Traveling at a Constant Rate

Graphs don’t just tell stories; they can also be very useful in making predictions. The lives of travelers on the Overland Trail often depended on their ability to foresee accurately what could happen to them. The data they worked with didn’t often fit neatly into simple rules, so they had to make approximations.

As travelers set out from Fort Laramie toward Fort Hall, Idaho, one of the decisions they made involved a shortcut called Sublette’s Cutoff. When you get to that activity, think about what choice you might have made.

Your experience up to this point on the journey has laid the groundwork for understanding that relationships can be represented in four interrelated ways—as situations, graphs, tables, and rules. Fortunately, you have modern technology, which was not available to help nineteenth-century emigrants see the connections among these mathematical representations.

Rodrick Rogers creates a graph based on data.
Previous Travelers

The first settlers had to make the long journey west without advice from previous travelers. However, later wagon trains used information from the early travelers to decide on the supplies to buy for their journey.

While in Fort Laramie, Wyoming, you get a letter from friends describing the supplies they and others used on the leg of the trip from Fort Laramie to Fort Hall, Idaho. Use the information in the letter to answer these questions.

1. Make a graph displaying all the data relating the number of people and the amount of beans they used. Use appropriate scales for the axes. Then make a similar graph for sugar and another for gunpowder.

2. Do these steps for each graph in Question 1.
   a. Sketch what you consider to be the line of best fit for the graph; that is, find the straight line that you think best fits the data.
   b. Make an In-Out table from your line. Determine a rule that fits your table or that comes reasonably close.
   c. Use either the In-Out table or your graph to find the quantity of each item needed for each of your group’s four family units.
Dear friends,

We’ve arrived in Fort Hall after many adventures, both good and bad. It would take me forever to describe all that happened. We can talk about all of that when we meet up again in California.

I know that you’re anxious for some practical information for your own trip. Several of the families on our wagon train kept track of the quantities of various goods they actually needed on the journey. The families were of different sizes, so this information should help you and your friends decide how the amounts vary from group to group. I know this won’t answer all your questions, but it’s a start.

<table>
<thead>
<tr>
<th>Number of people</th>
<th>Pounds of beans</th>
<th>Pounds of sugar</th>
<th>Pounds of gunpowder</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>61</td>
<td>20</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>95</td>
<td>50</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>56</td>
<td>30</td>
<td>2.5</td>
</tr>
<tr>
<td>7</td>
<td>75</td>
<td>23</td>
<td>4.1</td>
</tr>
<tr>
<td>11</td>
<td>125</td>
<td>60</td>
<td>5</td>
</tr>
<tr>
<td>10</td>
<td>135</td>
<td>40</td>
<td>5.8</td>
</tr>
<tr>
<td>5</td>
<td>80</td>
<td>39</td>
<td>1.8</td>
</tr>
<tr>
<td>7</td>
<td>100</td>
<td>44</td>
<td>3.8</td>
</tr>
<tr>
<td>10</td>
<td>103</td>
<td>53</td>
<td>4.3</td>
</tr>
<tr>
<td>6</td>
<td>75</td>
<td>35</td>
<td>3.6</td>
</tr>
<tr>
<td>8</td>
<td>100</td>
<td>35</td>
<td>3.2</td>
</tr>
<tr>
<td>7</td>
<td>105</td>
<td>36</td>
<td>3.1</td>
</tr>
<tr>
<td>9</td>
<td>125</td>
<td>45</td>
<td>4.7</td>
</tr>
<tr>
<td>12</td>
<td>150</td>
<td>55</td>
<td>6.1</td>
</tr>
<tr>
<td>10</td>
<td>125</td>
<td>31</td>
<td>5.2</td>
</tr>
</tbody>
</table>

Well, good luck to you all!

The Helmicks
Broken Promises

Over the years, Native Americans were forced onto smaller and smaller parcels of land. Though the U.S. government signed treaties with the native peoples, the government repeatedly broke those treaties.

The map labeled 1492 shows the outline of the contiguous 48 states of the United States. In each of the remaining maps, the dark portion represents that part of the land remaining to Native Americans in the given year.

1. The total area shown for 1492 is approximately 3,000,000 square miles. Using that estimate, approximate the area of Native American land at each of the given times. Suggestion: You may want to trace each map onto grid paper and use one square as the unit of area.

2. Make a graph showing the relationship between the passage of time and the area of Native American land. Be careful to use appropriate spacing on your time scale.

3. If it were 1861 and you were only looking at the part of your graph showing what had happened up to that year, what would you predict as the area of Native American land in the year 2020?

4. Based on the graph, what prediction might you have made in 1900 about Native American land in the year 2020?
Sublette’s Cutoff

As more and more emigrants made the journey west, scouts found shortcuts to lessen the travel time. Help provided by Native Americans in the area was also important in facilitating the journey.

The Shoshone, Assiniboin, and Crow nations were prominent in the area of what is now Wyoming and Idaho.

One shortcut between Fort Laramie and Fort Hall was known as Sublette’s Cutoff. The cutoff began just past South Pass in Wyoming, about 250 miles west of Fort Laramie. It ended near the Wyoming-Idaho border.

This shortcut saved 50 miles and a week of travel, but it crossed a dry and barren stretch of land. It was a grueling route of 15 days with little grass and no water.

Three families decide to attempt to cross Sublette’s Cutoff. The table shows how much water each family has left at the end of the second, fifth, and ninth days.

1. Graph the water supply data for all three families on the same set of axes. Colored pens or pencils might help.

2. Based on this information, who do you think will make it, and who will not? Explain your reasoning.

3. Is there a time when all three families will have about the same amount of water left? If so, when?

4. Estimate how much water each family used per day and how much water each family started with.

<table>
<thead>
<tr>
<th>Gallons of Water Remaining</th>
</tr>
</thead>
<tbody>
<tr>
<td>Family</td>
</tr>
<tr>
<td>Jones</td>
</tr>
<tr>
<td>Sanford</td>
</tr>
<tr>
<td>Minto</td>
</tr>
</tbody>
</table>
Who Will Make It?

Dagny Appel bought an almanac at the trading post in Fort Laramie. The almanac included predictions about the weather, crops, and livestock.

Dagny was a tireless planner. She shared the predictions with almost everyone in her wagon train.

One particular prediction worried Dagny: The Green River, some 330 miles away, was expected to flood in 30 days, which was about how long it might take to get there.

Three wagon trains kept track of their distances remaining from the Green River. The table shows how far each wagon train was from the river at the end of three different days.

<table>
<thead>
<tr>
<th>Distance to the River (in miles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wagon train</td>
</tr>
<tr>
<td>------------</td>
</tr>
<tr>
<td>Fowler</td>
</tr>
<tr>
<td>Belshaw</td>
</tr>
<tr>
<td>Clappe</td>
</tr>
</tbody>
</table>

1. Graph the data for all three wagon trains on the same set of axes. Colored pens or pencils might help. Sketch your line of best fit for each family’s data.

2. If the almanac is correct about when the flood will take place, who will make it to the Green River before the flood, and who will not? Explain your reasoning.

3. If the almanac is wrong and all three groups make it past the river, which wagon train do you think will arrive at the river first? Last? Explain your reasoning.

4. When the first of these wagon trains gets to the Green River, how far back is the next wagon train? What about the last wagon train? Explain how you found your answers.

5. Estimate the distance covered per day for each wagon train.
The Basic Student Budget

Cal, Bernie, and Doc are college students on budgets.

Sometimes the three have a little difficulty keeping to their budgets. Their biggest problem is the rent.

The total rent for their apartment is $900, which is split evenly among the three roommates. The rent is due on the last day of each month. The guys don’t get paid until the first day of the next month.

Their landlord has no tolerance for late payments.

Each student had a different amount of money after being paid on April 1. At the end of that day, Cal had $1,100, Bernie had $800, and Doc had $600. As the month goes by, they each occasionally note how much they had left at the end of the day.

The table shows their records so far.

<table>
<thead>
<tr>
<th>Date</th>
<th>Cal</th>
<th>Bernie</th>
<th>Doc</th>
</tr>
</thead>
<tbody>
<tr>
<td>April 3</td>
<td>996</td>
<td>766</td>
<td>570</td>
</tr>
<tr>
<td>April 10</td>
<td>704</td>
<td>698</td>
<td>490</td>
</tr>
<tr>
<td>April 17</td>
<td>440</td>
<td>626</td>
<td>430</td>
</tr>
</tbody>
</table>

---

*continued*
1. Sketch and label a graph that accurately represents this situation. Show all three students on the same graph.

2. Who will be able to pay his rent on time, and who will not? How do you know?

3. It’s April 21, and there’s a great concert on campus. This would be an extra cost, beyond the three students’ normal expenses. How much, if anything, can each one spend and still have enough for rent money on the morning of April 30?

4. Suppose each roommate actually starts May with the same amount of money with which he started April. Find an approximate rule for each roommate that will tell him how much money he should expect to have at the end of the xth day of May if his spending habits don’t change.
1. The Buck family crossed the Green River and continued on toward Fort Hall. Along the way, they decided to create a graph showing how far they had gone since crossing the Green River. They started their graph on the morning of July 12, at which time they had gone 50 miles since crossing the Green River. Over the next 10 days, they traveled 15 miles each day.

The Woods family crossed the Green River after the Bucks did. They also started a graph on the morning of July 12. As of that morning, they had gone 10 miles from the Green River. Their rate from that point on was 20 miles each day.

a. On a single set of axes, show what each family’s graph might have looked like over the ten days, beginning with the morning of July 12.

Suppose you are a historian and you just found their two graphs.

b. How would you know from the graphs that on the morning of July 12, the Buck family was farther from the Green River than the Woods family was?

c. How would you know from the graphs that the Woods family traveled faster than the Buck family traveled?

d. As a historian, you might build a database recording westward travel. For these records, you could use equations to document travel. Write an equation describing each family’s distance from the Green River after \(d\) days.
2. As of the morning of July 12, the Buck family has 100 pounds of coffee, while the Woods family has only 70 pounds. Each family consumes 5 pounds of coffee per day.

   a. On a single set of axes, make graphs showing how much coffee each family has remaining over the next ten days, beginning with the morning of July 12.

   b. How does the pair of graphs show that the Buck family starts with more coffee than the Woods family?

   c. How does the pair of graphs show that the two families are consuming coffee at the same rate?

   d. For each family, write an equation that describes the amount of coffee they have remaining after \(d\) days.

3. Is there a time when the two families are the same distance from the Green River? If so, when is it? How far are they from the Green River at that time? If not, why not?

4. Is there a time when the two families have the same amount of coffee? If so, when is it? How much coffee do they have at that time? If not, why not?
Graphing Calculator In-Outs

Some of the things you have been doing with paper graphs can also be done with technology.

You will probably decide that some problems are easier to do on a graphing calculator than on paper, whereas others are easier to do on paper. This activity will help you learn to use technology.

1. In *Previous Travelers*, you found a rule for estimating the number of pounds of beans needed for different numbers of people making the trip from Fort Laramie to Fort Hall. Here is a similar equation.

\[
\text{Number of pounds of beans} = 12 \cdot (\text{number of people})
\]

**a.** Enter and graph this function on a graphing calculator.

Now use the trace feature to answer parts b and c.

**b.** According to your graph, how many pounds of beans are needed for 20 people?

**c.** A certain family brought 155 pounds of beans. According to your graph, how many people can they feed?

2. In *To Kearny by Equation*, you were given an equation for the profit that the Papan brothers made from their ferry service. That equation depended in part on the amount of the captain’s travel time. Suppose the Papans decided to pay the captain $3 per day, regardless of the amount of business.

In that case, the profit the brothers would get for one day could be determined by this equation.

\[
\text{profit} = 2W - 3
\]
Here, $W$ is the number of wagons that the ferry captain takes across the river.

**a.** Enter and graph this function on your graphing calculator.

Now use the trace feature to answer parts b and c.

**b.** How much profit do the Papans make when 25 wagons use their ferry?

**c.** How many wagons will the ferry have to carry for the Papans to make a profit of $25?

3. The In-Out table shown here is for the function $Y = 3X^2 - 7X + 2$

Use technology to graph this function. Then use zoom and trace features to find the missing entries.

Where the Out is given, find all possible In values that will give the desired Out. If there aren’t any, write “none.”

Give your answers to the nearest tenth.
1. The Winstons were a large family seeking fortune in California, but they found the travels grueling. When they saw the beauty of Fort Hall and the opportunity to open a supply store in a rapidly growing community, they decided to settle there. From the time they opened the store, they had been able to add $50 to their bank account at the end of each month. They had some money in the account when they opened the store. Four months after the store opened, the account had grown to $360.

a. As the family tried to clean up its bookkeeping procedures, they realized they had lost their original deposit slip. Based on the information above, determine how much money was in the account when they opened the store.

b. Write a rule to express how much money would be in the Winstons’ account, using $x$ for the number of months since the store opened.

c. Determine when the Winstons’ account would reach $1,000. Explain your result.

2. There’s a great show at the Grand Old Theater, and tickets are all the same price. Shortly after the box office opens, the manager is told that so far, the theater has sold 20 tickets, and the cash register now contains $40. A little later he is told that the theater has now sold a total of 60 tickets and the cash register now contains $70.

a. How much money did the theater charge for each ticket?

b. How much money was in the register before any tickets were sold?

c. Write a rule for how much money was in the register after $p$ people bought tickets.

d. How much money was in the register after 250 people had bought tickets?
Sublette’s Cutoff Revisited

In Sublette’s Cutoff, you were given data showing the amount of water each of three families had at the end of certain days. You plotted the data and based your analysis on a pencil-and-paper graph.

Now, you will reexamine that situation, with a slightly different goal and using a different technique.

Here are the data.

<table>
<thead>
<tr>
<th>Family</th>
<th>Day 2</th>
<th>Day 5</th>
<th>Day 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jones</td>
<td>49</td>
<td>40</td>
<td>26</td>
</tr>
<tr>
<td>Sanford</td>
<td>68</td>
<td>48</td>
<td>25</td>
</tr>
<tr>
<td>Minto</td>
<td>30</td>
<td>21</td>
<td>14</td>
</tr>
</tbody>
</table>

For each family, use these steps to predict how much water they would have at the end of Day 15.

- Plot the data on a graphing calculator.
- Leave the data on the screen and graph a linear function that you think might approximate the data well.
- Examine how closely your function’s graph approximates the data, and adjust the function until you think it approximates the data as well as possible.
- Use your final choice of function to make a prediction.
The Basic Student Budget Revisited

In The Basic Student Budget, you were given certain data about a situation and asked to make a prediction. You plotted the data and based your prediction on a paper graph.

Now you will reexamine The Basic Student Budget data using technology.

The savings and spending information about Cal, Bernie, and Doc—the roommates from The Basic Student Budget—is given below.

- The total rent for their apartment is $900, which is split evenly among the three roommates.
- The rent is due on the last day of each month. The guys don’t get paid until the first day of the next month.
- At the end of payday April 1, Cal had $1,100, Bernie had $800, and Doc had $600.
- As the month goes by, they each occasionally note how much they have left at the end of the day. The table shows their records so far.

Here again is the technique you will use.

- Plot the data on a graphing calculator or other technology.
- Leave the data on the screen and graph a function that you think might approximate the data well.
- Examine how closely your function’s graph approximates the data. Adjust the function until you think it approximates the data as well as possible.
- Use your final choice of function to predict who will be able to pay rent on April 30.

<table>
<thead>
<tr>
<th>Date</th>
<th>Cal</th>
<th>Bernie</th>
<th>Doc</th>
</tr>
</thead>
<tbody>
<tr>
<td>April 3</td>
<td>996</td>
<td>766</td>
<td>570</td>
</tr>
<tr>
<td>April 10</td>
<td>704</td>
<td>698</td>
<td>490</td>
</tr>
<tr>
<td>April 17</td>
<td>440</td>
<td>626</td>
<td>430</td>
</tr>
</tbody>
</table>
When the Overland Trail travelers set out, they planned as well as they could, even though their information was limited.

In this POW, you will do some planning for a future stage of your own life—living on your own.

You may not yet have much information on what this part of your life might be like, so this is a research POW.

Imagine that you have just completed high school. There may have been adults who took care of many things for you before, but now you want to move out on your own.

For the purpose of this activity, assume that you need to provide your own financial support. What sorts of things do you need to plan for?

Be very detailed and accurate in your plan. If you are going to get your own apartment, then find an example of one and note the cost. You will need a job that you can enter with a high school education. You will need to know what you would get paid and how much of that is take-home pay.

It will probably be helpful to interview people for this POW. There is nothing like experience. What bills are you going to have to pay? Does your apartment rent include the cost of electricity? People already living on their own will be able to share with you how they manage the bills.

Your report should include a monthly budget, which is a plan showing how your money is going to be spent in a typical month.

Good luck!
Write-up

Because this is not a standard POW, you can’t use the standard POW write-up. Use these categories instead.

1. Description of the Task: Explain in your own words what you are trying to do in this POW.

2. Your Job: You can consider these questions.
   - What is the job?
   - How do you find it?
   - What are your hours and salary?


4. A Monthly Budget: Include more than just numbers. Discuss how and why you decided on your budget and where you got your information.

5. Self-assessment: What did you learn from this POW? In what ways do you think it will be helpful to you in the future?
All Four, One

In many activities, you have used different ways to represent a relationship between two variables.

- A situation
- A graph
- An In-Out table
- A rule for the table

This gives four different ways to think about a single problem. The connections among these four forms of representation are some of the most fundamental ideas in mathematics.

Explain how the four representations—situations, graphs, In-Out tables, and rules—relate to one another. Use examples from this unit and examples of your own to show how you can go from one form of representation to another.
Travel on the Trail

As you leave Fort Hall, you travel toward the region northwest of Reno, Nevada. You hear of a man who runs an inn and trading post along the route. You plan to spend a few days visiting him. The innkeeper is James P. Beckwourth.

1. As of this morning, July 28, your traveling companions—the Barker family—are a few miles behind you. They have gone 23 miles beyond Fort Hall toward Beckwourth’s trading post. Over the next portion of their journey, they will be traveling 12 miles a day.
   a. Make a graph that shows how far they will be from Fort Hall x days after July 28.
   b. Write a rule for your graph.

2. Your family is on its way from Fort Hall to Beckwourth’s trading post. As of the morning of July 28, you are farther beyond Fort Hall than are the Barkers, but your family is moving more slowly. Hoping to arrive at Beckwourth’s trading post before the Barkers, you must make some reasonable assumptions about the situation.
   a. Choose a specific value for your family’s distance from Fort Hall as of the morning of July 28, and a specific value for your rate of travel. State the values you choose.
   b. Describe how the numbers you chose in part a will make the graph and rule for your family different from the graph and rule for the Barkers.
   c. Will the Barkers catch up to your family? If so, how might you find out when this would happen?

3. If Beckwourth’s trading post is 140 miles away from where you are now, figure out the rate at which you will have to travel to arrive there 2 days before the Barkers. Use the distance you selected in Question 2a. Note: You may end up having to travel faster than the Barkers in order to do this.
James Beckwourth, born around 1798 in Virginia, was the son of an African American woman and her master, a white man. At 19, during a fight with one of his slave bosses, he slugged his way to freedom and traveled from one end of the continent to the other.

On the trail, he fought with and against several Native American nations and served as a scout for the U.S. Army in the war against the Seminole. A contemporary of Kit Carson and Davy Crockett, Beckwourth was part of the brutal frontier tradition. He was said to have “fought and killed with ease and pleasure.”

He also married often. After marrying a Crow woman, he was adopted into the Crow Nation and led the Crow in battle.

In 1850, Beckwourth located a pass through the Sierra Nevada into the American River Valley. This pass became a gateway to California during the gold rush. The mountain peak, the town, and the pass still bear his name.
Questions 1 through 3 use situations from this unit. The final problem uses the same mathematical ideas but without providing a situation.

1. In Previous Travelers, you worked with data showing the amount of beans needed (in pounds) by families of various sizes. Suppose your line of best fit goes through the points (4, 48) and (10, 120).

   a. Find an equation for this line.
   b. What is the rate of bean consumption per person? How is that value related to your equation? How did you find it from the data points?

2. In Sublette’s Cutoff, you were given data showing the amount of water (in gallons) that the Minto family had left at the end of certain days. Suppose your line of best fit goes through the points (0, 36) and (6, 24).

   a. Find an equation for this line.
   b. What is the Minto family’s rate of water consumption per day? How is that value related to your equation? How did you find it from the data points?
   c. How much water did the Minto family begin with? How is that value related to your equation? How did you find it from the data points?
3. *Who Will Make It?* provided data showing the distance the Fowler family was from the Green River in terms of the number of days they had been traveling. Suppose your line of best fit goes through the points (2, 300) and (10, 196).

   **a.** Find an equation for this line.

   **b.** What is the Fowler family’s rate of progress per day? How is that value related to your equation? How did you find it from the data points?

   **c.** How far from the Green River was the Fowler family initially? How is that value related to your equation? How did you find it from the data points?

4. Relate the data below to a context.

   **a.** Create a situation for a problem with a graph that is a straight line that includes the points (3, 12) and (7, 32).

   **b.** Explain what the points (3, 12) and (7, 32) represent in your context.

   **c.** Find an equation for the line through these points.

   **d.** Explain how the numbers in the equation relate to your situation and how you found them from the data points.
All Four, One—Linear Functions

During this unit you have explored the connections among four representations for linear functions.

A **linear function** is a function with a graph that is a straight line. A common form for writing a linear function is \( f(x) = ax + b \), where \( a \) is the **rate of change** and \( b \) is the **starting point**.

The four representations for relationships between two variables that have been explored in this unit are situations, tables, graphs, and rules.

Your Task

Create a how-to report for converting from any one of the four representations of a linear function to another. Begin by considering how many conversions you will have to address in your report. Be sure to discuss the role of \( a \) and \( b \) in each conversion.
Straight-Line Reflections

In this activity, you will summarize key ideas about situations that lead to straight-line graphs. You will describe how the situation is related to the rule or graph.

1. What is it about a situation that makes its graph a straight line?

2. Choose a situation from either Following Families on the Trail or Travel on the Trail. Describe in words how you translated the situation into a graph.

3. Graph the equation $y = 360 - 50(x - 4)$ on a graphing calculator or by hand. Notice this equation is linear.
   a. Use the tools you created in your how-to report to determine a linear equation in the form $y = ax + b$ from a graph or table.
   b. Demonstrate that the equation found in part a is equivalent to $y = 360 - 50(x - 4)$.

4. Graph the equation $3x + 2y = 9$ by calculator or by hand. Notice this equation is also linear.
   a. Use your tools to determine a linear equation in the form $y = ax + b$ from a graph or table.
   b. Demonstrate that the equation found in part a is equivalent to $3x + 2y = 9$. 
Reaching the Unknown

As you continue your journey across the country, you will see how the use of variables, graphs, and equations helps solve challenges along the trail. Some of the new situations faced by the emigrants are complex—setting a schedule for watching the wagons at night based on who is available to stand guard or paying hired hands fairly based on their experience and the money available for salaries. Further variables complicate matters—the nights grow longer or shorter depending on the season, or families have a little more money to pay hired hands.

Some situations require you to find numeric values that fit more than one condition. Others require you to determine when two different situations will lead to the same result.

Once you’ve finally arrived in California, you’ll see that life in this state wasn’t all golden. You’ll also see how the mathematics of expenses and profits played a role in the everyday decisions of early settlers.

Jeff Klein and Esteban Herevia map a route to California.
The Overland Trail: Reaching the Unknown

Group Activity

Fair Share on Chores

About 50 miles past Fort Hall, the California Trail splits from the Oregon Trail and heads into Nevada.

Two families, the Murphys and the Bensons, decided to continue on the Oregon Trail. You said good-bye and then ventured toward California.

Wagon trains often put their wagons in a circle to make a corral for the livestock. It was only in the movies that wagon trains created a circle to protect themselves from Native Americans.

Now that the Murphys and the Bensons have split from the wagon train, you have fewer wagons available.

The Washburn family decides that someone needs to keep an eye on their animals during the night. They announce that their children will take shifts each night, with one child at a time guarding the animals. Altogether, the animals need to be watched for ten hours. This family has two girls and three boys.

This sounds simple—two hours each. But the girls have other chores, and so do the boys. To balance out other assigned chores, the Washburn family decides that there should be one length of time for each girl’s shift and another length of time for each boy’s shift.
1. How long would you suggest that each type of shift be? Provide at least three different pairs of answers.

2. Using $G$ to represent the length of each girl’s shift and $B$ to represent the length of each boy’s shift, write an equation expressing the fact that the total of all their shifts is ten hours.

3. Suppose you know how long each girl’s shift is. Describe in words how you could find the length of each boy’s shift.

4. Write your sentence from Question 3 as a function, expressing $B$ in terms of $G$. That is, write an equation that begins $B =$ and has an expression using $G$ to the right of the equal sign.

5. Graph the function from Question 4 on your calculator. Check to see if your answers from Question 1 are on the graph.

6. Use the trace feature on your calculator to find three more pairs of possible shift lengths from your graph.
The Fulkerth family is large, and they have seven hired hands. The family has a total of about $20 per week available for salaries. Four of the hired hands are experienced at working on the trail. The other three are on their first trip. It seems fair that the experienced hired hands should get more pay than those without experience. So the Fulkerths decide that there will be one rate for the four experienced workers and another rate for the three without experience.

1. What should each weekly pay rate be? Suggest three possible combinations. The salary total can be a few cents more or less than $20 if that helps you avoid fractions of a penny.

2. Plot your three combinations from Question 1. Use $X$ for the pay rate of an inexperienced hired hand and $Y$ for the pay rate of an experienced hired hand.

3. Connect the points with a straight line. Use this graph to find two more possible pairs.

4. Describe in words how you could compute the weekly pay rate for an experienced hired hand if you knew the rate for an inexperienced hired hand. Assume the weekly total is exactly $20.

5. Express your sentence from Question 4 as an equation, giving $Y$ in terms of $X$.

6. Check to see if the two new pairs from Question 3 fit the equation from Question 5.
More Fair Share on Chores

As you saw in *Fair Share on Chores*, the Washburn family’s two girls and three boys are responsible for watching the animals in shifts during the night.

After some experience, the family has decided that to balance out other chores, the shift for each boy should be half an hour longer than that for each girl.

1. They have realized that as the season gradually changes, the total amount of time needed for the shifts is not always ten hours. Therefore, they want to know about combinations of shift lengths with different totals.
   a. What are some possible combinations of shift lengths in which the shift for each boy is half an hour longer than that for each girl? Give four possibilities.
   b. Describe *in words* how you could find the length of each boy’s shift if you knew the length of a girl’s shift.
   c. Use your answer to part b to write an equation in which $G$ represents the length of each girl’s shift and $B$ represents the length of each boy’s shift.
   d. Graph your equation on the calculator.
   e. For each combination that you gave in part a, state how much total time will be covered by all the children combined.

2. On a particular evening, it turns out that ten hours of animal watching is required after all. Find a pair of shift lengths that would total ten hours and still have the shift for each boy be half an hour longer than the shift for each girl.
More Fair Share for Hired Hands

Once again, the Fulkerth family is planning its budget. Times are a little better now, but they haven’t yet decided what the total budget should be for hired hands.

The same hired hands are still working for them. Now four hired hands are very experienced and three have only a little experience.

Although all the hired hands have some experience, the Fulkerths decide to continue having two pay rates. In the new pay scale, a very experienced hired hand will get $1 per week more than a less experienced hired hand.

1. Make several suggestions of how the Fulkerths could set up the two pay rates. Put these data in a table, with $X$ representing a less experienced worker’s weekly pay rate and $Y$ representing a more experienced worker’s weekly pay rate.

2. Graph the data from the table, and write an equation for your graph.

3. Find a set of pay rates that would make the total weekly pay for the hired hands approximately $30.
Water Conservation

Nevada seemed like a desert to the emigrants, who had been following large rivers most of the way from Westport. As you have seen, water was a very precious commodity on the Overland Trail. Travelers had to be careful not to run out.

They kept track of their water use, planning for the next opportunity to refill their water containers.

1. The Stevens family had a 50-gallon water container. In an effort to conserve water, they reduced their daily consumption to 3 gallons per day.

   If they began with a full container, how many gallons of water would they have left after 3 days? 8 days? 12 days? \( X \) days?

2. The Muster family was larger. They had a 100-gallon water container. Their daily consumption was 8 gallons per day. If they began with a full container, how many gallons of water would they have left after 3 days? 8 days? 12 days? \( X \) days?

3. Use your answers to the last part of Questions 1 and 2 to graph each family’s water supply. Use Number of days for the horizontal axis and Amount of water left for the vertical axis. Graph both functions on the same set of axes.

4. Is there a time when both families would have the same amount of water left? If so, when would it happen, and how much water would both families have at that time?

5. In how many days would each family run out of water?
The Big Buy

Just like families along the Overland Trail, modern families need to plan for expenses. In this activity, parents pay their son and daughter to do chores.

Max and Jillian Verde want to make some money over their school’s spring break. They ask their parents to let them work around the house to earn money.

Their parents agree, because Jillian and Max are saving to buy graphing calculators. Dad tells Jillian that he will give her a starting bonus of $10, and then pay her $5 an hour for the work she does around the house. Mom offers Max a slightly different deal. She will give him $40 to start, but only $3 an hour.

1. Write two separate equations—one for Jillian and one for Max—expressing how much money each will earn (including their starting money) in terms of time worked.

2. Graph both equations on the same set of axes.

3. If the graphing calculator costs $72, who will be able to buy a calculator with the least work time? Explain your answer.

4. If the graphing calculator costs $100, who will be able to buy one with the least work time? Explain your answer.

5. For what price must the calculator sell in order for Jillian and Max to earn that amount with the same number of hours of work? Explain your answer.
The California Experience

Who were the people in California in the 1850s?

Of course, Native Americans were there, probably for millennia. Some 300 different nations, including the Modoc, Washo, Maidu, Pomo, Cahuilla, and Miwok, held territory in what is now known as California.

Then came the Spanish. Out of their conquest and mixture with the Native Americans came a new culture and a new nation called Mexico, which became politically independent of Spain in 1821.

Indeed, that new Mexican nation claimed all of what is now the state of California, as well as all of Nevada and Utah and parts of Arizona, New Mexico, Colorado, and Wyoming. But in 1846, the United States provoked a war with Mexico in an attempt to gain territory. The United States won the war, which ended in 1848 with the Treaty of Guadalupe Hidalgo. In this treaty, Mexico was forced to cede much of what is now the southwestern United States.

Although the first wagon trains left Missouri for California in 1841, the great migration of the mid-nineteenth century was spurred by the discovery of gold in 1848, only nine days before the signing of the Treaty of Guadalupe Hidalgo.

continued
The arrival of hundreds of thousands of gold seekers and others permanently changed the lives of the people who had been living in California. Settlements led to the destruction of whole nations of native peoples.

During the gold rush, thousands of people were brought from China to work as laborers. Although in 1850, there were only a few hundred Chinese in California, by 1852, about ten percent of the population was Chinese. Many of them lived in slavelike conditions.

Those who traveled on the California Trail in search of gold often ended up destitute, and a number of women resorted to prostitution to survive.

So the California experience was a mixture of many things. Only a very few became rich from the mining of gold.
Getting the Gold

Many of those who made the long trek to California were in search of gold. Although few were able to get rich, many tried.

One of the most common ways to find gold was to pan for it in streams.

To pan for gold, all a person needed was a $9 shovel, a $50 burro, and a $1 pan. A person could get an ounce of gold each day, on average, by panning.

One ingenious person discovered a way to get gold from a stream by using a trough. The trough was a long chute that miners set in the stream and rocked back and forth to separate the gold from the silt of the stream.

Although it was more expensive to get started with the trough method, that technique produced about twice as much gold each day as did the pan method. To use a trough, a person needed a team of two burros, a shovel, and a trough. The trough cost $311.

At that time, gold was worth $15 an ounce. The following questions involve the amount of profit (income minus expenses) from each method after a certain number of days. A loss of money is considered a negative profit.

1. How much profit will each method yield
   a. after 16 days?
   b. after 30 days?
   c. after 5 days?
2. Make two graphs on the same set of axes. One graph should show the profit from panning, and the other should show the profit from using a trough.

3. Find rules to show the profit using each method.
   a. Find a rule showing how much profit the panning method will yield after \( x \) days?
   b. Find a rule showing how much profit the trough method will yield after \( x \) days.

4. How many days will it take for a miner using each method to break even?

5. After how many days will the two methods yield the same amount of profit?
The Mystery Bags Game

Once a prospector accumulated some gold, he brought it to a government office for weighing. That office had a pan balance, like the one shown here, and a collection of lead blocks of known weights.

The officials would put the gold on one side and then try various combinations of the lead weights until the two sides balanced. For example, the result here shows that the bag of gold on the left weighs 23 ounces.

The Game

The officials came up with a game to pass the time when things got slow. One official would take one or more empty bags and fill them each with the same amount of gold. These bags of equal weight were called the mystery bags.

Next, the official would play around with ways to place the mystery bags and some lead weights on the pan balance so that the two sides balanced. The game was to figure out the weight of each mystery bag.

For example, this situation shows three mystery bags balanced with three different lead weights.

Your Task

If all the examples were like this one, the game would have been boring. The group made things more challenging by sometimes combining mystery bags and lead weights on either side of the balance.
See if you can solve these mystery bag puzzles by figuring out how much gold there is in each bag. Explain how you know you are correct. You may want to draw diagrams to show what’s going on. You might consider how you could adjust what is on each side of the balance to simplify the situation but still be sure that the pans were equal in weight.

1. There are 3 mystery bags on one side of the balance and 51 ounces of lead weights on the other side.

2. There are 1 mystery bag and 42 ounces of weights on one side and 100 ounces of weights on the other side.

3. There are 8 mystery bags and 10 ounces of weights on one side and 90 ounces of weights on the other side.

4. There are 3 mystery bags and 29 ounces of weights on one side and 4 mystery bags on the other side.

5. There are 11 mystery bags and 65 ounces of weights on one side and 4 mystery bags and 100 ounces of weights on the other side.

6. There are 6 mystery bags and 13 ounces of weights on one side and 6 mystery bags and 14 ounces of weights on the other side. The playful government official could get in a lot of trouble for this one!

7. There are 15 mystery bags and 7 ounces of weights on both sides. At first, the official guessing thought this one was easy, but then he found it to be incredibly hard.

8. The official guessing the mystery bag weights wanted to be able to win easily every time, without calling you in for consultation. Therefore, your final task is to describe in words a procedure by which the official can find out how much is in a mystery bag in any situation.
More Mystery Bags

Here are some simple equations that might have come from mystery bag games. Find the weight of one mystery bag and explain how you got the answer.

1. \( M + 16 = 43 \)
2. \( 12M = 60 \)
3. \( 27 + 9M = 90 \)
4. \( 5M + 24 = 51 + 2M \)
5. \( 43M + 37 = 56M + 24 \)
6. \( 12M + 13 = 5M + 62 \)
7. \( 5M + 2M + 100 = 15M + 20 \)

8. Make up some equations of your own like those in Questions 1–7. Describe the pan balance setup for your equations, and find the weight of one mystery bag in each case.

Here are some equations that you have developed in recent activities. In each case, try to adapt the methods you used above to find a value for \( x \) that makes the equation true.

9. \( 10 + 5x = 40 + 3x \)
10. \( 50 - 3x = 100 - 8x \)
11. \( 15x - 60 = 30x - 420 \)
Scrambling Equations

Usually, the concept of equivalent equations is used to make things simpler. But in this activity, you’re going to make things more complicated. For example, look at the sequence of equations shown below.

\[
\begin{align*}
x &= 1 \\
6x &= 6 \\
6x - 3 &= 3 \\
\frac{6x - 3}{2} &= 1.5
\end{align*}
\]

All of these equations are equivalent, because they all have the same solution. You should be able to see what was done to each equation to get the one below it.

In this activity, you will begin by writing a very simple equation, such as \(x = 1\). Then you’ll write an equivalent equation that’s more complicated, and then something equivalent to that, and so on.

This activity has some very precise rules. You will be changing your equation exactly three times. At each stage, you can do any one of these things.

- You can add the same integer to both sides of the equation.
- You can add the same multiple of \(x\) to both sides of the equation.
- You can subtract the same integer from both sides of the equation.
- You can subtract the same integer from both sides of the equation.
- You can multiply both sides of the equation by the same nonzero integer.
- You can divide both sides of the equation by the same nonzero integer.
Remember that you are to do exactly three of these steps in any order. For instance, the example uses multiplication, then subtraction, and then division. At any time in the process, you’re also allowed to do arithmetic steps to simplify the right side of the equation.

When you are done with this process, copy your final, complicated equation onto one side of a sheet of paper and put your original equation on the reverse side. This sheet will be exchanged with another student. You will then have the opportunity to “uncomplicate” someone else’s scrambled equation.
More Scrambled Equations and Mystery Bags

Part I: More Scrambled Equations

This activity involves the same steps for getting equivalent equations that were described in *Scrambling Equations*.

1. The equations here show one sequence of three steps to “scramble” the equation \( x = 3 \).

\[
\begin{align*}
x = 3 \\
x - 5 &= -2 \\
10(x - 5) &= -20 \\
\frac{10(x - 5)}{4} &= -5
\end{align*}
\]

a. Describe what was done at each step.

b. Check that \( x = 3 \) is a solution to the final equation in the sequence, and show your work.

For Questions 2 through 4, do two things.

a. Uncomplicate each equation until you get back to a simple equation of the form “\( x = \) some number.”

b. Take the value of \( x \) you get from the simple equation and substitute it back into the original equation to check that it makes the “complicated” equation true.

2. \( 3x - 5 = -2 \)

3. \( \frac{x - 6}{4} + 1 = 7 \)

4. \( 4\left|\frac{x}{3} + 6\right| - 8 = 20 \)

*continued*
Part II: More Mystery Bags

Earlier in this unit, you used the idea of a pan balance to solve mystery bag problems. The equations here might come from such problems. Solve them using the concept of equivalent equations, but also think about how each step you do is related to the pan-balance model.

5. $11t + 13 = 7t + 41$

6. $12 + 7w = 4w + 21$

7. $8(x + 3) + 19 = 15 + 2(x + 35)$
Family Comparisons by Algebra

In *Following Families on the Trail*, you were given some information about travel and coffee consumption for the Buck family and the Woods family. Here are the basic facts.

- As of the morning of July 12, the Buck family had gone 50 miles since leaving the Green River and was traveling at 15 miles per day.
- As of the morning of July 12, the Woods family had gone 10 miles since leaving the Green River and was traveling at 20 miles per day.
- As of the morning of July 12, the Buck family had 100 pounds of coffee and was consuming 5 pounds of coffee per day.
- As of the morning of July 12, the Woods family had 70 pounds of coffee and was consuming 5 pounds of coffee per day.

Questions 1 and 2 here are the same as Questions 3 and 4 of *Following Families on the Trail*. But now you are to explain your answer using algebra. Set up and solve equations that represent the situations. Then interpret your results to answer the questions.

1. Is there a time when the two families are the same distance from the Green River? If so, when is it, and how far are they from the Green River at that time? If not, explain why not.

2. Is there a time when the two families have the same amount of coffee? If so, when is it, and how much coffee do they have at that time? If not, explain why not.
Starting Over in California

Hundreds of thousands of people traveled to California in the middle of the nineteenth century.

Some came across the Pacific Ocean from China. Some sailed from the Atlantic coast to Panama, crossed land there, and then sailed again to California.

Still others, as you know, came by boat around Cape Horn or came by wagon or by horse on the Overland Trail.

Biddy Mason walked.

About Biddy Mason

Biddy Mason walked to California behind her master’s 300-wagon train. Her job was to watch the cattle, but her master would not give her a horse, so she had to walk. She was one of the many enslaved African Americans brought to California by southern slave owners to work in the gold fields. Most remained enslaved.

Biddy Mason, however, broke away from her slave master and had the courage to sue for, and win, her freedom. She settled in California, working as a nurse and midwife. She became known for her generosity and great charity, taking in homeless children and supporting schools, churches, and hospitals.

Your Task

1. The Smith family had owned a blacksmith shop back in Pennsylvania. After arriving in California, the father, Richard, opened a blacksmith shop. The Smiths’ main business was shoeing horses. They borrowed money from a friend to get the equipment and agreed to use all the income from horse shoeing to pay off this debt. After 4 weeks of payments, the Smiths owed $101. After 12 weeks, they had reduced the debt to $53. Assume the Smiths paid the same amount each week.
   a. How much did the Smiths borrow to open the shop?
   b. Find an equation that gives the amount they will owe after \( x \) weeks.
   c. How long did it take to pay off the debt?

continued
2. Soon the Smiths heard about gold found in a river nearby. Richard decided to make pickaxes and shovels to sell to the miners. But Richard’s daughter Kaley wanted to pan for gold. Richard made a deal with Kaley: “I'll let you pan for gold for one year. If after a year you make more money prospecting than I do selling pickaxes and shovels, I'll let you continue panning for gold. But if you make less than I do, you agree to return to the family business, stay at home, and work with us.”

Surprise! On her first day of prospecting, Kaley found a gold nugget worth $150! After 2 months, Kaley had taken in $166 (including the $150), and her father had made only $42. After 5 months, Kaley’s total earnings had reached $190, and her father’s total was $105. If Kaley did not find any more large gold nuggets, do you think Kaley would get to keep panning for gold after a year?

3. The wagon train that your family is a member of has reached Sutter Creek in California. They decide to settle down in the area. Like Biddy Mason and the Smiths, you are now faced with the decision of how to make a living. Create a mathematical problem about some members of your family who either start a business or go for the gold.
Beginning Portfolios

California Reflections

*The California Experience* outlined some of the historical and social background of the gold rush.

Write about your own feelings concerning this period of American history.

You may want to talk about the group or groups you identify with, about issues of justice or injustice, or about the process of social change.

You may also want to comment on how your ideas about the period have changed over the course of this unit.

Graphs Along the Way

The meaning and use of graphs played an important role in this unit.

Select one activity that illustrates how graphs can describe a problem situation.

Select another activity that illustrates how graphs can be used to make a decision about a problem situation.

Explain how each of these activities helped you understand the meaning and use of graphs.
Now that *The Overland Trail* is completed, it is time to put together your portfolio for the unit.

- Write a cover letter that summarizes the unit.
- Choose papers to include from your work in the unit.
- Discuss your personal growth during the unit.

**Cover Letter**

Look back over *The Overland Trail* and describe the main mathematical ideas of the unit. This description should give an overview of how the key ideas were developed.

As part of compiling your portfolio, you will select activities that you think were important in developing the key ideas of this unit. Your cover letter should include an explanation of why you selected the particular items.

**Selecting Papers**

Your portfolio for *The Overland Trail* should contain these items.

- The items you selected in *Beginning Portfolios*
  Include your “California Reflection,” to reflect the historical elements of the unit and your reaction to them. Also include the two activities about graphs that you selected, along with the explanation you wrote about how these activities helped you understand the meaning and use of graphs.
- All Four, One—Linear Functions
  This assignment is included because it summarizes the connections among situations, graphs, tables, and rules—four different ways of representing functions.

*continued*
• An activity about starting value and rate
The unit included problems about starting value and rate in several
different contexts. Select an activity that illustrates these important
points, and explain your selection.

• An activity about solving equations
Several activities in this unit developed ways of thinking about and
finding solutions to equations. Select one of these activities, and write
about how it was meaningful for you.

• A Problem of the Week
Select one of the three POWs you completed during this unit: The
Haybaler Problem, Around the Horn, or On Your Own.

• Other quality work
Select one or two other pieces of work that represent your best
efforts. These can be any work from the unit—Problem of the Week,
homework, classwork, presentation, and so forth.

Personal Growth
Your cover letter for The Overland Trail describes how the
mathematical ideas developed in the unit. As part of your portfolio,
write about your own development during this unit. You may want to
address this prompt.

Many of the problems in this unit involved graphs, and you learned
how to use a graphing calculator to make graphs. Write about your
reactions to using this tool.

• What are the advantages of using the calculator to make graphs?
• What are the advantages of doing graphs by hand?
• How does each approach help your understanding of graphs and
  your ability to use them to solve problems?

Include any other thoughts you wish to share with a reader of your
portfolio.