3. Start at the 0.8 m mark and walk away from the sensor at a constant rate of 0.2 m/s.

4. a. The walker starts 2.5 m away from the motion sensor and walks toward it very slowly at a rate of 1 m in 6 s.
   b. The walker starts 1 m away from the motion sensor and walks away from it at a rate of 2.5 m in 6 s.

5. a. The walker starts 6 m away from the motion sensor and walks toward it at a rate of 0.2 m/s for 6 s.
   b. The walker starts 1 m away from the motion sensor and walks away from it at a rate of 0.6 m/s for 6 s.

6. The first graph, which shows a line, better represents the walk described because the walk is a continuous process. The walker is somewhere at every possible time in the 6 s.

7. Convert 1 mi/h to ft/s.

\[ \frac{1 \text{ mi}}{1 \text{ h}} = \frac{1 \text{ h}}{60 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ s}} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} = 1.46 \text{ ft/s} \]

8. a. 4 s
   b. Away; the distance is increasing.
   c. About 0.5 m
   d. \( \frac{2.9}{4} = 0.6 \text{ m/s} \)
   e. \( \frac{3.5 \text{ m}}{0.6 \text{ m/s}} = 5.83 \text{ s} \); approximately 6 s
   f. The graph is a straight line.

9. The graph is a straight line.

10. a. The rate is negative, so the line slopes down to the right.
    b. The rate is 0, so the line is horizontal.
    c. The line is not very steep.

11. a. ii
    b. iv
    c. iii
    d. i

12. Start walking at the 0 mark when the sensor starts, and walk 1 ft every second. Start walking at the 0 mark when the sensor starts, and walk 1 m every second. Because a meter is longer than a foot, 1 m/s is faster.

13. a. This is not possible because the walker would have to be at more than one distance from the sensor at the 3 s mark.
    b. This is possible; the walker just stands still at about 2.5 m from the sensor.
    c. This is not possible because the walker can’t be in two places at any given time.

14. a. \( x = \frac{21}{5} \), or 4.2
   b. \( x = \frac{22}{9} \), or 2.4
   c. \( \frac{x}{c} = \frac{d}{e} \)
      \( x = \frac{d}{e} \cdot c \) Multiply by \( c \) to undo the division.
      \( x = \frac{dc}{e} \)

15. a. \( \frac{24,901.55 \text{ mi}}{5 \times 365 + 2 \times 30.4 + 2} \text{ days} \approx 31.4 \text{ mi per day} \)
   b. \( 31.4 \text{ mi per day} \times \frac{1 \text{ day}}{365 \text{ days}} \approx 17,191.5 \text{ mi per year} \)
   c. Let \( t \) represent the number of days, and then write and solve the proportion.
      \( \frac{1 \text{ day}}{31.4 \text{ mi per day}} = \frac{t}{60,000 \text{ mi}^2} \)
      \( t \approx 1,911 \text{ days or about 5.24 yr} \)

16. a. \( \frac{175 \text{ mi}}{13.5 \text{ gal}} \), about 13 mi/gal, or about 0.077 gal/mi
   b. 13 mi/gal \cdot 5 gal = 65 mi
   c. \( \frac{1 \text{ mi}}{13 \text{ gal}} \) = \( \frac{0.077 \text{ gal}}{1 \text{ mi}} \); 0.077 gal/mi \cdot 100 mi = 7.7 gal

LESSON 3.4

Exercises

1. a. ii
   b. iv
   c. iii
   d. i

2. a. \( t \approx 0.18 \) h
   b. \( t \approx 0.47 \) h
   c. 24 represents the initial number of miles the driver is from his or her destination.
   d. 45 means that the driver is driving at a speed of 45 mi/h.
   e. \( 24 - 45t = 16 \) Original equation.
      \( 24 - 45t - 24 = 16 - 24 \) To undo adding 24, subtract 24.
      \( -45t = -8 \) Subtract.
      \( t = \frac{8}{-45} \) To undo multiplying by -45, divide by -45.
      \( t = \frac{8}{45} = 0.17 \)

3. a. \( d \approx 38.3 \text{ ft} \)
   b. \( d \approx 25.42 \text{ ft} \)
   c. The walker started 4.7 feet away from the motion sensor.
   d. The walker was walking at a rate of 2.8 feet per second.
4. a. \( x = 7.267 \)  
   b. \( x = 11.2 \)

5. a. \( 35 + 0.8 \cdot 25 = 55 \text{ mi} \)  
   b. 50 min. One way to find this answer is to write the equation \( 75 = 35 + 0.8 \cdot x \) and then solve it by working backward, undoing the operations. Another way is to make a calculator graph and trace it to find the \( x \)-value corresponding to the \( y \)-value 75.

6. a. The table shows that Louis burned 400 calories before beginning to run (400 is the \( Y_1 \)-value for \( X \)-value 0). The difference in consecutive \( Y_1 \)-values is 20.7, indicating that Louis burns 20.7 calories per minute while running. He wants to burn 700 calories.

   b. 400 \( \text{ ENTER, Ans} + 20.7 \text{ ENTER, ENTER...} \)

   c. \( y = 400 + 20.7x \)

   d. 700 \( \text{ ENTER, Ans} + 0 \text{ ENTER, ENTER...} \)

   e. \( y = 700 + 0x, \text{ or } y = 700 \)

   f. The \( y \)-intercept of \( Y_1 \), which is 400, is the number of calories burned after 0 min of running (that is, before Louis begins to run).

   [0, 100, 10, 0, 120, 10]

7. a. The \(-300\) could represent a start-up cost of \$300 for equipment and expenses, the 15 could represent the amount she earns per lawn, and \( N \) could represent the number of lawns.

   b. Possible questions and answers:

   How many lawns will Jo have to mow to break even? To answer this question, solve \(-300 + 15N = 0\). Jo must mow 20 lawns.

   How much profit will Jo earn if she mows 40 lawns? To answer this question, substitute 40 for \( N \). She would earn \(-300 + 15(40)\), or \$300.

8. a. The speed at 0 seconds is 5 m/s and the speed increases by 9.8 m/s every second, so the equation is \( s = 5 + 9.8t \), where \( t \) is the time in seconds and \( s \) is the speed in meters per second.

   b. \( s = 5 + 9.8(3) = 34.4 \text{ m/s} \)

   c. Solve the equation \( 83.4 = 5 + 9.8t \). It would take 8 seconds for the object to reach a speed of 83.4 m/s.

   d. Possible answer: It doesn’t account for air resistance and terminal speed.

9. a. \( y = 45 + 0.12x \), where \( x \) represents the dollar amount his customers spend and \( y \) represents his daily income in dollars

   b. [0, 840, 120, 0, 180, 30]

   c. \( y = 45 + 0.12 \cdot 312 = \$82.44 \)

   d. Solving \( 45 + 0.12x = 105 \) gives \( x = 500 \). Solving \( 45 + 0.12x = 120 \) gives \( x = 625 \). So customers would have to spend between \$500 and \$625 in order for Manny to earn \$105 to \$120.

10. a. Let \( x \) = the number of minutes Paula swims on Monday and \( y \) = the total number of calories she burns on Monday. She burns \( 3.8(30) = 114 \) calories biking and 6.9 calories for each minute she swims, so the equation is \( y = 114 + 6.9x \).

   b. Let \( x \) = the number of minutes Paula jogs on Wednesday and \( y \) = the total number of calories she burns on Wednesday. She burns \( 6.9(30) = 207 \) calories swimming and 7.3 calories for each minute she swims, so the equation is \( y = 207 + 7.3x \).

   c. Let \( x \) = the number of minutes Paula runs on Friday and \( y \) = the total number of calories she burns on Friday. She burns \( 6.9(15) = 103.5 \) calories swimming, \( 3.8(15) = 57 \) calories biking, and 11.3 calories for each minute she runs, so the equation is \( y = 160.5 + 11.3x \).
d. Monday: In her 60 min workout, Paula swims 60 – 30 = 30 min, so x = 30, and 
y = 114 + 6.9(30) = 321. Therefore, she burns 321 calories.
Wednesday: In her 60 min workout, Paula jogs 60 – 30 = 30 min, so x = 30, and 
y = 207 + 7.3(30) = 426. Therefore, she burns 426 calories.
Friday: In her 60 min workout, Paula runs 60 – 15 – 15 = 30 min, so x = 30, and 
y = 160.5 + 11.3(30) = 499.5. Therefore, she burns 499.5 calories.

11. Possible answer: Write the percent as one ratio of a proportion. Put the part over the whole in the 
other ratio.
a. \( \frac{8}{n} = \frac{15}{100} \), \( n \approx 53.3 \) 
b. \( \frac{15}{100} = \frac{n}{18.95} \), \( n \approx 2.8 \) 
c. \( \frac{p}{100} = \frac{326}{64} \), \( p \approx 509.4 \) 
d. \( \frac{10}{100} = \frac{40}{n} \), \( n \approx 400 \)

12. a. 22.4, 22.6, 22.1 
b. 363 mi + 342 mi + 285 mi = 22.3981 mpg 
\( \approx 22.4 \) mpg 
c. 22.4 mi 
\( \frac{1}{1} \) gal \( \cdot \) 17.1 gal = 383 mi 
d. 1 gal 
\( \frac{22.4}{22.4} \) mi \( \cdot \) 4230 mi \( \approx \) 189 gal

13. Sample explanation: For each equation, I looked for 
the graph with a rate of change that matched the recursive rule. I also checked that the starting value 
of the routine was the \( y \)-intercept of the graph.
a. ii \( \quad \) b. iv \( \quad \) c. iii \( \quad \) d. i

14. a. In 10 seconds, he cycled 140 meters, so he is riding at a rate of 14 m/s.
b. | Time (s) | Distance (m) |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14</td>
</tr>
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<td>2</td>
<td>28</td>
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<tr>
<td>9</td>
<td>126</td>
</tr>
<tr>
<td>10</td>
<td>140</td>
</tr>
</tbody>
</table>
c. Possible routines:
\{0, 14\} \( \text{ENTER} \), \{0, 14\} \( \text{ENTER} \), \{0, 14\} \( \text{ENTER} \), \ldots

2. a. Substitute 32 for \( t \) in the given equation.
\[ w = -29 + 1.4t \]
\[ w = -29 + 1.4(32) = 15.8 \]
At a wind speed of 40 mi/h, if the actual air temperature is 32°F, the wind chill temperature is 15.8°F.
b. Substitute -8 for \( w \) in the given equation.
\[ w = -29 + 1.4t \]
\[ -8 = -29 + 1.4t \]
To solve this equation, first undo subtracting 29 by adding 29.
\[ 21 = 1.4t \]
c. At a wind speed of 40 mi/h, the wind chill temperature changes by 1.4° for each change of 1° in actual temperature.

d. At a wind speed of 40 mi/h, if the actual temperature is 0°F, the wind chill temperature is −29°F.

3. a. The rate is negative, so the line goes from the upper left to the lower right.
    b. The rate is zero. The line is horizontal.
    c. The rate is positive, so the line goes from the lower left to the upper right.
    d. The rate for the speedier walker is greater than the rate for the person walking more slowly, so the graph for the speedier walker is steeper than the graph for the slower walker.

4. Possible answer:

5. a. In each case, the rate of change can be calculated by dividing the difference of two output values by the difference of the corresponding input values.
   i. 3.5  
   ii. 8  
   iii. −1.4
    b. The output value corresponding to an input value of 0 is the \( y \)-intercept.
   i. 6  
   ii. 1  
   iii. 23; the output value for input −3 is 27.2.
To get from input −3 to input 0, you add 3, so to get from output value 27.2 to the output value for 0, you must add \( 3 \cdot (−1.4) = 23 \).

6. a. The input variable \( x \) is the temperature in °F, and the output variable \( y \) is the wind chill temperature in °F.
    b. The rate of change is 1.4. For every 10° increase in temperature, there is a 14° increase in wind chill temperature.

c. \( y = −28 + 1.4x \); for every 5° increase in temperature, there is a 7° increase in wind chill temperature, so from the point \((-5, −35)\), add 5° to the temperature and 7° to the wind chill temperature to get the \( y \)-intercept of \((0, −28)\).

d. Both graphs show linear relationships with identical rates of change and identical \( y \)-intercepts. The graph of the points shows wind chill temperatures for temperatures of −5°F, 0°F, 5°F, 10°F, and so on. The graph of the equation shows wind chill temperatures for every temperature. The graph of the points is discrete and the graph of the equation is continuous.

7. a. \( \text{Distance from sensor} = 3.5 - 0.25 \cdot \text{time} \)
    b. Rewrite the equation as \( D = 3.5 - 0.25t \).
Substitute 0 for \( D \) and solve for \( t \).
\[
0 = 3.5 - 0.25t
\]
To undo adding 3.5, subtract 3.5.
\[
-3.5 = -0.25t
\]
To undo multiplying by −0.25, divide by −0.25.
\[
t = \frac{-3.5}{-0.25} = 14
\]
She would pass the sensor 14 s after she begins walking.

8. Because length times width gives area, 7.3 and \( x \) represent the length and width, respectively. The number 200 represents the area of the rectangle in square units. The solution is about 27.4 units. The rectangle is not drawn to scale. The length should be about 3.8 times the width.

9. a. 990 square units
    b. Possible answers:
\[
33x = 990; \quad x = \frac{990}{33}; \quad 33x = 1584 - 594; \quad x = \frac{1584 - 594}{33}
\]
c. The height is \( 990 \div 33 \), or 30 units.

10. Possible answer:
### Elevator Table

<table>
<thead>
<tr>
<th>Floor number</th>
<th>Height (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 (basement)</td>
<td>−4</td>
</tr>
<tr>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>22</td>
</tr>
<tr>
<td>3</td>
<td>35</td>
</tr>
<tr>
<td>4</td>
<td>48</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>
On the Road Again Table

Highway Distance from Flint

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Minivan (mi)</th>
<th>Sports car (mi)</th>
<th>Pickup (mi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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</tr>
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<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
On the Road Again Grid

Distance from Flint (mi)

Time (min)

(y)

(x)
On the Road Again Graph

Distance from Flint (mi)

Time (min)

- Minivan
- Pickup
- Sports car
1. You can use the following recursive routine to keep track of the monthly savings account balances for the three people listed in the table:

\[
\{0, 800, 1200, 2400\} \quad \text{ENTER} \\
\{\text{Ans}(1) + 1, \text{Ans}(2) + 25, \text{Ans}(3) - 42, \text{Ans}(4) - 85\} \quad \text{ENTER, ENTER, ENTER, \ldots}
\]

a. Complete the table for all three accounts.

<table>
<thead>
<tr>
<th>Time elapsed (mo)</th>
<th>Maria</th>
<th>Yolanda</th>
<th>Todd</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$800</td>
<td>$1,200</td>
<td>$2,400</td>
</tr>
<tr>
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<td></td>
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<td></td>
</tr>
<tr>
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<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. How many months elapse before Maria’s balance is larger than Yolanda’s balance?

c. The recursive routine generates the list \(\{5, 925, 990, 1975\}\). Explain the real-world meaning of each number in this list.

d. How many months will elapse before Yolanda’s balance is larger than Todd’s balance?

2. Refer to Problem 1. Make a scatter plot that shows the balance in Maria’s account over the 6-month period.

3. This graph shows Jon’s distance from the 0 mark as he walks along a measuring tape. The horizontal axis goes from 0 to 6 seconds. The vertical axis goes from 0 to 4 meters. Describe Jon’s walk, indicating the direction, walking speed, and time interval represented by each segment.

4. Match each recursive routine to a graph. Each square represents 1 unit.

a. \(3 \quad \text{ENTER}, \text{Ans} - 1 \quad \text{ENTER, ENTER, \ldots}\)

b. \(-1 \quad \text{ENTER}, \text{Ans} + 0.25 \quad \text{ENTER, ENTER, \ldots}\)

c. \(-1.5 \quad \text{ENTER}, \text{Ans} + 1.5 \quad \text{ENTER, ENTER, \ldots}\)

   i. 
   
   ii. 
   
   iii. 
   
   iv. 

   d. Write a recursive routine for the graph that does not match any of the recursive routines in 4a–c.
1. You can use the following recursive routine to keep track of the monthly savings account balances for the three people listed in the table:

\{0, 1000, 1200, 2200\} \text{ ENTER} \\
\{\text{Ans}(1) + 1, \text{Ans}(2) + 45, \text{Ans}(3) - 30, \text{Ans}(4) - 75\} \text{ ENTER, ENTER}, \ldots

a. Complete the table for all three accounts.

b. How many months elapse before Maria's balance is larger than Yolanda's balance?

c. The recursive routine generates the list \{4, 1180, 1080, 1900\}. Explain the real-word meaning of each number in this list.

d. How many months will elapse before Yolanda's balance is larger than Todd's balance?

<table>
<thead>
<tr>
<th>Time elapsed (mo)</th>
<th>Maria</th>
<th>Yolanda</th>
<th>Todd</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$1000</td>
<td>$1,200</td>
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</tr>
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<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>

2. Refer to Problem 1. Make a scatter plot that shows the balance in Maria’s account over the 6-month period.

3. This graph shows Jon’s distance from the 0 mark as he walks along a measuring tape. The horizontal axis goes from 0 to 6 seconds. The vertical axis goes from 0 to 4 meters. Describe Jon’s walk, indicating the direction, speed, and time interval represented by each segment.

4. Match each recursive routine to a graph. Each square represents 1 unit.

a. 0.5 \text{ ENTER}, \text{Ans} + 0.5, \text{ ENTER, ENTER}, \ldots

b. 3 \text{ ENTER}, \text{Ans} - 0.75, \text{ ENTER, ENTER}, \ldots

c. -1 \text{ ENTER}, \text{Ans} + 1, \text{ ENTER, ENTER}, \ldots

i. \hspace{2cm} ii.

iii. \hspace{2cm} iv.

d. Write a recursive routine for the graph that does not match any of the recursive routines in 4a–c.
Answer each question and show all work clearly on a separate piece of paper.

1. Leslie’s sister gave Leslie her collection of glass animal figurines when she left for college. At that time the value of the collection was $120. Leslie then bought several new glass animal figurines for $4.75 each.
   a. Write a recursive routine to find the value of Leslie’s collection after each new figurine is added. Assume each figurine does not increase or decrease in value.
   b. Write an equation in intercept form \((y = a + bx)\) to describe the relationship between the value of the collection, \(y\), and the number of new figurines, \(x\).
   c. Explain the real-world meaning of the values of \(a\) and \(b\) in your equation.
   d. Show how you can use the equation you wrote in 1b to find the value of Leslie’s collection if she buys 15 new figurines.
   e. If Leslie’s collection is now worth $224.50, how many figurines has she acquired since her sister left for college?

2. Suppose an automobile cost $15,400 when it was new, and each year its value decreases by $935.
   a. Complete the table of values.
   b. Write an equation relating the value of the car, \(y\), to the number of years elapsed, \(x\).
   c. Use your calculator to graph your equation from 2b. Use window settings that allow you to see where the graph crosses both axes. Sketch the graph and indicate the window you used.
   d. What is the coefficient of \(x\) in your equation? What does this coefficient mean in the context of the problem?
   e. What is the \(y\)-intercept of the graph? What does this point mean in the context of the problem?
   f. Where does the graph cross the \(x\)-axis? What does this point mean in the context of the problem?

3. Consider the sequence \(-14.3, -13.5, -12.7, -11.9, \ldots\)
   a. Write a recursive routine that will keep track of the term number and the term for this sequence.
   b. Find the term number of the first positive term of the sequence.
4. For each table, write an equation for $y$ in terms of $x$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
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<tbody>
<tr>
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<td>5.5</td>
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<td>10</td>
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<td>11.5</td>
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<table>
<thead>
<tr>
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<th>$y$</th>
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</thead>
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<table>
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<td>7.75</td>
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<td>-6.75</td>
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<td>-28.5</td>
</tr>
<tr>
<td>7</td>
<td>-43</td>
</tr>
</tbody>
</table>

5. Solve each equation using the method of your choice. Give the action taken for each step.

a. $-6(1 - 2x) - 12 = 0$

b. $\frac{9 + 4x}{3} = -2$

6. Solve the formula for $h$.

$V = \frac{\pi r^2 h}{3}$

**Challenge Problem**

A line contains the points $(-10, -20)$ and $(0, 30)$, as shown on the graph.

a. What is the rate of change for this line? How did you find it? What is the $y$-intercept for this line? How did you find it?

b. Write an equation in the form $y = a + bx$ for the line.

c. Name two other points on this line and show that their coordinates make your equation true.
Choose one or more of these items to replace part of the chapter test. Let students know that they will receive from 0 to 5 points for each item depending on the correctness and completeness of their answer.

1. (Lessons 3.1, 3.2)
   Consider the following recursive routine:
   \[
   \{0, 213\} \quad \text{ENTER} \\
   \{\text{Ans}(1) + 1, \text{Ans}(2) - 8\} \quad \text{ENTER}, \quad \text{ENTER}, \ldots
   \]
   a. Describe a real-world situation that can be modeled with this routine. Explain how the starting values and recursive rules fit your situation.
   b. Write two questions about your situation that can be answered using the recursive routine. Give the answer to each question.

2. (Lessons 3.1, 3.2)
   Write a recursive routine that meets each set of conditions. Your routine should generate both the term number and the value of the term. The starting value should be the 0th term.
   a. The starting value is 3, and the value of the 3rd term is 18.
   b. The rule is “subtract 6,” and the value of the 4th term is \(-26\).
   c. The starting value is positive, and the value of the 7th term is negative.
   d. The starting value is negative, the value of the 3rd term is \(-5\), and the value of the 4th term is positive.

3. (Lesson 3.3)
   You can think of this graph as a time-distance graph. Choose a scale and unit for each axis, and then write a story that fits the graph. Your story should include specific times, rates, and distances.
Chapter 3 • Constructive Assessment Options (continued)

4. (Lessons 3.4, 3.5)
   In this problem, you’ll look at graphs and equations representing trips to Chicago.

   a. A car and a bus travel to Chicago. The car starts out farther from Chicago and drives at a greater speed than the bus. Each vehicle travels at a constant speed. On the same axes, sketch graphs showing how each vehicle’s distance from Chicago changes over time. You do not need to include specific scale values. Indicate which graph represents which vehicle.

   b. The equation $y = 540 - 65x$ can be used to find a minivan’s distance from Chicago, $y$, after $x$ hours.
      i. If the minivan had started closer to Chicago and had traveled more quickly, how would the numbers in the equation be different?
      ii. If the minivan had started the same distance from Chicago but had traveled more slowly, how would the numbers in the equation be different?

   c. The graph shows how the distance of two cars from Chicago changed over time. Explain what the graph tells you about the two cars.
5. (Lessons 3.1–3.5)
Divide these tables, recursive routines, equations, and graphs into groups so the items in each group represent the same relationship. Show all your work and explain how you found your answers.

a.  
\[
\begin{array}{c|c}
 x & y \\
-3 & 7 \\
-1 & 3 \\
3 & -5 \\
5 & -9 \\
8 & -15 \\
\end{array}
\]

b.  
\[
\begin{array}{c|c}
 x & y \\
-5 & -11 \\
-4 & -9 \\
2 & 3 \\
4 & 7 \\
8 & 15 \\
\end{array}
\]

c.  
\[
\begin{array}{c|c}
 x & y \\
-2 & -3 \\
1 & -1.5 \\
4 & 0 \\
6 & 1 \\
10 & 3 \\
\end{array}
\]

d. \{0, 1\} \text{ ENTER} \\
\{\text{Ans}(1) + 1, \text{Ans}(2) - 2\} \text{ ENTER}, \text{ ENTER}, \ldots

e. \{0, -2\} \text{ ENTER} \\
\{\text{Ans}(1) + 1, \text{Ans}(2) + 0\} \text{ ENTER}, \text{ ENTER}, \ldots

f. \{0, -2\} \text{ ENTER} \\
\{\text{Ans}(1) + 1, \text{Ans}(2) + 0.5\} \text{ ENTER}, \text{ ENTER}, \ldots

g. \ y = -2 \\
h. \ y = -1 + 2x \\
i. \ y = 1 - 2x

j. 
\[
\begin{array}{c c c c c}
-5 & -4 & -3 & -2 & -1 \\
1 & 2 & 3 & 4 & 5 \\
1 & 2 & 3 & 4 & 5 \\
\end{array}
\]

k. 
\[
\begin{array}{c c c c c}
-5 & -4 & -3 & -2 & -1 \\
1 & 2 & 3 & 4 & 5 \\
1 & 2 & 3 & 4 & 5 \\
\end{array}
\]

l. 
\[
\begin{array}{c c c c c}
-5 & -4 & -3 & -2 & -1 \\
1 & 2 & 3 & 4 & 5 \\
1 & 2 & 3 & 4 & 5 \\
\end{array}
\]
6. (Lessons 3.4–3.6)
Rob runs a lemonade stand at the park. The table shows the profit he earns for different numbers of cups of lemonade sold.

<table>
<thead>
<tr>
<th>Cups sold</th>
<th>Profit ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>1.75</td>
</tr>
<tr>
<td>21</td>
<td>4.75</td>
</tr>
<tr>
<td>27</td>
<td>7.75</td>
</tr>
<tr>
<td>30</td>
<td>9.25</td>
</tr>
<tr>
<td>45</td>
<td>16.75</td>
</tr>
</tbody>
</table>

Tell whether each statement is true or false and explain how you know.

a. If Rob doesn’t sell at least 12 cups of lemonade, he loses money.

b. Rob makes about $0.12 for each cup of lemonade he sells.

c. The equation $p = -5.75 + 0.5c$ can be used to model this situation.

d. For Rob to make over $100, he must sell at least 200 cups of lemonade.

7. (Lessons 3.4–3.6)
Samantha drives a transport truck between Flint and other cities in the Midwest. Her truck is equipped with a GPS (global positioning system) that allows her dispatcher to keep track of where she is. She has been driving all afternoon. The dispatcher noted her distance from Flint at three times: at 1:12 she was 140.4 mi from Flint, at 2:36 she was 213.2 mi away, and at 4:36 she was 317.2 mi away. She is a very steady driver and right on schedule.

a. Make a table showing the time, measured in hours from noon, and the distance from Flint. Plot the points on a graph. Sketch the line that shows Samantha’s progress.

b. Write an equation modeling Samantha’s trip.

c. What is Samantha’s speed in miles per hour? Explain your thinking.

b. Where was Samantha at noon? How do you know?

e. At what time did Samantha leave Flint? Explain.

f. Samantha’s destination is a truck depot 416 mi from Flint. At what time is she expected to arrive? Show your work.
Lesson 3.1 • Recursive Sequences

1. Evaluate the expression \(\frac{2(3x + 1)}{-4}\) for each value of \(x\).
   a. \(x = 9\)  
   b. \(x = 2\)  
   c. \(x = -1\)  
   d. \(x = 14\)

2. Consider the sequence of figures made from triangles.

   a. Complete the table for five figures.
   b. Write a recursive routine to find the perimeter of each figure.
   c. Find the perimeter of Figure 10.
   d. Which figure has a perimeter of 51?

3. List the first six values generated by the following recursive routine:
   \(-27.4 \text{ ENTER}\)
   Ans + 9.2 \text{ ENTER}, \text{ ENTER}, \ldots

4. Write a recursive routine to generate each sequence. Then use your routine to find the 10th term of your sequence.
   a. 7.8, 3.6, −0.6, −4.8, . . .  
   b. −9.2, −6.5, −3.8, −1.1, . . .
   c. 1, 3, 9, 27, . . .  
   d. 36, 12, 4, 1.3, . . .

5. Ben’s school is \(\frac{3}{4}\) mile, or 3960 feet, away from his house. At 3:00, Ben walks straight home at 330 feet per minute.
   a. On your calculator, enter a recursive routine that calculates how far Ben is from home each minute after 3:00.
   b. How far is he from home at 3:05?
   c. At what time does Ben arrive home?
Lesson 3.4 • Linear Equations and the Intercept Form

1. Match the answer routine in the first column with the equation in the second column.
   a. \(2 \text{ ENTER} \), Ans \(-0.75 \text{ ENTER, ENTER}, \ldots\)
   i. \(y = -2 + 0.75x\)
   b. \(0.75 \text{ ENTER} \), Ans \(+2 \text{ ENTER, ENTER}, \ldots\)
   ii. \(y = 2 - 0.75x\)
   c. \(-0.75 \text{ ENTER} \), Ans \(-2 \text{ ENTER, ENTER}, \ldots\)
   iii. \(y = -0.75 - 2x\)
   d. \(-2 \text{ ENTER} \), Ans \(+0.75 \text{ ENTER, ENTER}, \ldots\)
   iv. \(y = 0.75 + 2x\)

2. A store could use the equation \(P = 6.75 + 1.20w\) to calculate the price \(P\) it charges to mail merchandise that weighs \(w\) lb. (1 lb = 16 oz)
   a. Find the price of mailing a 3 lb package.
   b. Find the cost of mailing a 9 lb 8 oz package.
   c. What is the real-world meaning of 6.75?
   d. What is the real-world meaning of 1.20?
   e. A customer sent $20.00 to the store to cover the cost of mailing. He received the merchandise plus $6.65 change. How much did his parcel weigh?

3. You can use the equation \(d = -10 + 3t\) to model a walk in which the distance \(d\) is measured in miles and the time \(t\) is measured in hours.
   Graph the equation and use the trace function to find the approximate distance for each time value given in 3a and b.
   a. \(t = 2.2\) h
   b. \(t = 4\) h
   c. What is the real-world meaning of \(-10\)?
   d. What is the real-world meaning of 3?

4. Undo the order of operations to find the \(x\)-value in each equation.
   a. \(9 - 0.75(x + 8) - 5 = -2\)
   b. \(\frac{15 - 8(x - 6)}{4} = -2.25\)

5. The equation \(y = 115 + 60x\) gives the distance in miles that a trucker is from Flint after \(x\) hours.
   a. How far is the trucker from Flint after 2 hours and 15 minutes?
   b. How long will it take until the trucker is 410 miles from Flint? Give the answer in hours and minutes.
Recursive Sequences

In this lesson you will

- find recursive sequences associated with toothpick patterns
- find missing values in recursive sequences
- write recursive routines that generate sequences

A recursive sequence is an ordered list of numbers generated by applying a rule to each successive number. For example, the sequence 100, 95, 90, 85, 80, 75, … is generated by applying the rule “subtract 5.” Example A in your book shows how to use your calculator to generate a recursive sequence. Work through the example and make sure you understand it.

Investigation: Recursive Toothpick Patterns

Steps 1–4   Draw or use toothpicks to build the pattern of triangles on page 159 of your book, using one toothpick for each side of the smallest triangle. For each figure, find the total number of toothpicks and the number of toothpicks in the perimeter.

Build Figures 4–6 of the pattern. This table shows the number of toothpicks and the perimeter of each figure.

<table>
<thead>
<tr>
<th>Number of toothpicks</th>
<th>Perimeter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 1</td>
<td>3</td>
</tr>
<tr>
<td>Figure 2</td>
<td>5</td>
</tr>
<tr>
<td>Figure 3</td>
<td>7</td>
</tr>
<tr>
<td>Figure 4</td>
<td>9</td>
</tr>
<tr>
<td>Figure 5</td>
<td>11</td>
</tr>
<tr>
<td>Figure 6</td>
<td>13</td>
</tr>
</tbody>
</table>

To find the number of toothpicks in a figure, add 2 to the number in the previous figure. To find the perimeter of a figure, add 1 to the perimeter of the previous figure. Below are the recursive routines to generate these number sequences on your calculator.

Number of toothpicks:   Perimeter:
Press 3 ENTER.             Press 3 ENTER.
Press ENTER to generate each successive term. Press ENTER to generate each successive term.

Build Figure 10, and find the number of toothpicks and the perimeter. Use your calculator routines to check your counts. (The tenth time you press ENTER, you will see the count for Figure 10.) There are 21 toothpicks in Figure 10 with 12 toothpicks on the perimeter.

(continued)
Lesson 3.1 • Recursive Sequences (continued)

**Steps 5–6**  Repeat Steps 1–4 for a pattern of squares. Here is what the pattern should look like.

![Figure 1](image1) ![Figure 2](image2) ![Figure 3](image3)

Look for rules for generating sequences for the number of toothpicks and the perimeter of each figure. You should find that the number of toothpicks in each figure is 3 more than the number in the previous figure and that the perimeter of each figure is 2 more than that of the previous perimeter. Notice that if you consider the length of a toothpick to be 1 unit, the area of Figure 1 is 1, the area of Figure 2 is 2, and so on.

**Steps 7–8**  Create your own pattern from toothpicks and, on your calculator, find recursive routines to produce the number sequences for the number of toothpicks, the perimeter, and the area.

Here is one pattern and the table and recursive routines that go with it.

![Figure 1](image4) ![Figure 2](image5) ![Figure 3](image6)

<table>
<thead>
<tr>
<th>Number of toothpicks</th>
<th>Perimeter</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 1</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Figure 2</td>
<td>14</td>
<td>12</td>
</tr>
<tr>
<td>Figure 3</td>
<td>20</td>
<td>16</td>
</tr>
<tr>
<td>Figure 4</td>
<td>26</td>
<td>20</td>
</tr>
<tr>
<td>Figure 12</td>
<td>74</td>
<td>52</td>
</tr>
</tbody>
</table>

Here are recursive routines that describe how the figures grow.

- **Number of toothpicks:**
  - Press 8 [ENTER]
  - Press +6.
  - Press [ENTER] repeatedly.

- **Perimeter:**
  - Press 8 [ENTER]
  - Press +4.
  - Press [ENTER] repeatedly.

- **Area:**
  - Press 3 [ENTER]
  - Press +3.
  - Press [ENTER] repeatedly.

For each routine, you can find the result for the figure with 40 puzzle pieces by pressing [ENTER] 40 times. You need 242 toothpicks to build the figure. The perimeter of the figure is 164, and the area is 120.

To find the number of pieces needed for a figure with area 150, use your area routine to generate numbers until you get to 150. You must press [ENTER] 50 times, so you would need 50 pieces. Now, use your number-of-toothpicks routine, pressing [ENTER] 50 times. The result is 302, so you need 302 toothpicks to build the figure.

Now, read Example B in your book, which gives you practice finding missing numbers in recursive sequences.
In this lesson you will
- write linear equations from recursive routines
- learn about the intercept form of a linear equation, \( y = a + bx \)
- observe how the values of \( a \) and \( b \) in the intercept form relate to the graph of the equation

Investigation: Working Out with Equations
Manisha burned 215 calories on her way to the gym. At the gym, she burns 3.8 calories per minute by riding a stationary bike.

Steps 1–3 You can use the following calculator routine to find the total number of calories Manisha has burned after each minute she pedals.

Press \{0, 215\} ENTER.
Press Ans + \{1, 3.8\}.
Press ENTER repeatedly.

In the list \{0, 215\}, 0 is the starting minutes value and 215 is the starting calories value. Ans + \{1, 3.8\} adds 1 to the minute value and 3.8 to the calorie value each time you press ENTER.

You can use your calculator routine to generate this table.

Steps 4–7 In 20 minutes, Manisha has burned 215 + 3.8(20) or 291 calories. In 38 minutes, she has burned 215 + 3.8(38) or 359.4 calories. Writing and evaluating expressions like these allows you to find the calories burned for any number of minutes without having to find all the previous values.

If \( x \) is the time in minutes and \( y \) is the number of calories burned, then \( y = 215 + 3.8x \). Check that this equation produces the values in the table by substituting each \( x \)-value to see if you get the corresponding \( y \)-value.

Steps 8–10 Use your calculator to plot the points from your table. Then, enter the equation \( y = 215 + 3.8x \) into the \( Y = \) menu and graph it. The line should pass through all the points as shown.

Manisha's Workout

<table>
<thead>
<tr>
<th>Pedaling time (min), ( x )</th>
<th>Total calories burned, ( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>215</td>
</tr>
<tr>
<td>1</td>
<td>218.8</td>
</tr>
<tr>
<td>2</td>
<td>222.6</td>
</tr>
<tr>
<td>20</td>
<td>291</td>
</tr>
<tr>
<td>30</td>
<td>329</td>
</tr>
<tr>
<td>45</td>
<td>386</td>
</tr>
<tr>
<td>60</td>
<td>443</td>
</tr>
</tbody>
</table>

Note that it makes sense to draw a line through the points because Manisha is burning calories every instant she is pedaling.
Lesson 3.4 • Linear Equations and the Intercept Form (continued)

If you substitute 538 for \( y \) in the equation, you get \( 538 = 215 + 3.8x \). You can work backward from 538, undoing each operation, to find the value of \( x \).

\[
\begin{align*}
538 &= 215 + 3.8x & \text{Original equation.} \\
323 &= 3.8x & \text{Subtract 215 to undo the addition.} \\
85 &= x & \text{Divide by 3.8 to undo the multiplication.}
\end{align*}
\]

Manisha must pedal 85 minutes to burn 538 calories.

Look back at the recursive routine, the equation, and the graph. The starting value of the recursive routine, 215, is the constant value in the equation and the \( y \)-value where the graph crosses the \( y \)-axis. The recursive rule, “add 3.8,” is the number \( x \) is multiplied by in the equation. In the graph, this rule affects the steepness of the line—you move up 3.8 units for every 1 unit you move to the right.

In your book, read the text and examples after the investigation. Make sure you understand the intercept form of an equation, \( y = a + bx \), and how the \( y \)-intercept, \( a \), and the coefficient, \( b \), are reflected in a graph of the equation. Here is an additional example.

**EXAMPLE**

A plumber charges a fixed fee of $45 for coming to the job, plus $30 for each hour he works.

a. Define variables and write an equation in intercept form to describe the relationship. Explain the real-world meaning of the values of \( a \) and \( b \) in the equation.

b. Graph your equation. Use your graph to find the number of hours the plumber works for $225.

c. Describe how your equation and graph would be different if the plumber did not charge the $45 fixed fee.

**Solution**

a. If \( x \) represents the hours worked and \( y \) represents the total charge, the equation is \( y = 45 + 30x \). The value of \( a \), which is 45, is the fixed fee. The value of \( b \), which is 30, is the hourly rate.

b. Here is the graph. To find the number of hours the plumber works for $225, trace the graph to find the point with \( y \)-value 225. The corresponding \( x \)-value, 6, is the number of hours.

![Graph of Linear Equation]

c. If the plumber did not charge a fixed fee, the \( a \)-value would be 0 and the equation would be \( y = 30x \). The line would have the same steepness, but because the charge for 0 hours would be $0, it would pass through the origin (that is, the \( y \)-intercept would be 0).
Lesson 3.4 • Working Out

In this demonstration you will explore relationships among recursive formulas, linear equations in intercept form, and graphs.

Manisha starts her exercise routine by jogging to the gym, which burns 215 calories. Then she pedals a stationary bike, burning 3.8 calories per minute.

Experiment

Step 1  Open a new Fathom document. Go to the Edit menu (PC) or the Fathom menu (Mac) and choose Preferences. Change the Linear Equation Form to $y = a + bx$.

Step 2  Make a new case table with attributes Time and Calories. Set up recursive formulas for the time for Manisha's workout in increments of 10 minutes, starting at 0, and the calories she burns. Don't forget to account for the calories she burns by jogging to the gym.

Investigate

1. After 20 minutes of pedaling, how many calories has Manisha burned?
2. How long did it take her to burn a total of 443 calories?

Experiment

Step 3  Make a scatter plot of the data. Drag the box to make it quite large.

Step 4  From the Graph menu, choose Add Movable Line.

Investigate

3. Drag the movable line that appears on the graph up and down by its middle. Rotate it by dragging one end. Get it to fit the data points as well as possible, watching how its equation changes at the bottom of the graph window.

4. Make a new attribute Guess in the case table and enter the right side of the equation of the line as its formula. How closely do these data values match those of the Calories attribute?

5. Adjust the formula for Guess until you get a perfect match.

Experiment

Step 5  Select the graph and choose Remove Movable Line from the Graph menu. Choose Plot Function from the Graph menu and graph the formula you got for a perfect match in the table.

Investigate

6. How closely does this graph match the data points?
Lesson 3.5 • Rate of Change

In this demonstration you will explore wind chill data using some of the graphing capabilities of The Geometer’s Sketchpad. The table below shows the wind chill temperatures for a wind speed of 35 miles per hour.

<table>
<thead>
<tr>
<th>Temperature (°F)</th>
<th>-5</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>35</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wind chill (°F)</td>
<td>-35</td>
<td>-28</td>
<td>-21</td>
<td>-14</td>
<td>0</td>
<td>21</td>
</tr>
</tbody>
</table>

Investigate

1. Define the input and output variables.
2. Find the rate of change for the data in the table.

Sketch

Step 1 Open a new Sketchpad document and choose Define Coordinate System from the Graph menu.

Step 2 Choose Plot Points from the Graph menu. Make sure Rectangular is selected. Enter the first point from the table, (-5, -35). Click Plot.

Step 3 Plot the rest of the points. When you are finished, click Done.

Investigate

3. Why can’t you see the points that you plotted? What scale should you use for each axis to show all of the points from the table?

Sketch

Step 4 To change the scale of your graph, drag the point at (1, 0) toward the origin. Continue to drag this point closer to the origin until you can see all of the points you plotted. Click in any blank space to deselect the point.

Step 5 Select the data point that lies on the y-axis and choose Coordinates from the Measure menu.

Investigate

4. What is the real-world meaning of the y-intercept?
5. Using your answers from Question 2 and Step 5, write an equation in intercept form.

Sketch

Step 6 Click in any blank space and then select two of the data points on the graph.

Step 7 Choose Line from the Construct menu, then choose Slope from the Measure menu.

Step 8 Select only the line and choose Equation from the Measure menu.
Lesson 3.5 • Rate of Change (continued)

Investigate
6. How does the measured slope relate to the rate of change you found in Question 2?
7. What is the real-world meaning of the slope, or rate of change?
8. Is the equation the same as the equation you wrote in Question 5? Explain.
9. What is the wind chill for 50°F? Describe how you found it.
10. What temperature would have a wind chill of −49°F? Describe how you found it.

Sketch
Step 9 Select the line. Choose Point On Line from the Construct menu.
Step 10 With the point selected, choose Coordinates from the Measure menu.
Step 11 Drag the point on the line and watch the coordinates change.

Investigate
11. What point represents the answer to Question 9? Question 10?
Lesson 3.7 • Tying Knots

In this demonstration you will make predictions from modeled data that are not exactly linear.

Experiment

Step 1 Either enter class data from the Tying Knots Investigation into a case table or use the sample data in Knots.ftm.

Step 2 Drag a new graph from the shelf and make a scatter plot, with Knots on the horizontal axis and Length on the vertical axis.

Step 3 Choose Add Movable Line from the Graph menu and adjust the line to pass close to all the data points.

Investigate

1. What is the approximate rate of change for this data set? What is the real-world meaning of the rate of change? What factors have an effect on it?

2. What is the $y$-intercept of the graphical model? What is its real-world meaning?

3. What is the difference between the actual measurement of the rope with seven knots and the length predicted by the equation of the movable line?

Experiment

Step 4 From the Graph menu, choose Make Residual Plot. A new graph appears below the first, showing differences between the $y$-coordinates of the data points and the movable line.

Step 5 Adjust the movable line as needed to make as many of the residuals as close to 0 as possible.

Investigate

4. Now what is the difference between the actual measurement of the rope with seven knots and the length predicted by the graph?

5. Use the equation of the line to predict the length of a rope with 23 knots. How much do you trust this prediction? Explain.
CHAPTER 3 Calculator Notes for the TI-83 and TI-83/84 Plus

Note 3A • Recursion on a List

Refer to Note 0D to review recursion and Note 1B to review entering a list into the Home screen.

When defining a recursive routine, the value for the last answer can come either from a list or it can be a single number, as in Note 0D. If you want to generate two patterns at once or keep track of the term numbers of a sequence, using recursion on a list can be useful. When using recursion on a list, you must refer to the number of the term in the list that you want to use. So, Ans(2) does not mean the last answer times 2, but rather the value of the second term in the previous list.

To use recursion on a list, enter a list of initial values and press ENTER. The list must be enclosed in braces. Then create a formula line that is a list, with each term being a formula using the value of a term(s) in the original list. Finally, press ENTER repeatedly to generate a sequence of lists.

The screen here shows a sequence of lists, each containing two terms. The pattern of the first term in each list is a sequence that starts with 1 and increases by 1 each time. This sequence gives an index number for the second term in each list. The pattern of the second terms in the lists is a sequence that starts with 4 and increases by 7 each time. So, for example, {4 25} indicates that 25 is the fourth term in the sequence that begins with 4 and increases by 7 with each new term.

Note 3B/App • Collecting Distance Data Using the EasyData App

You must have a TI-83 Plus or TI-84 Plus to use this Note. If you have a TI-83, see Note 3B on pages 32–33. You can also use the programs CBRSET and CBRGET with any calculator. See page vi.

You will need a CBR (Calculator-Based Ranger).

Connect the CBR to the calculator. Press and select EasyData. The CBR will immediately begin collecting distance data, which is displayed on your calculator screen.

To collect distance data, press Setup (WINDOW) and select 2:Time Graph…. You will be shown the default settings for time interval and number of samples. Press Edit (ZOOM) to edit these settings. Enter 0.2 for the sample interval and press Next. Enter 30 for the number of samples and press Next. Then press OK.

The calculator will resume collecting data. Press Start (ZOOM) to collect the 6 seconds of data you have specified. You will be told that this function will overwrite the current list data. Press OK to continue. The calculator will collect the data and graph it.
To end the Application, press `Main` then press `Quit`. You will get a message telling you where the data is stored. Time data is in `L1`, distance data is in `L6`, velocity data is in `L7`, and acceleration data is in `L8`.

When you press `STAT`, then select 1:Edit, `L1` and `L6` are shown (you’ll need to arrow to the right to see `L6`.) If you want to see `L7` and `L8`, you’ll need to recall those lists. See Note 1B for help recalling or moving a list.

**Note 3B • WALKER Program**

This program can be used with the TI-83, TI-83 Plus, and TI-84 Plus. However, if you have a TI-83 Plus or TI-84 Plus, you should consider using the EasyData App described in Note 3B/App on the preceding page.

This program will collect and graph distance-time data or velocity-time data using a motion sensor. Each WALK command allows you to see the graph being constructed as the data are collected. To start, you must select one of the following commands.

1. **FREE FORM** gives you a blank screen for simple data collection.
2. **WALK LINE** draws a random line for you to try to walk.
3. **WALK PATH** draws a random path of three segments for you to match.
4. **WALK FUNCTION** expects you to enter an equation into `Y1` before you start the program and then to match the path it makes.
5. **WALK SPEED** graphs meters per second and time. You must try to match three horizontal segments.
6. **REPEAT LAST** repeats the previous command on the same graph.
Note 3B • WALKER Program (continued)

PROGRAM: WALKER
FULL
ClrHome
Disp "NOW CHECKING THE"
Disp "CALCULATOR-CBR"
Disp "LINK CONNECTION."
Disp "PLEASE WAIT..."
{1,8}→L₁
Send(L₁)
{0}→L₂
Lbl M
{7}→L₂
Send(L₂)
Get(L₁)
If dim(L₁)=1 and L₁(1)=0
Then
ClrHome
Disp "***LINK ERROR***"
Disp "PUSH IN THE LINK"
Disp "CONNECTORS"
Disp "FIRMLY THEN HIT"
Disp "[ENTER]."
Pause
Goto M
End
Disp "" Output(6,1," STATUS: O.K.")
Output(8,10,"[ENTER]")
Pause
6→T:4→D:2→A
{47,20,6}→WFFT
1→B
{1,8}→CLEAR
{1,11,2}→SONIC
{3,1,-1,8}→READ
GridOn:Func
FnOff
Lbl 0:8+F:
PlotsOff 1:FnOff
Menu "" "" WALKER "" "" FREE
FORM",7," WALK LINE",8," WALK
PATH",9," WALK FUNCTION",A," WALK
SPEED",2," REPEAT LAST",3," QUIT",4
Lbl 1:F:Fn0 n:1→U
Lbl 7:→F
Lbl 0:F→→F
Lbl 1
8→Hmin:Umax
8→Vmin:0:5→Vmax
1→Hsc1:1→Vsc1
PlotsOff
randInt(1,20,4)/2→L₄
{0,T/3,2T/3,T}→L₃
If F=-1:3→L₃
Plot2(xyLine,L₃,L₄,L₅)
If F=1:PlotsOff
Lbl 5
ClrDraw:DispGraph
Text(0,2,"M")
Text(55,83,"S")
Lbl 9:Pause
Send(L CLEAR)
Send(L SONIC)
Send(L READ)
int(RT)+1→N
N→dim(L₁)
For(J,2,N)
Get(L₁(J))
L₁(J)→H
If F=2:7→T
 If F=-2→T
Pt-On(I(J-1),H)
End:Send(L CLEAR)
seq(I,J,0,N-1)→L₆
Pause
Plot1(Scatter,L₁,L₂,L₃)
Goto 0
Lbl 2:2+F
GridOff
0→Xmin:–3
→Xmax:3
→Ymin
→Ymax
1→Xscl:1→Yscl
PlotsOff
randInt(0,4,6)-1.9
L₆(1)→L₂
L₆(2)→L₃
L₆(3)→L₄
L₆(4)→L₅
L₆(5)→L₆
{0,2,4,6,4,6}→L₆
Plot2(xyLine,L₅,L₆)
Lbl 6:2+F
ClrDraw
DispGraph:
Text(0,2,"M/S")
Text(55,83,"S")
Goto 9
Lbl 3
If U=99:Then:Fn0 n:1→U:End
If Vmin=8:Goto 5
Goto 6
Lbl 4
ClrHome
Note 3C • INOUT Program

In this program, you write a linear rule or expression that links a set of input values to their corresponding output values. Before executing the program, choose a level of difficulty that is easy, medium, or hard. We recommend that you choose “EASY” until you get a string of rules correct on the first try. If you make an incorrect guess, the program displays your results and allows you to try again. In the beginning, enter your guess in the form \(a \cdot \text{L1} + b\), where \(a\) is the starting value and \(b\) is the recursive rule. Later you may wish to write your rule in other ways. But always use list \(\text{L1}\) as the variable in your expression. List \(\text{L1}\) is the input list.

PROGRAM: INOUT

\begin{verbatim}
: ClrHome
: Disp "INOUT PROGRAM","EASY",1,
: "MEDIUM",2,"HARD",3,"QUIT",0
: Menu("INOUT PROGRAM","EASY",1,
: Lbl 1
: \{0,1,2,3,4,5\}→L₁
: randInt(–5,5,2)→L₂
: Goto 5
: Lbl 2
: seq(X,X,–3,8)→L₁
: rand(12)→L₁
: SortA(L₁,L₁)
: 6+dim(L₁)
: SortA(L₁)
: randInt(–7,7,2)→L₂
: Goto 5
: Lbl 3
: seq(X,X,–9,9,.5)→L₁
: randInt(9,9,2)→L₂
: L₁(1)→L₁(2)→L₂
: List→matr(L₁,L₁,\[A\])
: Disp "INOUT"
: Disp \[A\]
: Repeat sum(L₃–L₃)=0
: If R:Then
: Input "START VALUE:",S
: Input "CHANGE:",C
: S+CL₁→L₁
: Else
: Input "GUESS:",L₃
: End
: List→matr(L₃,L₁,L₂,\[A\])
: ClrHome
: Disp "INOUT YOU"
: Disp \[A\]
: End
: Goto 8
: Lbl 0: ClrHome
: Disp "PRESS ENTER","TO REPLAY","PRESS","1 AND","ENTER","TO QUIT"
\end{verbatim}

To enter \([A]\), press 2nd [MATRX] and choose \([A]\); don’t use 2nd ( or ALPHA A.

Note 3D • LINES Program

You can use this program to practice writing the equation for a line. Before you begin, clear any equations in the Y= screen, and set the graph style to “line with trail” by moving to the left of Y₁ and pressing ENTER four times. When you run the program, the graph of a line will appear. In the “EASY” option, the
Note 3D • LINES Program (continued)

Run the program and follow these steps:

a. Press 1 ENTER to select the “EASY” option, or 2 ENTER to select the “HARD” option.

b. Press ENTER to see the graph.

c. Study the graph and determine values that will help you write the equation for the line. You can trace on the line to see the coordinates of individual points.

d. Press Y= and enter the linear equation that you think matches the line on the graph.

e. Press GRAPH to compare the graph of your line to the program’s line.

f. If the two lines don’t match, repeat steps c, d, and e until they do. You can enter new equations for other Y= lines or you can clear your old guess before you enter a new one.

g. When the graphs match, you will see them as one line. Trace and switch from one line to the other. The equation in the upper-left corner will indicate whether there really are two lines. Pressing ENTER after you’ve traced a line will only regraph that same line.

h. When you are finished, press 2nd [QUIT]. Press ENTER, then press 1 or 2 and ENTER to replay, or 2 ENTER to quit.

Clean-Up

When you are finished running the program, you will probably want to turn the grid off. You should also turn the expressions off. Press 2nd [FORMAT] and select GridOff and ExprOff. You might also want to clear an equation that is hidden in the Y= screen in Y0.

```
PROGRAM:LINES
ClrHome
A*®
Lbl 2:1®
Lbl 1
Disp "PRESS ENTER\n", "TO SEE GRAPH"
Disp "AFTER YOU ARE\n", "DONE, PRESS Y=":Pause
Func:FnOff:Plots Off
GridOn:
-9.4<Xmin:9.4
1<Xscl:1
-6.2<Ymin:6.2
1<Yscl:1

Y0=6.2*Ymin:6.2*Ymax
ClrHome
Disp "PRESS ENTER", "TO CONTINUE,"
If A=1:randInt(-9,8)*A
randInt(A+1,9)*C
randInt(-6,6)*B
randInt(-6,6)*D
(D-B)/(C-A)*M
"B+M(X-A)"*Y
GraphStyle(0,5)
DispGraph
Stop
Lbl 3

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```
Linear Equations

At this point, you might think about how you and your student are interacting. For example, are you being a student to your student? Do you explain as little as possible or just enough so that your student is becoming an independent learner and thinker? Is the pencil or calculator in your student’s hands? Do you try not to answer questions that your student hasn’t asked? Telling your student too much can waste time, because your student might not understand. It can lead to your student feeling overwhelmed. Deeper understanding can result when you allow your student to teach the concept or skill to you and others.

Content Summary

Chapter 3 focuses on equations of lines. Students expand their idea of linearity, and they learn to work backward.

Linearity

You can think of linearity in several ways. Linearity as constant rate of change. One way to think about linearity is that the rate of change of one variable in relation to the other is constant. You start somewhere and advance by the same amount at each step. This kind of change is called linear growth, although the values will be shrinking instead of growing when the rate of change is negative. With variation in Chapter 2, growth always started at 0; in this chapter, growth can start with any value.

Linearity as equations of the form \( y = ax + bx \). Another way of thinking about linearity is through equations that relate variables. In this book, linear equations have the intercept form \( y = ax + bx \). This form indicates the starting point, \( a \), and what’s added to it, \( b \), each time \( x \) increases by one unit. The traditional slope-intercept form \( y = mx + b \) that you might remember is mentioned in Chapter 4, but the intercept form introduced in this chapter better reflects the idea of growing at a constant rate from a starting point.

Linearity as graphs of lines. You can also understand linearity through graphs. The equation \( y = ax + bx \) is graphed by starting at point \( (0, a) \) and moving vertically by amount \( b \) for each unit moved across from left to right.

Working Backward

Many real-life situations call for predicting when a quantity will grow to a certain value. Ways of making that prediction reflect the three ways of thinking about linearity. From the growth perspective, you might think of counting the steps as you repeatedly add on to the starting point until you reach the desired value. This can be done by hand, or you can use home-screen recursion or sequences on a graphing calculator.

If the situation is represented by an equation, there might be two methods of solving it: the undoing method and the balancing method. If you know that \( 3x + 2 = 17 \), you can use the undoing method and think, “I multiply \( x \) by 3 and add 2 to get 17. To find \( x \), I can undo that process, beginning with 17. I subtract 2 (to get 15) and then divide by 3 (to get 5).” You’ll need the balancing method if the unknown appears more than once. Applying the balancing method to the equation \( 3x + 2 = 17 \), you subtract 2 from both sides to get \( 3x = 15 \), and then divide both sides by 3 to get \( x = 5 \).

(continued)
Chapter 3 • Linear Equations (continued)

Summary Problem
You and your student might discuss this summary problem from Chapter 3. It’s a good problem to revisit several times while working through the chapter.

Here’s a table showing the heights above and below ground of different floor levels in a 25-story building (taken from page 158):

| Floor number | Basement (0) | 1 | 2 | 3 | 4 | … | 10 | … | … | 25 |
|--------------|-------------|---|---|---|---|----|----|----|----|----|----|
| Height (ft)  | −4          | 9 | 22| 35| … | … | 217| … | … | … | … |

What floor has a height of 282 feet?

Questions you might ask in your role as student to your student include:

- How far apart are the floors?
- What could the negative number mean?
- Could you solve this problem by recursion on a graphing calculator?
- Is it possible to represent the height by an algebraic expression?
- Does the distance between floors appear in the expression?
- Does the height of the basement appear in the expression?
- Can the whole problem be represented by an equation?
- Can you graph the equation?
- What are various ways of solving the equation?
- Could you make an equation that tells how long it takes an elevator to reach various floors?
- In the Empire State Building in New York City, the floors vary in heights. Could you still write an equation that might be useful for either the heights or the elevator time?

Sample Answer
The floors, which start with a negative number (possibly meaning that the basement floor is below ground level), are 13 feet apart. To solve the problem on a graphing calculator, you could start with −4 and repeatedly add 13 until you reach 282. It would be most efficient to use recursion on a list [see Calculator Note 3A] to keep track of both the floor number and the height.

Or you can solve the equation \( \text{height} = -4 + 13 \cdot (\text{floor number}) \), or \( 282 = -4 + 13x \). Using the balancing or the undoing method, you can solve the equation to get \( x = 22 \).

Ask what 22 represents (the floor with a height of 282 feet). If you know the time it takes for the elevator to travel one floor, you can use that number in place of 13 to find the time it takes the elevator to travel from the basement to any other floor.

For buildings with irregular floor heights, you might use an equation containing the average height to make estimates of height or time. Or you might use different equations for different parts of the building.
1. (Lessons 3.1, 3.2) Plot the first six points represented by each recursive routine.
   a. \{-4, 2\} \{Ans(1) + 1, Ans(2) + 3\}
   b. \{0, 1.5\} \{Ans(1) + 1, Ans(2) - 0.25\}
   c. \{2, -2\} \{Ans(1) + 1, Ans(2) + 0.5\}

2. (Lessons 3.3, 3.4) The table at right shows a person’s distance from a motion sensor at various times.
   a. Describe the walk shown in the table. Include where the walker started, how quickly the walker walked, and in what direction the walker walked.
   b. Write a linear equation for the walk, in intercept form. Graph the equation and plot the points from the table.

3. (Lessons 3.5, 3.6) A local theater company has a yearly membership fee, and members pay a reduced per-ticket cost. The equation \(C = 25 + 8n\) expresses the total cost \(C\) of purchasing \(n\) tickets in a single year.
   a. Based on the equation above, what is the yearly membership fee? What is the cost per ticket?
   b. If a person does not want to buy a membership, theater tickets cost $10 each. Write an equation for the total cost \(C\) of purchasing \(n\) tickets for someone without a membership.
   c. Graph both equations for the cost of \(n\) tickets. What is the rate of change of the cost for a member? For a non-member?
   d. Christina looked at the schedule for the upcoming theater year, and she found 12 shows that she would like to attend. Should she buy a membership?
   e. Last year, Christina bought a membership, and she spent a total of $137 that year on the membership fee and theater tickets. How many tickets did she buy?

4. (Lesson 3.6) Give the additive inverse of each number.
   a. 1  b. \(-1.25\)  c. \(\frac{3}{4}\)  d. \(-\frac{6}{5}\)

5. (Lesson 3.6) Give the multiplicative inverse of each number in Exercise 4.
1. a. The graph should include the points \((-4, 2), (-3, 5), (-2, 8), (-1, 11), (0, 14),\) and \((1, 17)\).

![Graph diagram]

\([-5, 2, 1, 0, 20, 5]\]

b. The graph should include the points \((0, 1.5), (1, 1.25), (2, 1), (3, 0.75), (4, 0.5),\) and \((5, 0.25)\).

![Graph diagram]

\([-1, 6, 1, -0.5, 2, 0.5]\]

c. The graph should include the points \((2, -2), (3, -1.5), (4, -1), (5, -0.5), (6, 0),\) and \((7, 0.5)\).

![Graph diagram]

\([0, 8, 1, -3, 2, 1]\]

2. a. The walker started 0.3 m away from the sensor, and walked away from the sensor at a rate of 0.4 m/s.

\(y = 0.3 + 0.4x\)

![Graph diagram]

\([0, 6, 1, 0, 4, 1]\)

2. a. The graph should include the points \((0, 1.5), (1, 1.25), (2, 1), (3, 0.75), (4, 0.5),\) and \((5, 0.25)\).

![Graph diagram]

\([-1, 6, 1, -0.5, 2, 0.5]\]

c. The graph should include the points \((2, -2), (3, -1.5), (4, -1), (5, -0.5), (6, 0),\) and \((7, 0.5)\).

![Graph diagram]

\([0, 8, 1, -3, 2, 1]\]

2. a. The walker started 0.3 m away from the sensor, and walked away from the sensor at a rate of 0.4 m/s.

\(y = 0.3 + 0.4x\)

![Graph diagram]

\([0, 6, 1, 0, 4, 1]\)

3. a. $25; $8. The total cost is \((\text{membership fee}) + (\text{cost per ticket}) \cdot (\text{number of tickets})\).

\(C = 10n\)

b. Graph \(y = 25 + 8x\) and \(y = 10x\); Rate of change for member: $8 per ticket; for non-member: $10 per ticket.

![Graph diagram]

\([0, 20, 2, 0, 200, 20]\]

d. Christina should not buy a membership. Compare the total cost under each option. With membership,

\[
\frac{25}{\text{number of tickets}} + \frac{8}{\text{number of tickets}} \cdot 12 \quad \text{and} \quad \frac{12}{\text{number of tickets}} \cdot 25 \quad \text{for member.}
\]

Without membership,

\[
\frac{10}{\text{number of tickets}} \cdot 12 \quad \text{for non-member.}
\]

She will save $1 by not getting a membership.

e. 14 tickets. Solve the equation \(25 + 8n = 137\). This solution uses the balancing method; students might also solve by undoing.

\[
\begin{align*}
25 + 8n &= 137 & \text{Original equation.} \\
25 + 8n - 25 &= 137 - 25 & \text{Subtract 25 from both sides.} \\
8n &= 112 & \text{Combine like terms.} \\
n &= 14 & \text{Divide both sides by 8.}
\end{align*}
\]

4. Take the opposite of each number. The additive inverse is the number that is added to the given number to equal 0.

\[
\begin{align*}
a &= -1 & b &= 1.25 & c &= \frac{3}{4} & d &= \frac{6}{5} \\
\end{align*}
\]

5. Find the reciprocal of each number. The multiplicative inverse is the number that is multiplied by the given number to equal 1.

\[
\begin{align*}
a &= 1 & b &= -0.8 & c &= \frac{4}{3} & d &= -\frac{5}{6} \\
\end{align*}
\]
Quiz 2
(For use anytime after Discovering Algebra, Lesson 3.4.)

Name _________________________  Period ___________  Date ______________

1. A student group is hosting a nature trail cleanup project for 80 volunteers. They plan to pick up trash along 18 kilometers of trail. If each volunteer cleans the same length of trail, how many meters of trail will each volunteer be responsible for?
   A) 144 m  B) 180 m  C) 270 m  D) 444 m  E) None of these

2. The owner of a skating rink rents the rink for private parties. The owner charges a flat fee of $150.00 plus an additional $3.75 for each skater. Let s represent the number of skaters. Which equation could be used to determine c, the total cost of renting the rink?
   F) c + 3.75s = 150  
   G) c = 150 + 3.75s  
   H) c = (150 + 3.75)s  
   I) c(150 + 3.75) = s  
   J) c + 150 = s + 3.75

3. In 2001, Alaska had a population of 635,000 people. In the same year, 265,000 people lived in Anchorage Borough, Alaska. To the nearest whole percent, what percent of Alaska’s population lived in Anchorage Borough in 2001?
   A) 24%  B) 29%  C) 37%  D) 42%  E) 68%

4. A video store is giving away used videotapes to its most loyal customers. The table shows the type and number of videotapes to be given away.

   If Miguel chooses a videotape at random, what is the probability that he will choose either a comedy or a sci-fi movie? Express the probability as a decimal, and give mathematical evidence to justify your answer.

   **Videotapes**
   
<table>
<thead>
<tr>
<th>Type</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Action</td>
<td>13</td>
</tr>
<tr>
<td>Drama</td>
<td>11</td>
</tr>
<tr>
<td>Comedy</td>
<td>10</td>
</tr>
<tr>
<td>Sci-fi</td>
<td>6</td>
</tr>
</tbody>
</table>

   Answer ____________________________________________

(continued)
5. The table shows the price per share of two stocks over a seven-week period.

   a. Make a double-line graph on the grid that shows how the prices of the two stocks change for the weeks shown in the table. Include a title, labels for the axes, appropriate scales, and a key.

   b. During which week was the price difference between the two stocks the greatest? Use information from your graph to justify your answer.

      Answer: ______________________________________
               ______________________________________
               ______________________________________
               ______________________________________

   c. Predict what the price of Stock Y will be in week 8 if it continues to increase at the same rate. Give mathematical evidence to justify your answer.

      Answer: __________________