graphs are the same. So, $y = (x + 4.5)^2 - 6.25$ is the vertex form of $y = x^2 + 9x + 14$.

CONDENSED LESSON 10.3

From Vertex to General Form

In this lesson you will

- draw diagrams to **square expressions**
- draw diagrams to write **trinomials** as squares of expressions
- convert a quadratic equation from **vertex form** to **general form**

In the general form of a quadratic equation, $y = ax^2 + bx + c$, the right side is the sum of three terms. A **term** is an algebraic expression that represents only multiplication and division between variables and constants. A sum of terms with positive integer exponents is called a **polynomial.** Read about polynomials on page 544 of your book.

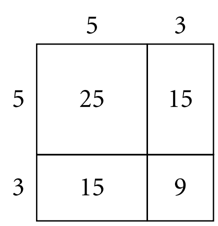
Example A gives you practice identifying polynomials. Read and follow along with this example. Then, read the text about *like terms* that follows the example.

Investigation: Sneaky Squares

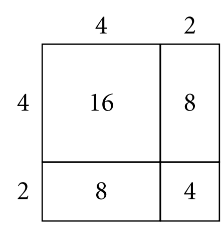
The diagram in Step 1 in your book expresses 7^2 as $(3 + 4)^2$. The sum of the areas of the inner rectangles is $9 + 2(12) + 16 = 49$. This verifies that $7^2 = (3 + 4)^2$.

Now, draw and label a similar diagram for each expression in Step 2. Here are the results for parts a and b.

a. $(5 + 3)^2 = 25 + 2(15) + 9 = 64$

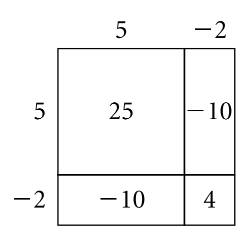


b. $(4 + 2)^2 = 16 + 2(8) + 4 = 36$

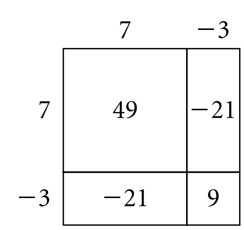


You can use similar diagrams to show the squares of differences, even though negative lengths are involved. The diagram before Step 3 in your book shows 7^2 as $(10 - 3)^2$. Draw and label a diagram for each expression in Step 3. Here are the results for parts a and b.

a. $(5 - 2)^2 = 25 + 2(-10) + 4 = 9$



b. $(7 - 3)^2 = 49 + 2(-21) + 9 = 16$



The expressions in Step 4 involve variables. Draw and label a diagram for each expression. Combine like terms to express each answer as a trinomial. The results for parts a and b are on the next page.

(continued)

a. $(x + 5)^2 = x^2 + 10x + 25$

	x	5
x	x^2	$5x$
5	$5x$	25

b. $(x - 3)^2 = x^2 - 6x + 9$

	x	-3
x	x^2	$-3x$
-3	$-3x$	9

For certain types of trinomials, you can make a rectangular diagram and then write an equivalent expression in the form $(x + h)^2$. Try this for the expressions in Step 5. Here are the results for parts a and b. In each case, the diagram is divided into a square with area equal to the first term, a square with area equal to the last term, and two rectangles, each with area equal to *half* the middle term.

a. $x^2 + 6x + 9 = (x + 3)^2$

	x	3
x	x^2	$3x$
3	$3x$	9

b. $x^2 - 10x + 25 = (x - 5)^2$

	x	-5
x	x^2	$-5x$
-5	$-5x$	25

You can use the results from Step 5 to solve the equations in Step 6. Here are the results for parts a and b.

$$
\begin{aligned}
x^2 + 6x + 9 &= 49 \\
(x + 3)^2 &= 49 \\
\sqrt{(x + 3)^2} &= \sqrt{49} \\
x + 3 &= \pm 7 \\
x &= -3 \pm 7 \\
x &= 4 \text{ or } x = -3
\end{aligned}
$$

$$
\begin{aligned}
x^2 - 10x + 25 &= 81 \\
(x - 5)^2 &= 81 \\
\sqrt{(x - 5)^2} &= \sqrt{81} \\
x - 5 &= \pm 9 \\
x &= 5 \pm 9 \\
x &= 14 \text{ or } x = -4
\end{aligned}
$$

Numbers such as 25 are called **perfect squares** because they are the squares of integers, in this case, 5 and -5. The trinomial $x^2 - 10x + 25$ is also called a perfect square because it is the square of $x - 5$. If the coefficient of the x^2-term is 1, then a trinomial is a perfect square if the last term is the square of half the coefficient of the x-term. Use this idea to identify the perfect squares in Step 7. Here are the results.

a. Because 49 is equal to the square of half of 14, this is a perfect-square trinomial: $x^2 + 14x + 49 = (x + 7)^2$.

b. Because 81 is equal to the square of half of -18, this is a perfect-square trinomial: $x^2 - 18x + 81 = (x - 9)^2$.

c. This is not a perfect-square trinomial because 25 is not equal to the square of half of 20.

d. This is not a perfect-square trinomial because -36 is not equal to the square of half of -12.

You can use your skills at squaring binomials to convert equations from vertex form to general form. This is illustrated in Example B in your text. Read that example and the text that follows it.

CONDENSED LESSON

10.4 Factored Form

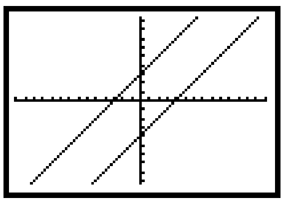
In this lesson you will

- work with quadratic equations in **factored form**
- learn the connection between the factored form of a quadratic equation and the equation's roots
- write the equation of a parabola in three different forms

You have worked with quadratic equations given in vertex form and in general form. In this lesson you will learn about the **factored form** of a quadratic equation.

Investigation: Getting to the Root of the Matter

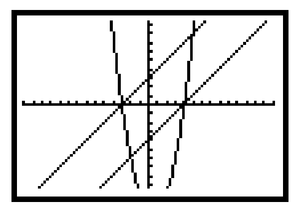
Steps 1–4 In the same window, graph $y = x + 3$ and $y = x - 4$.



$[-14.1, 14.1, 1, -9.3, 9.3, 1]$

The x-intercept of $y = x + 3$ is -3. The x-intercept of $y = x - 4$ is 4.

Now, in the same window, graph $y = (x + 3)(x - 4)$. The graph is a parabola.



The x-intercepts of the parabola are -3 and 4—the x-intercepts of $y = x + 3$ and $y = x - 4$, respectively. This makes sense because the product $(x + 3)(x - 4)$ is zero when $x + 3$ is zero or when $x - 4$ is zero.

You can use a rectangular diagram to expand the expression $(x + 3)(x - 4)$ and then rewrite $y = (x + 3)(x - 4)$ as $y = x^2 - x - 12$. Verify that the two equations are equivalent by graphing both on the same axes. Because the equations are identical, you know that the *roots* of $y = x^2 - x - 12$ are -3 and 4.

	x	-4
x	x^2	$-4x$
3	$3x$	-12

Steps 5–8 If you are given a quadratic equation in general form, you can sometimes use a rectangular diagram to rewrite it in factored form and then find its roots.

Consider the equation $y = x^2 + 5x + 6$. The diagram shows that you can rewrite the left side as $(x + 3)(x + 2)$. So, in factored form, the equation is $y = (x + 3)(x + 2)$. Use a calculator graph or table to verify that $y = x^2 + 5x + 6$ and $y = (x + 3)(x - 2)$ are equivalent.

	x	2
x	x^2	$2x$
3	$3x$	6

From the factored form, you can see that the roots of $y = x^2 + 5x + 6$ are -3 and -2.

(continued)

Now, use rectangle diagrams to rewrite each equation in Step 8 in factored form and find its roots. Here are the results.

a. $y = (x - 5)(x - 2)$; roots: 5 and 2

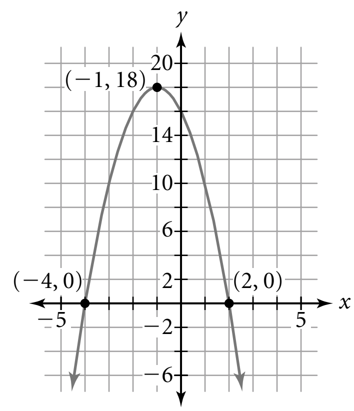
b. $y = (x + 8)(x - 2)$; roots: -8 and 2

c. $y = (x - 6)(x + 8)$; roots: 6 and -8

d. $y = (x - 7)(x - 4)$; roots 7 and 4

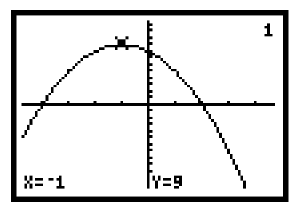
Now, read the text after the investigation on page 552 in your book, which summarizes the three forms of a quadratic equation. Then, read the example, which shows you how to write an equation of a parabola in all three forms, and read the text that follows the example. Here is an additional example.

EXAMPLE Write the equation for this parabola in vertex form, general form, and factored form.



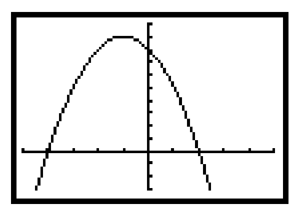
▸ *Solution* From the graph, you can see that the x-intercepts are -4 and 2. So, the factored form contains the binomial factors $(x + 4)$ and $(x - 2)$. You can also see that, because the parabola is "upside down," the coefficient of x^2 must be negative.

If you graph $y = -(x + 4)(x - 2)$ on your calculator, you'll see that it has the same x-intercepts as the graph above, but a different vertex.



$[-4.7, 4.7, 1, -12.4, 12.4, 1]$

The vertex of $y = -(x + 4)(x - 2)$ is $(-1, 9)$, while the vertex of the original parabola is $(-1, 18)$. So, the original parabola is a vertical stretch of the graph of $y = -(x + 4)(x - 2)$ by a factor of $\frac{18}{9}$, or 2. This means the factored form is $y = -2(x + 4)(x - 2)$. Graph this equation on your calculator to verify that its graph looks like the original graph.



$[-5, 5, 1, -6, 20, 2]$

You now know that the value of a is -2 and the vertex is $(-1, 18)$, so you can write the vertex form of the equation, $y = -2(x + 1)^2 + 18$. To get the general form, expand either the factored or vertex form.

$y = -2(x + 1)^2 + 18$	The vertex form.
$y = -2(x^2 + 2x + 1) + 18$	Use a rectangle diagram to expand $(x + 1)^2$.
$y = -2x^2 - 4x - 2 + 18$	Use the distributive property.
$y = -2x^2 - 4x + 16$	Combine like terms.

CONDENSED LESSON

10.6 Completing the Square

In this lesson you will

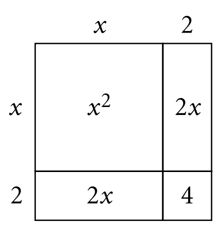
- solve quadratic equations by **completing the square**
- rewrite quadratic equations in vertex form by completing the square
- work with a quadratic equation that has **no real-number solutions**

You can find approximate solutions to quadratic equations by using calculator graphs and tables. If you are able to write the equation in factored form or in vertex form, you can use symbolic methods to find the exact solutions. In this lesson you'll learn a symbolic method called **completing the square** that you can use to find exact solutions to any quadratic equation in the general form, $y = ax^2 + bx + c$.

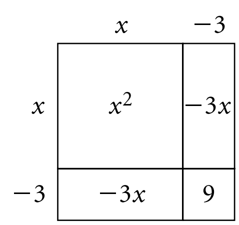
Investigation: Searching for Solutions

Complete each rectangular diagram in Step 1 so that it is a square. Here are the results for parts a, b, and d.

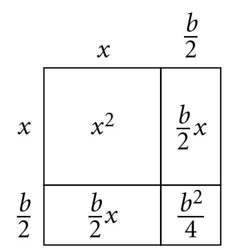
a.



b.



d.



Using the diagrams, you can write equations of the form $x^2 + bx + c = (x + h)^2$. Here are the equations for parts a, b, and d. Write a similar equation for part c.

a. $x^2 + 4x + 4 = (x + 2)^2$ **b.** $x^2 - 6x + 9 = (x - 3)^2$ **d.** $x^2 + bx + \frac{b^2}{4} = \left(x + \frac{b}{2}\right)^2$

Notice that the operations on the right side of each equation can be "undone" to get x. For example, you can undo $(x + 2)^2$ by taking the square root of $x + 2$ and then subtracting 2. You cannot undo the operations on the left side of the equation to end up with x, so the $(x + h)^2$ form is used to solve problems.

If you assume the area of each square in Step 1 is 100, you can write equations that can be solved by undoing the order of operations. Here are the equations for parts a and b. Write a similar equation for part c.

a. $(x + 2)^2 = 100$ **b.** $(x - 3)^2 = 100$

Here are the symbolic solutions of the equations above. Write a similar solution of the equation for part c.

a.
$$\begin{aligned} (x + 2)^2 &= 100 \\ \sqrt{(x + 2)^2} &= \sqrt{100} \\ x + 2 &= \pm 10 \\ x &= -2 \pm 10 \\ x &= -12 \text{ or } x = 8 \end{aligned}$$

b.
$$\begin{aligned} (x - 3)^2 &= 100 \\ \sqrt{(x - 3)^2} &= \sqrt{100} \\ x - 3 &= \pm 10 \\ x &= 3 \pm 10 \\ x &= 13 \text{ or } x = -7 \end{aligned}$$

(continued)

Lesson 10.6 • Completing the Square (continued)

When an equation cannot easily be rewritten in factored or vertex form, you can solve it by completing the square. The example below illustrates this method. Make sure you understand what is happening at each step.

$x^2 + 6x - 1 = 0$	The original equation.
$x^2 + 6x = 1$	Add 1 to both sides.
$x^2 + 6x + 9 = 1 + 9$	Add 9 to both sides, making the left side a perfect-square trinomial.
$(x + 3)^2 = 10$	Rewrite the trinomial as a squared binomial.
$x + 3 = \pm\sqrt{10}$	Take the square root of both sides.
$x = -3 \pm \sqrt{10}$	Add -3 to both sides.

The solutions are $-3 + \sqrt{10}$, which is about 0.162, and $-3 - \sqrt{10}$, which is about -6.162. You can check these solutions by graphing $y = x^2 + 6x - 1$ and finding the x-intercepts or by making a table and finding the x-values that correspond to the y-value 0.

Here are the steps for solving $x^2 + 8x - 5 = 0$.

$x^2 + 8x - 5 = 0$	The original equation.
$x^2 + 8x = 5$	Add 5 to both sides.
$x^2 + 8x + 16 = 5 + 16$	Add 16 to both sides, making the left side a perfect-square trinomial.
$(x + 4)^2 = 21$	Rewrite the trinomial as a squared binomial.
$x + 4 = \pm\sqrt{21}$	Take the square root of both sides.
$x = -4 \pm \sqrt{21}$	Add -4 to both sides.

The solutions are $-4 + \sqrt{21}$, which is about 0.583, and $-4 - \sqrt{21}$, which is about -8.583.

The key to solving a quadratic equation by completing the square is to express one side as a perfect-square trinomial. In the investigation the equations were in the form $y = 1x^2 + bx + c$. Example A in your book shows how to solve a quadratic equation when the coefficient of x^2 is not 1. Read this example carefully. Then, read Example B, which shows how to complete the square to convert an equation to vertex form.

In the solution of $2(x + 2)^2 + 3 = 0$ in Example B, the final step is $x = -2 \pm \sqrt{-\frac{3}{2}}$. Because a negative number does not have a real-number square root, the equation has no real-number solutions. The graph of the equation $y = 2(x + 2)^2 + 3$ on page 563 can help you understand why there are no solutions. The graph does not cross the x-axis, so there is no real-number value of x that makes $2(x + 2)^2 + 3 = 0$ true.

CONDENSED LESSON 10.7

The Quadratic Formula

In this lesson you will

- see how the **quadratic formula** is derived
- use the quadratic formula to solve equations

If a quadratic equation is given in vertex form or if it has no x-term, you can solve it by undoing the operations or by keeping the equation balanced. If the equation is in factored form, you can solve it by finding the x-values that make the factors equal to zero. If the equation is a perfect-square trinomial, you can factor it and then find the solutions. Look at the six equations given at the beginning of the lesson, and think about how you would solve each one.

In the previous lesson you learned how to solve quadratic equations by completing the square. Unfortunately, this method can sometimes be messy. For example, try solving $-4.9x^2 + 5x - \frac{16}{3} = 0$ by completing the square.

Consider the general quadratic equation $ax^2 + bx + c = 0$. In this lesson you will see how completing the square in this general case leads to a formula that can be used to solve any quadratic equation.

Investigation: Deriving the Quadratic Formula

Consider the equation $2x^2 + 3x - 1 = 0$. This equation is in the general form, $ax^2 + bx + c = 0$, where $a = 2$, $b = 3$, and $c = -1$.

In this investigation, we will show the steps for solving the general equation, $ax^2 + bx + c = 0$, on the left and the steps for solving the particular equation, $2x^2 + 3x - 1 = 0$, on the right.

First, group all the variable terms on the left side of the equation.

$ax^2 + bx = -c$	$2x^2 + 3x = 1$

To complete the square, the coefficient of x^2 must be 1. So, divide both sides of the equation by the value of a.

$x^2 + \frac{b}{a}x = -\frac{c}{a}$	$x^2 + \frac{3}{2}x = \frac{1}{2}$

Complete the square of the left side of the equation by adding the square of half the coefficient of x. Add this same value to the right side.

$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$	$x^2 + \frac{3}{2}x + \left(\frac{3}{4}\right)^2 = \left(\frac{3}{4}\right)^2 + \frac{1}{2}$

Rewrite the trinomial on the left side of the equation as a squared binomial. On the right side, rewrite the fractions with a common denominator.

$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{4ac}{4a^2}$	$\left(x + \frac{3}{4}\right)^2 = \frac{9}{16} + \frac{8}{16}$

Take the square root of both sides.

$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{\sqrt{4a^2}}$	$x + \frac{3}{4} = \pm\frac{\sqrt{9 + 8}}{\sqrt{16}}$

(continued)

Get x by itself on the left side.

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} \qquad x = -\frac{3}{4} \pm \frac{\sqrt{17}}{4}$$

Express the solutions as fractions.

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \qquad x = \frac{-3 + \sqrt{17}}{4}$$

$$\text{or } x = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \qquad \text{or } x = \frac{-3 - \sqrt{17}}{4}$$

In decimal form, the solutions to $2x^2 + 3x - 1 = 0$ are approximately 0.281 and -1.781.

Look at the solutions to the general form. Notice that, for the solutions to be real numbers, the value of a cannot be zero (because division by zero is undefined) and the value of $b^2 - 4ac$ must be greater than or equal to zero (because negative numbers do not have real square roots).

The quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, gives the solutions to a quadratic equation written in the general form, $ax^2 + bx + c = 0$. To use the formula, you need only know the values of a, b, and c. The example in your book illustrates how to use the formula. Here is another example.

EXAMPLE Use the quadratic formula to solve $2x^2 - 9 = x$.

▶ ***Solution*** First, write the equation in standard form by subtracting x from both sides. The result is $2x^2 - x - 9 = 0$. For this equation, $a = 2$, $b = -1$, and $c = -9$. Now, substitute these values into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-9)}}{2(2)}$$

$$x = \frac{1 \pm \sqrt{1 - (-72)}}{4}$$

$$x = \frac{1 \pm \sqrt{73}}{4}$$

So, the solutions are $\frac{1 + \sqrt{73}}{4}$, or about 2.386, and $\frac{1 - \sqrt{73}}{4}$, or about -1.886.

Read the rest of the lesson in your book.

CONDENSED LESSON

10.8 Cubic Functions

In this lesson you will

- determine whether given numbers are **perfect cubes**
- discover the connection between the factored form of a **cubic equation** and its graph
- write equations for **cubic functions** based on their graphs

Read the text at the bottom of page 572 of your book. It explains that the cubing function, $f(x) = x^3$, models the volume of a cube with edge length x. Functions in the family with parent function $f(x) = x^3$ are called *cubic functions.*

The volume of a cube with edge length 5 is 5^3, or 125. The number 125 is a **perfect cube** because it is equal to an integer cubed (that is, raised to the third power). The number 5 is called the **cube root** of 125. You can express this by writing $5 = \sqrt[3]{125}$.

EXAMPLE A Determine which numbers are perfect cubes.

a. 24,389 **b.** 1,428 **c.** 270 **d.** 4,913

▸ ***Solution*** Find the cube root of each number. (See **Calculator Note 10B.**)

a. $\sqrt[3]{24{,}389} = 29$ **b.** $\sqrt[3]{1{,}428} \approx 11.261$

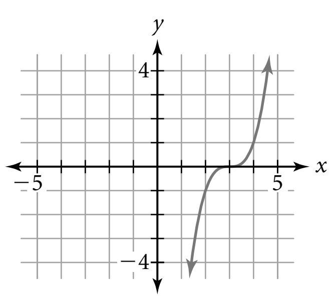
c. $\sqrt[3]{270} \approx 6.463$ **d.** $\sqrt[3]{4{,}913} = 17$

So, 24,389 and 4,913 are perfect cubes because their cube roots are integers.

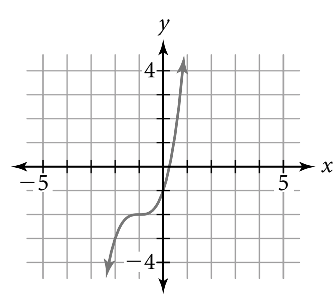
The graph of the parent function $y = x^3$ is shown on page 572 of your book. You can use what you know about transformations to write equations for other cubic functions.

EXAMPLE B Write an equation for each graph.

a.



b.



▸ ***Solution***

a. The graph is a translation of the graph of $y = x^3$ right 3 units. So, the equation is $y = (x - 3)^3$.

b. The graph is a translation of the graph of $y = x^3$ left 1 unit and down 2 units. So, the equation is $y = (x + 1)^3 - 2$.

(continued)

Lesson 10.8 • Cubic Functions (continued)

Investigation: Rooting for Factors

In this investigation you'll discover the connection between the factored form of a cubic equation and its graph.

Steps 1–3 List the x-intercepts for each graph in Step 1. Here are the results.

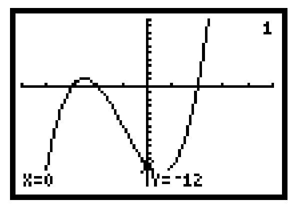
Graph A: $-2, 1, 2$	Graph B: $-2, 1, 2$	Graph C: $-1, 0, 2$
Graph D: $-1, 0, 2$	Graph E: $-2, 1, 3$	Graph F: $-3, -1, 2$

Use tables and graphs to help you match the equations in Step 2 with the graphs in Step 1. Here are the results.

a. Graph F	**b.** Graph C	**c.** Graph A
d. Graph D	**e.** Graph B	**f.** Graph E

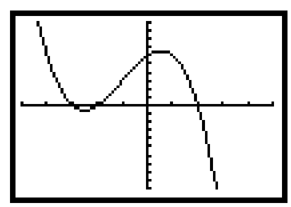
The x-intercepts are the roots of the equation. So, if the graph of a cubic equation has x-intercept a, the factored form of the equation includes the factor $(x - a)$. For example, Graph A has x-intercepts at -2, 1, and 2, and its equation includes the factors $(x + 2)$, $(x - 1)$, and $(x - 2)$.

Step 4 Now, you will find an equation for the graph in Step 4. It has x-intercepts -3, -2, and 2. The equation $y = (x + 3)(x + 2)(x - 2)$ has the same x-intercepts. Here is its graph.



$[-5, 5, 1, -15, 10, 1]$

Now, make adjustments to the equation until the graph looks like the one in Step 4. The point $(0, -12)$ corresponds to the point $(0, 6)$ in the original graph. So, you need to reflect the graph over the x-axis and apply a vertical shrink by a factor of 0.5. The equation then becomes $y = -0.5(x + 3)(x + 2)(x - 2)$. If you graph this equation on your calculator, you'll see that the result matches the graph in Step 4.


$[-5, 5, 1, -10, 10, 1]$

Read Example C in your book, which shows how to find an equation for another cubic graph.

If you are given a cubic equation in general form, you can graph it and then use the x-intercepts to help you write the equation in factored form. The text after Example C shows how to rewrite $y = x^3 - 3x + 2$ in factored form. The graph of this equation touches the x-axis at $x = 1$ but doesn't actually pass through the axis at this point. This indicates a *double root*. This means that the factor $x - 1$ appears twice in the equation. The factored form of the equation is $y = (x + 2)(x - 1)^2$.

