# Discovering Algebra
## An Investigative Approach

# A Guide for Parents


Key Curriculum Press
Innovators in Mathematics Education

**Teacher's Materials Project Editor:** Elizabeth DeCarli

**Project Administrator:** Aaron Madrigal

**Contributing Writers:** Larry Copes, Kendra Lockman

**Accuracy Checker:** Dudley Brooks

**Project Manager:** Rose M. Rummel-Eury, Interactive Composition Corporation

**Editorial Production Supervisor:** Christine Osborne

**Production Supervisor:** Ann Rothenbuhler

**Production Director:** McKinley Williams

**Text Designer:** Jenny Somerville

**Composition, Technical Art, Prepress:** Interactive Composition Corporation

**Cover Designers:** Jill Kongabel, Marilyn Perry, Jensen Barnes

**Printer:** Data Reproductions

**Textbook Product Manager:** James Ryan

**Executive Editor:** Casey FitzSimons

**Publisher:** Steven Rasmussen

© 2007 by Key Curriculum Press. All rights reserved.

Cover Photo Credits: Background image: Pat O'Hara/DRK Photo. Bottom image: Marc Epstein/DRK Photo. All other images: Ken Karp Photography.

**Limited Reproduction Permission**
The publisher grants the teacher whose school has adopted *Discovering Algebra,* and who has received *Discovering Algebra: An Investigative Approach, A Guide for Parents* as part of the Teaching Resources package for the book, the right to reproduce material for use in his or her own classroom. Unauthorized copying of *Discovering Algebra: An Investigative Approach, A Guide for Parents* constitutes copyright infringement and is a violation of federal law.

TMThe Discovering Mathematics logo is a trademark of Key Curriculum Press. All other registered trademarks and trademarks in this book are the property of their respective holders.

Key Curriculum Press
1150 65th Street
Emeryville, CA 94608
510-595-7000
editorial @keypress.com
www.keypress.com

Printed in the United States of America
10 9 8 7 6 5 4 3 11 10 09 08 ISBN 978-1-55953-774-2

# Contents

# The *Discovering Algebra* Approach to Learning

*Discovering Algebra: An Investigative Approach* covers the topics offered in traditional algebra courses, but the teaching style as well as the learning experience might be different from what you remember from your own high school algebra course.

In the past, and probably in your own school experience, students were asked to spend a lot of time manipulating symbols—moving *x*'s, *y*'s, and numbers around in expressions and equations—before they got the chance to understand what they were doing. For example, you might recognize this scenario: After going over homework, your teacher showed a new type of problem and a method for solving it. You worked alone with pencil and paper and practiced solving problems of that type. For homework, you worked on more problems of the same type. The next day, the class went through that same process with a new type of problem. At some point, you took a test with a lot of problems on it. You had to remember the methods and figure out what method to use for each problem. If you did well on all the tests, you were "good at math." If you didn't do well, you might have thought you "just couldn't do math."

Many students cannot succeed in such an environment. Perhaps you had a hard time yourself. The teacher and textbook cannot furnish enough examples to apply to every new situation or problem. As a result, many students are limited in their understanding, unable to do more than mechanical manipulations. They don't know when to apply a particular problem-solving strategy. They don't come away from their math course with a set of ideas that weave together into "the big picture." They doubt that mathematics will be relevant to their careers and they don't see what others like about it. Even students who pass are reluctant to continue on in mathematics. Some develop "math phobia"—the fear of math—and avoid courses in science or business that require math. Ultimately, their fear limits their career choices and their life income.

But all students can learn math better, have a good time doing it, and come away with an appreciation of its value as a tool for science, business, and everyday life. *Discovering Algebra* is a program that helps all students reach a deep understanding of math by encouraging them to investigate interesting problems in cooperative groups, use technology where appropriate, and practice skills that make routine problems automatic.

## All Students Benefit

From their own teaching experience, the *Discovering Algebra* authors know that all students can experience more success in mathematics. When the focus is on understanding concepts and problem-solving strategies instead of just memorizing formulas and procedures, students with concentration, attention, and memory issues can be more successful. Passive or reluctant students will learn to communicate better. To say that all students can learn math does not mean the course has been watered down. In fact, even very successful math students will find they are challenged, learn more, and remember longer with the *Discovering Algebra* approach. That's because the concepts and methods are not isolated from

real-world applications, or from previously learned ideas, or from information they are getting in other classes. The mathematics that students study is closer to what is needed by both students seeking employment after high school and students preparing to attend college.

## Deep Understanding Is Important

In your own math classes, you might have been told: "Just do it—don't ask why." But there are logical reasons behind mathematical methods and ideas, and the people who understand these reasons succeed at math and, ultimately, at science and business. *Discovering Algebra* helps more students understand these reasons. Because the concepts make sense to students, students remember the methods (or reinvent them if they've forgotten them) and can apply them to new problems. To help develop that kind of flexible understanding, *Discovering Algebra* offers a more visual approach, with clearer and more frequent illustrations and graphs, and thoughtful integration of text captions to lead students through examples. *Discovering Algebra* also acknowledges the need for a gradual development of mathematical ideas. Students are helped to see where the text is leading, and full-blown explanations are delayed until all the groundwork is laid. Once a topic has been made part of what students are expected to know, it is reviewed and referred to again whenever appropriate. Understanding the math can make the math more fun, will increase pride and confidence, bolster capacity for critical and abstract thinking, and increase the chance that students will use math in their lives.

## Students Learn Better in Cooperative Groups

Students are not expected to do all this learning by themselves. Many students make sense of mathematical ideas best in interaction with other people, using informal language. They think best out loud, or they get ideas from others. And they understand better from seeing other students' viewpoints. They learn that nothing bad happens if they make mistakes or misapply a procedure and that trial and error is also a respected strategy. This helps quiet or insecure students learn to contribute. When students are working in groups, the teacher circulates and observes, poses questions, and intervenes when necessary to assist. He or she works as a partner to student groups, monitoring the back-and-forth, modeling good communication, and drawing out clues that students are confused or on the right track. Group work helps students learn better, and it also teaches essential teamwork skills. In their groups, students will be asked to demonstrate their understanding both orally and in writing.

## Investigation Is Motivating

Some students learn better by seeing, some by hearing, and some by reading, so an explanation that makes sense to one student might not make sense to another. These different "learning styles" are addressed by the investigations in *Discovering Algebra*. Because most students are more interested in class if the problems they investigate are related to the real world, many of the hands-on investigations involve problems that students might see in their lives outside school. Some investigations use very familiar scenarios, and others are career-oriented. Some investigations allow students to get up and move around, many use graphing calculators or other technology, and some involve pure mathematical ideas. In your student's *Discovering Algebra* textbook is a career-oriented investigation on

page 103, an on-your-feet investigation on page 172 that uses motion sensors, and an activity using a familiar object—the bicycle—on page 132. The Multiply and Conquer Investigation on page 97 is an example of a pure mathematics activity. The teacher might have students work in groups on the investigation, and later lead a whole-class discussion. Each student develops his or her own understanding and benefits from sharing ideas and suggestions offered by others. Students learn that there are many approaches to solving problems. They also learn that they are individually responsible for describing orally or in writing what they have learned.

## Problem Solving Is Important

In life, we all need to be good at solving problems that don't exactly fit into a model we know. This is an important job skill and career asset as well: People who "think outside the box" to solve problems at work move up faster and are seen as leaders. To help prepare students to use math in their lives, many investigations in *Discovering Algebra* pose problems that students haven't already been told how to solve. They learn to brainstorm, consider subproblems, come at a problem from a unique angle, and make diagrams and models. In this way, they learn problem-solving skills rather than learning how to solve only particular types of problems.

## Using Technology Helps

Computers and calculators surround us, and students will use them at work, sometimes with custom-designed software, so working with them in these classes teaches students skills that will be useful later on. Whether you are computer literate yourself or strictly "low-tech," your student is probably fascinated with technology, and using technology in class will help keep your student interested.

Technology is not used as a substitute for learning basic arithmetic. When used appropriately, technology can make mathematics more visual, more logical, and more fun. Most importantly, technology tools allow students to investigate many more situations and examples than they can explore by using pencil and paper. Getting fast results on numerous examples helps students see patterns, form generalizations, and test conclusions. That leads to a deeper understanding of concepts and a greater willingness to explore further and tackle larger problems. If your student's teacher does not have access to technology, or doesn't have access on certain days, the *Discovering Algebra Teacher's Edition* suggests low-tech alternatives for technology-dependent investigations. Calculator Notes for various calculator models are available online at www.keymath.com. These notes give the keystroke instructions to perform the functions needed for class activities. Homework exercises that require a graphing calculator are noted in the text. The teacher can give you advice about which graphing calculator to purchase if you decide to buy one for your student.

## Practicing Skills Is Essential

As students investigate a new concept, they develop and practice new skills. After students learn why a process works, they apply their new skills in the Practice Your Skills exercises in the student text. They extend these skills in the Reason and Apply exercises. Finally, each lesson has Review exercises so that students retain and extend their understanding of skills and concepts they learned in previous lessons. For further practice, your student's teacher has probably received

a copy of *Discovering Algebra: More Practice Your Skills*. You can access these worksheets online at www.keymath.com/DA.

*Discovering Algebra* supports an approach to mathematics that brings about better understanding of concepts and skills. Instead of solving one type of problem after another, students engage in investigations, examples, and exercises that help them build up their own bank of skills and concepts. Students learn to describe how and why something is true. Instead of working alone, students bounce ideas off their peers. Because students are actively involved in acquiring skills and concepts, they can successfully attack and deal with test problems even if they forget a particular process or formula. Your student's teacher also has access to a wide range of support materials that will help him or her to respond to an individual student's pace, language issues, and need for additional assistance or enrichment.

# Working with Your Student

Begin by taking stock of how your student uses his or her after-school time. Evaluate whether there is a suitable place with good light to make homework a comfortable activity, and whether distractions in the homework environment are manageable. Your support and praise are as important to your student's success as the teacher's guidance and the quality of the learning materials. You'll want to make your support effort as thoughtful as possible.

## Your Own Experience with Math Is a Big Influence

Did you do well in math when you were in school? If math was hard for you, you might actually find it easier to help your student than if it came easily to you, because you'll be especially sympathetic. You've probably also developed some practical understanding since you left school. The important thing is to work hard to keep from passing on negative ideas about math. You have the chance to help your student have a better attitude toward math. Your message must be "mathematics is important for everyone." To be successful in our society, everyone must be able to recognize when a situation needs a mathematical solution, to tell what quantities are involved, and to understand how to work toward a solution. Your student has the benefit of a better approach and better materials than you probably had.

What if you're good at math? You will have to work hard to keep from dominating your student's learning. It's sometimes very hard to resist explaining an idea or giving an answer *you* understand, but holding back is necessary if your student is to remember the idea and ultimately become an independent learner. Praise all your student's honest efforts and support his or her attempts to explain, question, or break down the problem.

No matter how comfortable you are with math, you can help your student reach the goals of the *Discovering Algebra* approach and learn algebra. Try to establish two habits when you work with your student.

First, *be a student to your student.* Keep asking him or her for explanations. Ask questions as if *you* were the student trying to learn. No matter how well you understand things yourself, asking "Why does that work?" is better than saying, "Here's how to do that."

Second, *be curious and enthusiastic.* Offer comments like, "I haven't seen this idea before, but it seems interesting" rather than "It's beyond me!" or "This isn't important." Ask what happened in class, ask what your student contributed and how well he or she understood, and be curious about the homework. Showing this kind of interest says that you expect your student to be actively involved in class and to work on homework every day.

## Learn About and Use Other Resources

Use this guide in conjunction with the *Discovering Algebra* textbook. Refer to the notes on individual chapters. References are made to specific examples and exercises in the text. By all means, be aware of resources your student has at school and what he or she can access from home.

## Use Tried and True Strategies

Some classic problem-solving strategies can help your student, and you can assist him or her to use them.

1. *Make an organized list.* Stating the facts given in a problem one at a time is especially helpful for a student with low reading skills, an attention deficit disorder, or a simple case of impatience. Be sure your student understands what is being asked for. Have your student do the writing. Another approach is to build a table of values, prices, or corresponding numbers. This will help your student *look for patterns* that give a clue to the answer or that lead to a solution process. Flow charts can also help a student work step by step.
2. *Draw a diagram.* This is very helpful for real-world problems, or problems that have geometric figures or a coordinate grid. Be sure your student is the one doing the drawing. You can coach, ask questions, or make suggestions: "Why not draw a line for the wall?" "Where is the person standing?" Encourage your student to *label parts of the diagram* with quantities that represent distance or other measurements, and use arrows for motion. Use stick figures or smiley faces to represent people.
3. *Eliminate possibilities.* Deciding what kind of solution is improbable or not at all possible can spark your student's thinking process. If the "ingredients" for a process he or she wants to apply are not present in the problem, or if not enough information is available, a different line of reasoning is necessary.
4. *Solve an easier, related problem.* Substitute easier numbers in the given problem to make the process clearer. Then put the "harder" or "messier" numbers back in and apply the same process. Or, just *work on one stage of a problem.* This might help your student recognize a process that he or she remembers and understands. It also reestablishes a climate of success. Be sure to praise success on easier problems or success with partial answers to demonstrate your support and to prove to your student that he or she has some level of ability and achievement.
5. *Work backward.* Start at the end of a series of steps and see how it feels to work toward the beginning. This is a good way to check whether a guessed answer is right and to understand why it was a good guess. *Guess-and-check* is a good strategy in itself if the student gets closer and closer to the right answer in successive steps.

If your student continues to have homework trouble even when you have tried to help, you can guide him or her to list questions for the teacher. This list will help the teacher know whether the student feels that he or she basically understood the lesson and is simply stuck on a single problem, or whether the student feels completely off track and hasn't understood the lesson or even the last several lessons. Are there particular symbols that your student doesn't understand? Is there an example in the book that he or she cannot follow? Helping your student script questions to the teacher will reduce anxiety or shyness about asking for help. If, finally, your student feels unable to ask for assistance, you should intervene with a note or call to the teacher.

Your student will be using a graphing calculator in class. You might want to purchase a graphing calculator for your student at home. Ask your student's teacher for advice about the kind of calculator to purchase and where to purchase it. Unless the teacher recommends another calculator, a good choice is the TI-83 Plus or TI-84 Plus manufactured by Texas Instruments.

If you have Internet access, you can enrich your student's experience by having your student follow Web links and view the Dynamic Algebra Explorations available for *Discovering Algebra.* You can also download Calculator Notes, Condensed Lessons, and More Practice Your Skills worksheets. Find these at www.keymath.com/DA.

If the teacher has registered, you can access the online version of *Discovering Algebra.* Discovering Algebra Online is a service that provides students with access to all the content of their printed textbook page by page, in an easy-to-use format. The online textbook has an interactive glossary and an index, and direct links to the chapter-specific resources just mentioned.

*Discovering Algebra* has been designed with an investigative approach to engage your student in doing mathematics—understanding, learning, remembering, and applying algebra skills. With your student's growing sense of responsibility for his or her own learning, a teacher's professional guidance, and your earnest support, your student will make gains in mathematics and have a positive experience with algebra.

# Overview of Topics in *Discovering Algebra*

The arrangement of topics in *Discovering Algebra: An Investigative Approach* is carefully planned to help students develop connections between new and previously learned material and to build their understanding.

In Chapter 0, students review and increase their facility with some arithmetic skills by using them to solve problems. Students are introduced to the idea of recursion, an intuitive procedure of doing a process over and over again, each time building on the last step. Recursion will be used throughout the course.

In Chapters 1 through 5, students learn to set up and solve linear equations—equations whose graphs are lines—which is the heart of a beginning algebra course.

- In Chapter 1, students use graphs and statistical measures to organize and make sense of data.
- In Chapter 2, students work on proportional reasoning and learn how to solve equations by undoing, a powerful method that works for many types of equations.
- Chapter 3 allows the idea of linear expressions to grow out of proportional reasoning. Students are also introduced to the balancing method for solving equations.
- In Chapters 4 and 5, students see linear equations in other contexts. In Chapter 4 students deepen their understanding of linear equations by fitting lines to data (building on ideas in Chapter 1). Chapter 5 focuses on expanding the ideas of linear equations through systems of these equations, and through the introduction of inequalities.

In Chapters 6 through 9, students study nonlinear growth.

- In Chapter 6, students learn about exponential growth and equations.
- Chapter 7 generalizes linear and exponential growth to the idea of function.
- Chapter 8 shows how graphs of functions can be transformed.
- In Chapter 9, students investigate relationships between quadratic functions and their graphs and equations.
- Chapter 10 introduces probability and counting techniques.
- Chapter 11 previews geometry.

## Chapter Summaries

In chapter summaries, the chapter content is briefly summarized, and important new words are italicized. A summary problem is presented, along with question prompts that you can use to get your student thinking. The summary problem is a comprehensive problem that will give you and your student a lot to talk about. The question prompts are followed by sample answers. Review exercises and solutions are provided at the end of the material for each chapter.