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This book will help clarify your role as the facilitator of your students’ learning in the investigation-based, technology-rich *Discovering Mathematics* classroom. *Discovering Mathematics* challenges both students and teachers to be imaginative, creative, and curious. You will find suggestions for planning lessons and structuring investigations, for assessment, and for communicating with parents. You will be led through a sample lesson from *Discovering Algebra*, *Discovering Geometry*, and *Discovering Advanced Algebra* to give you ideas for guiding an investigation and make you more comfortable with the *Discovering Mathematics* approach. We feel confident that you’ll be happy with your students’ results, the classroom atmosphere, and your own teaching satisfaction as you implement the *Discovering Mathematics* series.
Learning by Doing

As a mathematics teacher, you know the importance of getting your students involved. When you engage your students in an interesting problem, they are motivated to learn the ideas, terminology, and techniques they need to talk about and solve the problem. With engaged students, you become less of an instructor and more of a facilitator or guide: Your students’ curiosity drives their questions while you support their explorations and steer them toward the answers they’re seeking. As your students seek answers to their questions, they willingly practice mechanical skills as they explore possible outcomes. And they take ownership of the concepts because the ideas have come as a result of their own efforts. Your students thus understand these ideas more deeply and are better equipped to apply them to solve other problems than if they had used a more traditional approach.

Skills and Mechanics

You want to be sure that your students learn the basic mathematics skills they will need to go on in mathematics. With Discovering Mathematics, you will find that your students are practicing computation and mechanics as they work on the in-class investigations and complete the exercises following each lesson.

Students meet standards. Your students get quality practice working with numbers, equations, and geometry and algebra concepts. In the course of doing investigations and completing the exercises, they are practicing skills as they learn mathematics concepts. On standardized tests of basic skills, you will find that your students will do as well as or better than students using a traditional mathematics curriculum.

Number sense is developed. The books of the Discovering Mathematics series are carefully planned to develop number sense along with facility at symbol manipulation. As students complete the investigations, you’ll see that they will try various approaches requiring them to estimate, compute, graph relationships, make and verify conjectures, apply what they know about functions, solve equations, and think about whether or not a result makes sense.

Skill practice and application are provided. Your students practice these and other skills as they investigate and in the exercises that end each lesson; these skills are also reviewed periodically throughout the book. Because your students practice skills in the context of investigations, they are learning to select among their stock of skills, identify appropriate problem-solving methods, and apply their skills in ways they will use throughout their lives. This type of skill application is both realistic and motivating. While investigating, students are encouraged to make sense of ideas and concepts for themselves. As they innovate their own processes, they are acquiring and reinforcing essential mathematical skills. With the confidence that they understand the concepts and have strategies to solve problems on their own, students don’t need to memorize algorithms. What is more, students who have strategies for approaching problems are less likely to panic on a test when they can’t remember the “right way” to solve a problem.
If some students have come to your class lacking basic mathematics skills, or if your assessment reveals some students who need more practice, the Teaching Resources package provides extra practice materials. *More Practice Your Skills with Answers* contains blackline masters for worksheets related to the skills learned in each lesson. For example, the extra practice for *Discovering Algebra* Lesson 4.1, A Formula for Slope, relates slope to earlier work students have done with linear equations.

### Lesson 4.1 • A Formula for Slope

<table>
<thead>
<tr>
<th>Name</th>
<th>Period</th>
<th>Date</th>
</tr>
</thead>
</table>

1. Find the slope of each line using a slope triangle or the slope formula.
   - a.
   - b.
   - c.

2. Find the slope of the line through each pair of points.
   - a. (0, 4), (5, 8)
   - b. (−4, 3, 8); (2, 7, −1, 4)
   - c. \[ \left( \frac{1}{2}, \frac{3}{4} \right) \]
   - d. (−8, 2); (−8, −5)

3. Given one point on a line and the slope of the line, name two other points on the line. Then use the slope formula to check that the slope between each of the two new points and the given point is the same as the given slope.
   - a. (3, 1); slope \[ \frac{2}{3} \]
   - b. (4, 2); slope 1
   - c. (5, 3); slope \[ −1 \frac{2}{3} \]
   - d. (−3, 6); slope 0
   - e. (−4, −7); slope −2
   - f. (8, −5); slope \[ \frac{5}{7} \]

4. Write the equation of each line in intercept form.
   - a.
   - b.
   - c.
   - d.

A consumable version of *More Practice Your Skills* without answers is also available. Parents and students can find these worksheets at [www.keymath.com](http://www.keymath.com). As needed, you might use the TestCheck Test Generator and Worksheet Builder to quickly create additional worksheets on any lesson objective.
Investigation

Student discovery is at the heart of the Discovering Mathematics series. The Discovering Mathematics books, therefore, embody the essential component of discovery: investigation. That is, whenever possible, the lessons engage your students in exploring a problem before telling them how to solve the problem. Your students work in groups, exchanging ideas as they investigate, then share their reasoning and results with the class. You give your students mathematical terminology when they need it for their investigation, which they in turn use to summarize and communicate their findings. As your students ask questions, see patterns, make conjectures, and ask more questions, they gain the confidence to approach new problems and apply new concepts to solve them.

Investigations encourage discovery. To better understand the investigation-based approach, consider the topic of direct variation. A traditional textbook might first define the term direct variation then give examples that reveal a linear equation with no constant term and a graph of a line through the origin. Students might then do an activity or solve some exercises to reinforce the definition they have just learned. In Discovering Algebra (Lesson 2.4, page 115), on the other hand, your students actively engage in an investigation before direct variation is defined and before any examples are presented.

### Investigation

#### Step 10
Use the calculator table function to find the missing quantity.

#### Step 11
In this investigation you used several ways to find missing values—

#### Step 12
You can also use that rate as a conversion factor between kilometers and miles.

#### Step 13
Using a direct variation equation or graph is an alternative to solving proportions.

---

**Calculator Note 1K**

See the T refinery and the T rollhätte Canal. How is using this equation to show how miles and kilometers are related.

**Calculator Note 2A**

To learn about the table function.

**Practice Your Skills**

Let’s represent distance in miles and $y$ represent distance in kilometers. Solve the equation $y = 1.6x$ into your calculator (Use $y$-intercept $100, x = 1$).

1. Take the graph of $y = 1.6x$ to find each missing quantity. Adjust the window settings as necessary.
   - 20 miles $\rightarrow$ distance \( y \)
   - 120 kilometers $\rightarrow$ miles

2. Use the calculator table function to find the missing quantity.
   - 10 miles $\rightarrow$ distance \( y \)
   - 400 kilometers $\rightarrow$ miles

3. Find for missing values in the table. Record each value to the nearest tenth.
By the time you draw your students’ attention to the formal definition of *direct variation*, they have already worked with a table, a graph, and an equation for a direct-variation relationship. With your guidance, they have discovered that the graph is a line through the origin, that the ratio of *y*-values to *x*-values is constant, and that the relationship is related to rates. They also have written and used an equation for the relationship. When you discuss the term *direct variation* with them, you are simply codifying the approach they have already discovered and giving them vocabulary they can use to discuss their ideas.

**Investigations provide opportunities to use real data.** Not all investigations of the *Discovering Mathematics* series ask your students to discover a new concept. In some investigations, your students use the mathematics they already know to analyze data—either given data or data they gather themselves. As your students look at data and try to make sense of the relationships they see, they are learning to use mathematics to model phenomena in the real world. This process deepens their understanding of the mathematical concepts involved. For example, in *Discovering Advanced Algebra* (Lesson 3.3, page 131), your students learn about line of fit by investigating how long it takes different-size groups to do “the wave.” After collecting nine pieces of data for number of people and amount of time, your students plot the points then find the equation of the reasonable line of fit. Here, you help your students as they use the concepts of slope and equation of a line to construct meaning from the data they have gathered. These concepts are then taken a step further as they use interpolation and extrapolation to make predictions about the real world.

**Investigations encourage a variety of problem-solving approaches.** The investigative philosophy acknowledges that you can approach most mathematics problems in a variety of valid ways. This acknowledgment helps include your students who might otherwise feel they “can’t do math.” Engaged students of all abilities naturally discover different ways to solve a problem; as their guide in discovery, you support their creativity in finding their own way to a solution. And as your students gain confidence in their mathematical abilities, you are helping them become independent learners by releasing them from their dependence on you for the answer. When your students understand that there are several valid approaches to solving a problem and have the confidence to pursue those approaches, they can perform better on tests because they’re not immobilized if they can’t remember the “right method” for solving a problem.
Technology

If you were going to your friend’s house down the street, you would walk because the distance is short and a bicycle or car would not save you much time. If your friend lived 200 miles away, though, you would drive to save time. This idea is analogous to technology use in mathematics. If your students need to add two small numbers, a graphing calculator will not save them time. If you want your students to explore the graph of a quadratic equation, however, a graphing calculator would allow them to manipulate the graph more quickly than they could with only pencil and paper. As with cars, mathematics technology allows you to go faster and farther than conventional means do. For this reason, technology is integrated throughout the Discovering Mathematics curriculum. Graphing calculators, The Geometer’s Sketchpad®, and Fathom Dynamic Data™ allow your students to work faster, explore further, and understand the concepts more deeply.

Technology includes hand-held and computer-based applications. Graphing calculators are used extensively throughout Discovering Algebra and Discovering Advanced Algebra, giving your students the opportunity to explore relationships and to construct meaning from concepts and procedures that are often presented in traditional textbooks without explanation and are memorized without understanding. The visual presentation and computation speed of graphing calculators allow your students to grasp the nature of functions quickly and thoroughly and to collect and analyze large amounts of data. Sketchpad™ and Fathom are integrated into all three books. Sketchpad allows your students to see the effects of transformations and changes in parameters, helping them develop a deeper understanding of the properties of shapes and the structure of equations. Your students can make conjectures and test them immediately, enabling them to generalize properties and thus understand concepts more quickly and effectively. Fathom makes it easier for your students to investigate alternative data displays, to compare statistics, and to see the effects of removing outliers. With Fathom, your students more easily grasp the significance of various statistics and more fully appreciate the judgment involved in interpretation and analysis.

Technology can be implemented in short, simple steps. We know that many teachers have not used technology to this extent before, so we provide informative, clearly written support materials to guide you through the technology sections of each book. Discovering Algebra and Discovering Advanced Algebra have calculator guides for the most popular Texas Instruments and Casio graphing calculators. These guides walk you and your students through the calculator use in each lesson, giving specific keystroke instructions, screen captures, and hints for shortcuts. Discovering Algebra Technology Demonstrations, Discovering Geometry with The Geometer’s Sketchpad, and Discovering Advanced Algebra Demonstrations with Fathom and The Geometer’s Sketchpad give you ideas, instructions, and files for using Fathom, Sketchpad, and Calculator-Based Laboratory (CBL 2™) technology in activities and demonstrations. Discovering Algebra with TI-Navigator™ provides activities for use with the Texas Instruments TI-Navigator classroom learning system.
Tools for Investigating

As you work with your students to investigate and discover mathematical concepts, you use a variety of tools and materials in addition to technology. For example, in geometry, your students use rope to investigate circumference-to-diameter relationships or the Pythagorean Theorem. While in algebra, your students use rope to explore the functional relationships that emerge in knot-tying or pendulum experiments. Sand is useful for helping your students discover volume relationships in geometry. In algebra, your students can use sand to investigate functions involving rate and time. These materials along with other classic and nontraditional tools—meterstick, protractor, compass, straightedge, patty paper—engage your students in hands-on mathematics explorations.
Teaching with *Discovering Mathematics*

Teaching an investigation-based program, such as *Discovering Mathematics*, is quite different from teaching a more traditional program. When teaching a traditional program, you present new concepts, work through some examples, then assign problems that give students an opportunity to practice what you have just taught them. When teaching an investigation-based program, you pose a problem *without* telling students how to solve it. Students discover the new concepts themselves as they explore the problem while you facilitate, assess progress, and provide information on a need-to-know basis. After students have worked on the problem, you guide a class discussion that summarizes the key ideas. Some investigations in *Discovering Mathematics* vary from this model by providing an opportunity for students to explore a familiar concept more deeply, rather than to discover something new. In those lessons, students are exploring the world with mathematics that has been explained to them.

Transitioning from a traditional program to an investigation-based program requires you to change the way you think about yourself, your students, and the teaching process. For example, you will

- view yourself as a facilitator and guide, rather than as a disseminator of information.
- become comfortable with a more active and busier classroom.
- allow students to make mistakes as they try to figure things out.
- trust students to help and correct each other.
- be ready to take advantage of “teachable moments.”
- be open to the possibility that the process will lead you to a deeper understanding of the mathematics.

Once you become used to this new type of teaching, you may find it quite liberating. For example, you

- don’t have to know all the answers; in fact, you can be open to the validity of students’ answers even if they are different from your own.
- don’t have to plan every minute of a class period; indeed, you can be quite flexible in response to student needs.
- can learn more about students’ understanding without spending more time.
- can elicit and pursue questions that extend the mathematics of the lesson.
- can see students are learning as you watch them investigate and as you test them.
- can have more fun teaching.

**Cooperative Learning**

The *Discovering Mathematics* series engages students by giving them an opportunity to investigate mathematics in cooperative groups. As they work, group members will learn to plan together, brainstorm, determine and organize tasks, and communicate their individual and collective results.
Cooperative learning has many benefits.

- As students articulate ideas for other group members, they develop a deeper understanding of mathematical concepts and practice their oral communication skills.
- Students are exposed to more ideas for solving problems. When solving challenging problems, “two heads are better than one.”
- Students learn and practice the essential life skill of working with others. In fact, developing cooperative skills is an important part of the curriculum.
- Students who have not been successful in mathematics, but who are comfortable in social situations, gain confidence in their mathematical abilities.
- Students learn to solve more complex problems than they otherwise could, without a group to contribute different areas of expertise.
- Group members can provide more immediate feedback to students than the teacher can.
- Students learn to respect and appreciate differences in ethnicity, physical and mental abilities, and learning styles.

FREQUENTLY ASKED QUESTIONS ABOUT COOPERATIVE LEARNING

Teachers who have not used cooperative groups in their classrooms often have questions and concerns.

Will the strongest or most motivated student(s) in each group do all the work and therefore most of the learning? If you leave groups to their own devices, one or two students may well do most of the work, especially if the entire group is receiving the same grade. These suggestions will help ensure that all students in the group do their share of the work.

- Assign tasks to group members then rotate those tasks.
- Require each student to produce a separate investigation report.
- Let groups know that the student from each group who will share the group’s results with the class will be chosen at random when Sharing begins.
- Make group participation a part of each individual grade.

Is it fair to give everyone in a group the same grade? At the beginning of the school year, before all students are contributing to the work, you should not give group grades. Instead, you might assign grades in part according to the extent to which each student contributes to the group. Once you see group members are all participating, there may be an investigation, a project, or a group quiz for which you assign a group grade. It is never advisable to give a term grade to a group or to count group grades more than individual work on homework, quizzes, and tests.

Having students work in groups seems time-consuming. Will I be able to cover all the required course material? The Discovering Mathematics series is designed so that students will encounter all the major ideas of algebra and geometry through
investigations. As you watch students work in groups, you may sometimes get frustrated that it seems to take them more time to discover an idea for themselves than for you simply to tell it to them. When this happens, remind yourself of how many times you would need to repeat your explanation of an idea before students understood it deeply. Then note how well students remember it when they arrive at it through an investigation.

Won’t my class become loud and out of control? While your classroom may be a little noisy, it is important to remember that a noisy class is not necessarily an out-of-control class. Guide students to understand what is productive, acceptable noise. You will probably have more control of your class if all your students are actively engaged in doing mathematics.

How can students teach each other? A great deal of learning mathematics is seeing patterns and making and testing conjectures. Students learn as they are exposed to different approaches and as they ask questions and talk with others about what they are thinking. They will not always discover a classic or efficient method for solving a problem, but they will understand the value of methods presented by others after they have struggled with their own methods. And when you introduce terms and help students formalize relationships after they have seen the patterns, they find it much easier to remember what you say. For example, the sample lesson from Discovering Algebra on page 17 shows students’ development of the concept of slope before slope is defined.

Does having students work in cooperative groups require me to spend more time planning? Teaching in a cooperative-group setting requires about the same planning time as teaching in a traditional classroom setting. Some aspects of planning will take more time—you will need to think through the investigation and read the teacher’s edition descriptions for guiding the investigation and helping students share their results. Some aspects of planning will take less time—you will not need to plan what you will be saying and doing every minute. (For more information on planning, see pages 14–16.)

FORMING GROUPS

Most investigations in Discovering Mathematics are appropriate for groups of three to four students. However, if you or your students have little experience working in groups, you might assign groups of four, but ask students to work in pairs for the first few investigations.

There are several strategies for assigning students to groups. At the beginning of the school year, when you don’t know much about your students, it is probably easiest to assign groups randomly. As you get to know your students better, you can think about who might work well with whom. Mixing students of different ability levels can work. Or you might feel it is important to balance the groups for other factors, such as ethnicity, gender, or potential discipline problems.

Assigning students to groups might take an extra hour of planning each time you do it. Fortunately, you don’t need to reorganize the groups very often. In fact, it is usually a good idea to keep students together long enough to get to know each other and learn to work together. Change the composition of groups occasionally, though, for variety of experience. You might reorganize groups at the beginning of each chapter or grading period or halfway through a grading period.
VARIETIES OF GROUP ORGANIZATION

Groups should have a chance to work together in a variety of ways. The teacher’s editions suggest ways of dividing tasks within or between cooperative learning groups.

**Pair-share.** A group of four splits into two pairs. Each pair does one part of the investigation or completes the investigation for one situation. The pair can divide up their work in various ways; perhaps one measures and the other keeps track of measurements, one reads the description of a situation while the other makes a diagram, one graphs an equation and the other checks. At the end of the investigation, the two pairs share and combine their work. This often works well when the group needs to gather a lot of data or explore several values of a parameter.

**Jigsaw within a group.** The work of the group is divided and assigned to different group members. They work individually on their part of the investigation, then they come together to share results. For example, each member might experiment with a different number, a different equation or family of functions, or a different geometric figure. After students finish their individual explorations, the group discusses and compares their findings and makes a conjecture or agrees upon a solution that they will share with the class.

**Jigsaw among groups.** Sometimes an exploration or sequence of investigations can be broken up among groups so that each is working on a separate part. Your class might explore the sum of the angles of a polygon by having one group concentrate on quadrilaterals, another on pentagons, and so on. As groups share, your students look for patterns and find a formula that will apply to an \(n\)-sided polygon. Groups prove different theorems, then learn from each other’s work when they share ideas.

FACILITATING EFFECTIVE GROUP BEHAVIOR

There are several things you can do to help group work go smoothly.

**Arrange furniture appropriately.** The type and arrangement of classroom furniture should be conducive to group work. Tables work best because they give your students a lot of work space, but it is also possible to have an effective
cooperative setting with desks. When using desks for group work, be sure the
desktops are brought together to form as close to a single surface as possible, with
no gaps between them. If possible, arrange the tables or desks so all students can
see the front of the classroom by turning their heads rather than their entire bodies
or chairs.

**Discuss with your students why they are working in groups.** Try to counter
students’ notions that learning is competitive, that a math course involves listening
to information delivered by an all-knowing teacher, and that stronger students will
suffer by working with weaker students. Emphasize that students will learn from
each other when they see different ways of approaching a problem and that
combining the different skills and ideas of group members can make solving a
complex problem easier. Also, point out that good group skills are life skills that
will help students succeed at work and at home.

**Work with the class to develop specific guidelines for productive group work.**
Here are a few suggestions for group members.

- Speak in a low (two-foot) voice that can be heard easily by the group, but that
does not disturb the rest of the class.
- Be considerate of others in your group.
- Listen without interrupting.
- Stay on task.
- Help others in your group.
- Be supportive.
- Ask questions if you don’t understand.
- Criticize ideas, not people.
- Make sure everyone in the group understands the ideas well enough to present
  them to the class.

**Give appropriate assignments.** Good group assignments, such as the
investigations in *Discovering Mathematics*, elicit many ideas or are large enough for
group members to divide the tasks. The teacher’s editions suggest ways to divide up
steps of some investigations among groups. In *Discovering Geometry*, for instance,
the *Teacher’s Edition* suggests dividing theorem proofs among groups, then sharing,
so all members of the class can add the theorem to the list of theorems they can
use in further proofs.

**Hold each group fully accountable.** End each group session with group
presentations to the class. You probably won’t have time for all groups to present
every time, but each group should be prepared.

**Hold individuals responsible for group participation.** Make part of the
individual grade dependent on how the student contributes to the group.
“Numbered heads together” is one method for ensuring that all group members
understand the ideas from an investigation. Assign each group member a number
from 1 through 4. Roll a die or spin a spinner to see which member of each group
will present the group’s ideas to the class. You can control the order in which the
ideas are presented by choosing the order in which you call on groups.

**Trust the group process.** Give groups plenty of time to correct their mistakes. If a
student asks you a question, turn it back to the group if possible. If some students
are causing behavior problems, try to facilitate a group solution. For example,
review the guidelines with the group or help students clarify their roles within the group. In extreme cases, you may need to call a student aside for a discussion of poor behavior. When you do so, also talk one-on-one with the other group members about expectations. It may help to point out that the groups will change soon, so students will have a chance to work with a different set of classmates.

Your Role in an Investigation-Based Classroom

What do you do while your student groups investigate? After you have given students a few minutes to settle down and get started, begin to circulate among the groups, observing and encouraging as necessary.

- To facilitate groups, say little as you move around the room. Don’t be too quick to jump in and correct a student who makes an error. If the other group members don’t notice the error and if the error is a common one, make note of it and have a student present the error later so the entire class can learn from it. If students ask you a question, deflect it back to the group.

- Devise a plan to make use of students’ ideas during Sharing time. Which group should present what? What questions should be raised? What points should be made? Which problem-solving strategies should be shared with the class and in what order should they be presented? Look for students who seem to understand various key steps particularly well. Be sensitive to the fact that students with poor calculation skills may have creative problem-solving ideas. Keep in mind the suggestions in the Sharing Ideas section of the teacher’s notes.

You may find it helpful to think of yourself as an experienced lead investigator and manager, rather than as the person with all the answers. Here are some suggestions that might help you in this role.

Remove attention from yourself. As your students work and also when they present their results, consciously try to disappear from their awareness. Deflect questions back to students and wait for them to answer each other’s questions. Move toward letting your students be more independent thinkers and learners. As students become better at investigating, they may not even notice you are visiting. Although you may feel unappreciated, independent student learning and critical thinking represent your ultimate success as a teacher!

Avoid micromanaging. Trust your students to think creatively and correct one another. Focus on helping them work more effectively. When students present ideas to the class, withhold your judgment, letting other students critique each presentation before you ask clarifying questions. After asking a question, give plenty of “wait time” for students to think and make sense of the ideas.

Do not assume you know the outcomes. Yes, you can solve the mathematics problem yourself, but you cannot really anticipate what sense students will make of it. Every mathematics problem can be approached validly in more than
one way. As you learn to resist your students’ requests to tell them the “right”
solution or the “best” solution and encourage them to think creatively, yet critically,
they will begin to produce ideas you have not anticipated—ideas you can use to
bring about deeper understanding. Ask students what led them to their approach,
what reasoning process they followed. You want students to realize you are more
interested in their answers than in the answer. Avoid comments and questions that
suggest you know the right answer and are leading students to it.

**Make constructive suggestions.** Rather than giving omniscient hints, suggest
good thinking strategies. Rather than telling students how to get unstuck, make
suggestions such as “Sometimes you can understand the problem better if someone
reads it aloud” or “One good problem-solving technique is to make a diagram.” Be
prepared to ask follow-up questions.

**Help groups to stay on task.** Go to any groups that aren’t on task. If your simply
joining them doesn’t get them back on track, ask about their progress. If they think
they’ve finished the task, look at their work, ask questions, make suggestions, and
challenge them to extend their results. If students say they’re stuck, ask one of
them to describe what they’ve done, and ask others for their ideas about it.

**Give appropriate praise and encouragement.** You cannot praise students for
being “on the right track” when you don’t know all the outcomes. You can,
however, praise them for good thinking and cooperation. You can also encourage
your students to learn from mistakes and persevere despite setbacks.

**Take a long-term perspective.** Note where the teacher’s edition foreshadows future
ideas, and think about the problem you have just assigned in that context. Be aware
of the extent to which you must transcend the focus on solving the particular
problem at hand and instead build your students’ problem-solving and group skills
so they can deal successfully with the mathematics that lies ahead. Focus on the
process skills and habits of mind that will help students throughout their lives.

**Manage students’ interactions, not their thought processes.** Control the working
environment so students feel confident that they can contribute and so no student
distracts others while they are working. Determine the order of presentations so
that students get ideas from each other. Remember that students who may be
labeled “underachievers” often achieve a great deal in this kind of environment.

**Don’t attempt to follow a “script.”** Following a script is impossible when you are
acting as a manager rather than as a font of wisdom and knowledge. You must
instead rely on your own skills as an investigator. Even if you have had very little
experience investigating mathematics, your life experiences and knowledge of
mathematics put you ahead of your students.

**Work to dispel the notion that mathematics is a collection of facts and procedures.**
Help your students see that developing and applying appropriate problem-solving
techniques and justifying ideas and results with sound reasoning are more
important than getting the correct answer every time. Students should experience
mathematics as a process of finding and connecting ideas. Let them know that the
thinking and problem-solving skills they develop will serve them in all aspects of
their life. They are learning far more from you than mathematical facts.
Structuring a Class Period

The instruction for a typical lesson includes these stages.

- **Introduction.** You set the context for the investigation, pose the problem, and make sure students understand the terminology. This introduction is called out in the teacher’s edition only when there is additional material that is not in the student book.

- **Investigation.** Students work on the multistep investigation in their book or on the one-step problem from the teacher’s edition while you observe, assess, encourage, and make plans for Sharing and summarizing the mathematics.

- **Sharing.** Selected students or groups share their findings with the class. You lead the asking of questions, elaborate on students’ ideas, and praise good work.

- **Example(s).** You might lead the class through the examples, perhaps as they follow along with calculators or geometry tools. Or students might read the examples to themselves or with their groups. In some cases you might extend the solution using ideas from the teacher’s edition.

- **Closing.** You remind your students of the key mathematical concepts and where they arose in the lesson. At the beginning of the year when cooperative learning is new to your students, you might lead a quick discussion on how the groups are functioning, what they can improve, and what they do well.

- **Exercises.** Students begin to work on the homework exercises, either in groups or individually.

The order of the stages and the time spent on each will vary depending on the particular lesson, the class’s experience with investigation-based learning, and your students’ familiarity with the subject matter. At times lessons will take two days with the investigation taking one class period and Sharing following on the next day. The Lesson Outline in the Planning section of the teacher’s notes for each lesson provides a suggested structure, including time estimates for each stage.

Planning

The *Discovering Mathematics* teacher’s editions include many ideas for planning. For each chapter and lesson, they list objectives and materials and outline how to spend class time. The commentary on the lesson includes alerts, based on teacher experiences, about difficulties students might have, but may not be able to articulate, and suggestions for how you might respond to them. But every class is different. Don’t limit your planning to reading through the comments. Think about each chapter and lesson with your own students and schedule in mind.

Although careful planning is necessary, don’t let it get in the way of your teaching. What your students do with an investigation can be different from what you anticipate. Each day adjust your plans for the next few days. The farther ahead you can see, the better you will be able to decide how to balance between allowing your students to explore and answer their own questions and getting the class through the course.

**PLANNING A CHAPTER**

To allow flexibility, each chapter contains more material than most classes will have time to do. Although all the investigations are valuable, you probably won’t be able
to explore all of them in the same depth. You may choose to work through some investigations as a whole class following suggestions for investigations that make good demonstration. Or you might omit some investigations because of time constraints. Each chapter interleaf in your teacher’s edition includes a Using This Chapter section that can help you decide which parts of the chapter are more important for you to complete to meet your curriculum goals.

PLANNING A LESSON

When you are planning a lesson, it is a good idea to start by reading the entire student lesson. Then ask yourself a few questions.

What are the important mathematical ideas in the lesson? Before you teach a lesson, you need to know what key concepts your students are expected to learn. The Lesson Objectives and Closing the Lesson sections of the teacher’s notes provide a summary of the main ideas. You also should keep in mind how the lesson relates to the conceptual story line of the chapter. If you are not sure about this, review the section titled The Mathematics in the chapter interleaf.

What problem will students be investigating? The next step is to look at the multistep investigation presented in the student book. In many lessons, the teacher’s editions provide one-step problems that can be used as alternatives to the multistep investigations. One-step investigations, problems whose solution leads students to discover a concept, give students more responsibility for structuring their work than multistep investigations do. If your class has experience conducting investigations from earlier courses or if they have become good investigators in this course, these problems are probably appropriate for them. In some lessons, the one-step problem is close enough to the multistep investigation that you could ask some groups to do one and the rest to do the other. Do not assume that only “better” mathematics students will succeed at one-step problems; more accurate predictors of success are a willingness to think creatively and a tolerance of ambiguity. These characteristics can be found in students with a wide range of mathematical abilities. Working on one-step problems can help all students acquire these dispositions. An example of teaching a lesson using the one step can be found on pages 20–22.

Read the investigation or one-step problem carefully and try to anticipate terminology your students, especially ELL students, may not understand. The margin notes in the teacher’s editions provide suggestions, marked [language] or [ELL]. Aural learners benefit from hearing the problem instructions read aloud. Determine who will read them, you or the students, and whether they will be read to the whole class or in groups.

How might students approach and respond to the problem during the investigation stage? To anticipate what you can expect from students during the investigation stage of the lesson, work the problem yourself in as many ways as you can. If possible, show it to friends or colleagues and observe how they deal with it. Read through the teacher’s notes, focusing on the [alert]s and on suggestions for questions you might ask. Think about other questions you could ask to help your students as they investigate. The Sharing Ideas section might give you some ideas.
What types of inquiry should I encourage during Sharing? To prepare for the Sharing stage, read the Sharing Ideas section in the teacher’s edition. Think about other questions you might ask. Here are some question openers that might inspire you.

- What happens if . . .?
- Why . . .?
- How many . . .?
- In general, . . .?
- What do we mean by . . .?
- Is there a relationship . . .?
- What if the . . . were not . . .?
- Under what conditions . . .?
- What's the largest/smallest . . .?
- What are the properties of . . .?
- What other . . .?
- How do you know . . .?
- Is it possible . . .?
- How can you . . .?

What homework exercises should I assign? Each Discovering Mathematics book contains many more exercises than most classes will have time to do. Select those you think will generate interest among your students and satisfy your curriculum goals. The Assigning Homework chart for each lesson offers some guidance about the appropriateness of the exercises for particular goals.

How many days will my students need for this lesson? Can I take more than one day to build toward extended problem-solving ability? Yes, some lessons are recommended for two days and others that some teachers might do in a day, you’ll want your students to work on for two days.

How much time should I spend on each part of the lesson? See the Lesson Outline in the Planning section of the teacher’s notes for a suggested time line. You may need to adjust the time spent on each stage according to the needs of your class.

What materials will I need? You will find a list of materials for the book just before Chapter 0, for each chapter in the chapter interleaf, and in the Planning section of each lesson. If materials are needed for the homework exercises, they are listed in the Building Understanding section.

What might I be able to assess about students’ understanding and thinking? A good way to assess student understanding is to ask them to talk about the processes they follow when solving a problem. As students talk with their groups and share with the whole class, you will see what they understand and where they are having trouble.
Teaching a Discovering Algebra Lesson

Here and in the next two sections are concrete examples of how you might plan and teach a Discovering Mathematics lesson.

PLANNING LESSON 4.1

- First determine what the key mathematical concepts of the lesson are and read the student lesson.
- Then read the Lesson Objectives and Closing the Lesson sections of the teacher’s notes.
- Next, decide which problem to use—the multistep investigation in the student book or the one-step problem in the teacher’s notes. Read both carefully. Which would work better for your students at this point in the course? Which are you more comfortable dealing with? You might want one or two groups to do the one-step problem while other groups work through the steps of the investigation.
- For this example, we’ll assume you decide to use the investigation. Look closely at the steps of the investigation. Draw some slope triangles, and try to predict the various ways students will approach the problem. Look over the margin notes in the teacher’s notes. Read both carefully. Which would work better for your students at this point in the course? Which are you more comfortable dealing with? You might want one or two groups to do the one-step problem while other groups work through the steps of the investigation.

### LESSON OBJECTIVES

- Investigate and solve real-world problems that involve the slope of a line
- Learn how to calculate slopes with slope triangles and the slope formula
- Learn about slopes of rising, falling, horizontal, and vertical lines

### Closing the Lesson

As needed, say that the slope of a line can be calculated by finding two points on the line and dividing the vertical change between those points by the corresponding horizontal change. You can use the Four Slope Types transparency.

### One Step

Point out Hector’s bill and ask students to write careful instructions for finding the slope of a line between any two given points. As students work, suggest that they use slope triangles.

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**Step 1**

Is there a linear relationship between the time in hours that Hector uses the Internet and his total fee in dollars? If so, why do you think such a relationship exists?

Use the numbers in the table to find the hourly rate in dollars per hour. Explain how you calculated this rate.

**Step 2**

Choose a different pair of points on your graph. Create a slope triangle between them and use it to find the slope of the line. How does this slope compare to your answers in Step 2 and Step 3?

**Step 3**

Think about what you have done with your slope triangles. How could you use the coordinates of any two points to find the vertical change and the horizontal change of each arrow? Write a single numerical expression using the coordinates of two points to show how you can calculate slope.

**Step 4**

Draw a pair of coordinate axes on graph paper. Use the x-axis for time in hours and the y-axis for total fee in dollars. Plot and label the three points the table of data represents. Draw a line through the three points. Does this line support your answer in Step 1?

**Step 5**

How do the arrow lengths relate to the hourly rate that you found in Step 2? Use the arrow lengths to find the hourly rate of change, or slope, for this situation.

What units should you apply to the number?

In Step 4, you used arrows to show the vertical change and the horizontal change when you moved from one point to another. The right triangle you created is called a slope triangle.

**Step 6**

Write a symbolic algebraic rule for finding the slope between any two points \((x_1, y_1)\) and \((x_2, y_2)\). The subscripts mean that these are two distinct points of the form \((x, y)\).
Look over the exercises and decide which ones to assign using the advice in the Building Understanding section of the *Teacher’s Edition*. You might decide that there are enough essential exercises for one homework assignment, but that you will save Exercise 10 for a review and perhaps use it as a group performance assessment.

For this lesson you decide to assign the essential exercises plus one more that is a good candidate for a portfolio entry and that gives your students a chance to write about the relationships they see between graphs and the application they model.

**TEACHING LESSON 4.1**

*Open the lesson with a reminder that lines have steepness.* Mention that steepness is often called *slope* in mathematics. Ask students if they know other uses of the word *slope*, and tie each suggestion to steepness. Then have your students start work on the investigation in their groups. After they have started, begin to circulate among groups.

A typical conversation with a group might go like this:

**Jamie:** What are we supposed to be doing?
**You:** The investigation. How far have you gotten?
**Rory:** We’re stuck on the first step.
**You:** Sometimes it helps to read the problem out loud. Have you tried that?
**Jamie:** No.

**Chantel:** I’ll do it. (reads aloud)
**You:** (waiting)
**Rory:** So?
**Jamie:** (to you) What’s a linear relationship?
**You:** (looking at Chantel)

**Chantel:** That’s that line stuff, right?
**Jamie:** Like, if we graph those points, they’ll be in a line?
**Rory:** What points?
**Jamie:** The points with numbers in the table. You know, 40 and 16.55; 50 and 19.45.

**Rory:** (to you) Is that what we’re supposed to do?
**You:** One good problem-solving technique is to make diagrams, so it sounds to me like a good approach. Carry on!

Note that, in this sample dialog, you are acting as an experienced investigator who knows good problem-solving techniques. You are not dispensing information or even giving hints.

As groups work through the investigation, note interesting mathematical approaches that arise. Decide who will present what ideas during Sharing. Give blank transparencies to the students who will be sharing so they can draw diagrams to help with their presentations.
Sharing might go like this. A student draws a slope triangle and finds the lengths of its legs. You suspect that some of your students probably do not see why the vertical length of one leg is the difference in y-coordinates and the horizontal length of the other is the difference in x-coordinates. You press, as if you’re confused, for an explanation that relates the sides of the slope triangle to parallel, congruent segments on the axes until a student provides a good explanation.

The next presenter defines the slope as the horizontal change over the vertical change. You say nothing, but wait for another student to offer an alternative. When nobody says anything, you make a remark about the units of the slope in the Internet question and if they represent the correct rate. Ask another student to present the idea of dividing vertical change by horizontal change. You make sure everyone realizes that these methods give reciprocal results, and ask which is better.

Students begin arguing, and you listen carefully, assessing their understanding of slope as representing rate, their awareness of the units, and their ability to refer back to the text to support their arguments. Because some students aren’t convinced, you ask the class to read the example in the student book to help them figure out what the units of the rate should be. After the issue is settled, discuss the idea from the Dynamic Concept of Slope section of the teacher’s notes and ask about slopes of vertical lines.

Close the lesson by restating the formula. Put the emphasis on change rather than on variables. Let students begin work on the homework while you make a few notes about students who surprised you with their ideas and about what you’ll do differently the next time you teach the lesson.
PLANNING LESSON 5.4

- Read the student lesson, focusing on the new conjectures.

- Write a few sentences encapsulating what you want to teach:
  The three midsegments of a triangle divide it into four congruent triangles. A midsegment of a triangle is parallel to the third side and half as long. The midsegment of a trapezoid is parallel to the bases and is equal in length to the mean (average) of their lengths.

- Read the Closing the Lesson section of the teacher’s notes.
  Note that the triangle can be thought of as a special case of a trapezoid, in which one of the bases is shrunk to a point.

- Think about how this lesson fits in with the chapter. Recall that the chapter concerns properties of polygons, and make a note to tie the lesson to the rest of the chapter.

Closing the Lesson

The midsegment of any trapezoid is parallel to the bases, and its length is the average of their lengths. If one of the bases shrinks to a point to make a triangle, the midsegment remains parallel to the remaining base, and its length becomes half the length of that side of the triangle. The three midsegments of a triangle divide it into four congruent triangles.
Although you could tell your students about these properties of triangles and trapezoids very quickly, remember that the goal is for them to understand the properties deeply by discovering the properties themselves. Look over the investigation and the one-step problem and think about which is more appropriate for your class.

- For this example, we’ll assume you decide to use the one-step problem. Work on the problem yourself, imagining how different students in your class will approach it. Present the problem to co-workers, family, or friends and see what they do with it. Look over the margin notes in the Discovering Geometry Teacher’s Edition for ideas about questions and insights students might have. Review the suggestions in the Sharing Ideas section.

- Plan the details. The problem statement is fairly complicated. Would it be a good idea to suggest that one person in each group read it aloud while the others draw pictures, then compare the pictures? Perhaps you should make a copy of the problem for each group, rather than use a transparency, because the groups will need to study the problem intently.

- Look over the exercises and decide which ones to assign, guided by the advice you find in the Teacher’s Edition. The nine essential exercises will be easy for your students to complete, so you decide to assign Exercise 11 as well, which will give students a chance to write about their reasoning. When you give the assignment, you might mention to students that they may want to include Exercise 9 in their chapter portfolio.

TEACHING LESSON 5.4

Open the lesson by briefly reminding the class that they have been studying properties of polygons. Then hand out the copies of the problem. Suggest that one member of each group read the problem slowly while each of the other members draws a diagram. When the reader has finished, the group members should compare and discuss the diagrams.

After students begin working, take attendance, then start to circulate among the groups. A typical conversation with a group might go like this:

Pedro: You and I got the same diagram, Terry, but yours is different, Kim.
  
Kim: Yeah. You made these lines parallel, but there’s nothing in the problem that says there are east-west streets.

Pedro: Oh.

Terry: But the problem refers to east and north and all. Those directions are parallel.

Kim: Or perpendicular.

Pedro: And streets always run north and south or east and west.

Kim: But not in the mountains where they build tunnels.
Terry: So maybe these aren’t really streets, but directions. Does it matter?

Kim: I guess not.

You: Your discussion is interesting and, I think, should be shared with the whole class when we reconvene. Kim, would you mind summarizing it for us then?

If some groups seem to be ahead of the others, wonder aloud what would happen if you shrink one side of the trapezoid to a point. Decide which groups you will ask to do a presentation and plan the order of their presentations.

The Sharing stage might go like this. Kim summarizes the discussion about streets and directions. Then another student draws a trapezoid. No other student speaks up, but you’re pretty sure some don’t understand very deeply, so you ask, as if confused, how the student knows it’s a trapezoid. A discussion ensues.

One student mentions that there are no parallel lines on Earth, and you are impressed, but you point out that in mathematical modeling, assumptions must be made and, in this problem, we need to assume we are working in a plane.

You ask a student who drew everything to scale to present her group’s conjecture that the tunnel’s length is “halfway between” the lengths of the trapezoid’s bases, and you choose another student to talk about averages. You wonder aloud if the average of two numbers is always “halfway between” them, and a student offers an algebraic explanation.

You ask a student to present ideas about shrinking one base of the trapezoid to a point, making sure that the Triangle Midsegment Conjecture is stated. Then you wonder aloud about what would happen if all three midsegments of a triangle were drawn and write down the various conjectures the class proposes.

Close the lesson by restating the three main conjectures of the lesson. Emphasize how the Trapezoid and Triangle Midsegment Conjectures are related.

Have students begin work on the homework while you make a few notes about students who surprised you with their ideas and about what you’ll do differently the next time you teach the lesson.
Teaching a Discovering Advanced Algebra Lesson

PLANNING LESSON 5.3

- As always, you begin by asking yourself, “What’s the mathematics of this lesson?”
- You consult the Discovering Advanced Algebra Teacher’s Edition to find that it’s about rational exponents, primarily fractional exponents, and their use in solving equations.
- The lesson also introduces the point-ratio form of exponential equations.
- You start by writing out sentences to use in your summary of the lesson:
  “The point-ratio form of an exponential equation, \( y = y_1 \cdot b^{x-x_1} \), allows you to find the exponential equation given one point and the common ratio.
  Fractional exponents can be used to solve for bases in power equations, although they won’t necessarily produce all solutions. \( a^{m/n} \) means the \( n \)th power of the \( n \)th root of \( a \).
  \( a^{m/n} \) also equals the \( n \)th root of the \( m \)th power of \( a \).” Although you find these last two statements very familiar, you wonder if your students will believe the equality is obvious, then you yourself begin to wonder why it holds.
- You read the Sharing Ideas section of the Teacher’s Edition. It says the expressions are equivalent but doesn’t explain why. You then spot the string of equations purportedly showing that \( 1 = -1 \), and you realize that perhaps the equality is not obvious, so you take a couple minutes to look back at The Mathematics section that opened the chapter.
Next you ask yourself, “What problem should I use to carry the mathematics?” Because you’re suddenly uncomfortable with your own understanding, you decide to use the investigation rather than the one-step problem. You pull out copies of Calculator Notes 3B and 5B.

You look over the examples and note the questions to be raised. You appreciate for the first time how the point-ratio form is analogous to the point-slope form of a linear equation and decide to emphasize that connection for your students.

You consult the Teacher’s Edition for suggestions on how to bring up the mathematical concepts during Sharing. You decide to break up the Sharing, doing part of it after the investigation and the rest after the examples.

Now you ask what other inquiry or reflection you might encourage. The main question you have is why does the $n$th root of the $m$th power equal the $m$th power on the $n$th root? You write it in symbols: Why does $\sqrt[n]{a^m} = (\sqrt[n]{a})^m$? You suspect the reason may be obvious, but you decide to raise the question with your students—or better yet, let them raise it—and see what they say.

You consult the Teacher’s Edition again to decide what homework to assign.

**TEACHING LESSON 5.3**

*You begin the lesson by reminding students that they have found meaning for $a^n$ for positive and negative integer values of $n.*  
Ask if there is any meaning for an exponent that’s a fraction. You think to add “such as $\frac{1}{2}$,” remembering the investigation. A couple of students hazard guesses, for which you thank them even though they’re pretty vague. Chris, who has always seemed to know everything in the course already, states authoritatively that “the $\frac{1}{2}$ power means the square root.” You respond as closely to the way you responded to the others as you can, realizing that most of the other students aren’t ready to learn from that contribution. When no other ideas are forthcoming, you ask students to begin the investigation.

Here’s part of your conversation with a group that’s working on Step 4:

**Robin:** We’re fine.

**You:** Good.

**Eduardo:** So we’re just multiplying by 5 each time.

**Robin:** Yeah.

**You:** Could I see your data, please? (Eduardo passes over a calculator; as they continue talking, you note that the table begins with $x = 4$.) You sure are bright to realize that from these big numbers.

**Robin:** It’s obvious.

**You:** Not to me. Is there some way to see the table for smaller numbers?

**Katie:** You mean set it to begin earlier, like at 1?

**Leo:** Oh. You can scroll up, can’t you? (demonstrates)

**Katie:** I didn’t realize the table was bigger than what you set.

**Leo:** Oh. I see now.

**You:** You see what?

**Leo:** That you’re multiplying by 5.

**Katie:** Wait. (to you) Why is $y$ equal to 1 when $x$ is 0?

**You:** (look around group)

**Robin:** That’s because anything to the 0 power is 1.
You: (wait)

Robin: Well, it just is.

You: Was there something about that in an earlier lesson?

Katie: (looking through book) Oh, yeah. Here: exponents of 0. \( a^0 = 1 \).

Eduardo: OK.

You: But why, Robin?

Robin: I don’t know.

Leo: Didn’t we do that in the investigation right before that? Look, Step 9. I don’t remember what we did, though.

Eduardo: Oh, sure. Like, \( 4^2 \) is 1, right? But if you subtract exponents, you get \( 4^{5-5} \), which is \( 4^0 \).

Robin: Sure.

Leo: Oh, yeah, right. OK.

You: Back to the question about this table. Why do you multiply by 5 to get from each \( y \)-value to the next?

Katie: Because 5 is the square root of 25?

You: Well, keep at it, team!

At some point during Sharing, you ask Katie to explain about powers of 0.

After students have shared their results, you remember your question. Your students aren’t far enough along to come to know why, so you focus on coming to know that: “Does the \( n \)th power of the \( m \)th root really equal the \( m \)th root of the \( n \)th power?” When Chris says, “Sure,” you ask, “Always, sometimes, or never?”

Students begin working with examples, but they get nowhere. Without seeing too far ahead, you remind them that a good problem-solving strategy is to go back to basic definitions. Someone suggests writing \( 4^{3/2} \) as \( \sqrt{4} \cdot 4 \cdot 4 \) and as \( \sqrt{4} \cdot \sqrt{4} \cdot \sqrt{4} \). Both equal \( 2 \cdot 2 \cdot 2 \), and some students seem satisfied. You’re not sure you are, but you have to move on to the examples.

You work through the examples. Students raise some of the questions you wished to see raised. You stress, showing the excitement of your own discovery, the analogy between the point-ratio form of an exponential equation and the point-slope form of a linear equation. You think of a good way to say it: “Instead of starting with a point and adding the slope repeatedly, you start with the point and multiply by the base repeatedly.” Toward the end of the period, you raise the rest of your questions. There’s a little time for discussion before you summarize the main points of the lesson and students begin work on the exercises.

Allow students to begin work on the homework. You’ll make a few notes about students who surprised you with their ideas and about what you’ll do differently the next time you teach the lesson.
Supporting Your Teaching

In addition to the teacher’s edition, each volume of the Discovering Mathematics series has an array of support materials for teachers, including many ancillaries, a Web site, and a training institute.

Ancillaries, Books, and CDs

These ancillaries are available for all books in the Discovering Mathematics series.

- **Teaching and Worksheet Masters**: Masters for worksheets called for in the student book and transparencies you might use during class discussion
- **Assessment Resources**: Blackline masters of Form A and B quizzes, tests, and exams, including answers; constructive assessment options and rubrics
- **Condensed Lessons**: Abbreviated versions of the lessons for students who miss class or need additional support
- **Practice Your Skills with Answers**: Blackline masters of skills exercises, similar to the practice exercises in the student books
- **Practice Your Skills Student Workbook**: Workbook of skills exercises, similar to the practice exercises in the student books
- **Teaching Resources on CD**: All the ancillaries in PDF format
- **TestCheck Test Generator and Worksheet Builder CD**: A test generator that includes all the chapter text and constructive assessment items in the assessment resources plus hundreds of other test items, many in multiple choice format

**DISCOVERING ALGEBRA AND DISCOVERING ADVANCED ALGEBRA**

Both textbooks provide calculator notes and programs and data on CD for major graphing calculator models. Discovering Algebra has notes for Texas Instruments TI-73, TI-83, and TI-83/84 Plus, and for Casio fx-7400G Plus, fx-9750G Plus, and CFX-9850GC Plus. Discovering Advanced Algebra has notes for Texas Instruments TI-83, TI-83/84 Plus, TI-89, TI-92, and Voyage 200. The calculator notes and the contents of the data and programs CD are also available at [www.keymath.com](http://www.keymath.com).

**DISCOVERING ALGEBRA**

- **Technology Demonstrations**: Demonstrations using The Geometer’s Sketchpad, Fathom, and the Calculator-Based Laboratory (CBL 2) to dynamically illustrate concepts in the student book
- **Discovering Algebra with TI-Navigator**: Activities and assessments for using the Texas Instruments TI-Navigator classroom system with Discovering Algebra
- **Parent Guide**: Includes tips for working with students, chapter summary exercises, and review exercises with complete solutions

**DISCOVERING ADVANCED ALGEBRA**

- **Demonstrations with Fathom and The Geometer’s Sketchpad (with CD)**: Demonstrations
you can use to dynamically illustrate examples or other material in the student book

**DISCOVERING GEOMETRY**

*Discovering Geometry with The Geometer’s Sketchpad* (with CD): Lessons based on The Geometer’s Sketchpad that can be used in place of lessons in the student book, and demonstrations you can use to dynamically illustrate examples or other material in the student book.

*Tracing Proof in Discovering Geometry*: Describes *Discovering Geometry*’s approach to developing proof skills and the research that supports that approach.

**Web Sites**

Each book in the *Discovering Mathematics* series has a Web site to which you can direct students who need additional help or information.

*Discovering Algebra*:  [www.keymath.com/DA](http://www.keymath.com/DA)


*Discovering Advanced Algebra*:  [www.keymath.com/DAA](http://www.keymath.com/DAA)

These sites contain Dynamic Explorations that allow students to visually explore a variety of concepts. Also, through these sites, students can download PDF files of a *Parent Guide, More Practice Your Skills, Calculator Notes,* and *Condensed Lessons* and follow links to additional information about topics in the text. The links are organized by chapter and lesson. For example, the links for *Discovering Algebra* Chapter 7: Functions might lead students to Web pages on Alan Turing, electronic encryption, graphing functions, graphing functions on the TI-83 or TI-83/84 Plus, the vertical line test, independent and dependent variables, Albert Einstein, function notation, temperature conversions, the New York Stock Exchange, local temperatures, top-grossing movies, graphing absolute values, absolute-value inequalities, parabolas in the real world, inverse functions, and graphing inverse functions. Because Web sites change frequently, these links are updated often.

The *Discovering Mathematics* series also has a Web site for registered teachers, [www.keypress.com/keyonline](http://www.keypress.com/keyonline). This site contains general information, downloads of all teacher ancillaries, and additional resources.

**Professional Development Center Institutes and Workshops**

Key Curriculum Press offers one-day workshops, three-day courses, and week-long institutes to model teaching strategies, classroom management, and discovery-based learning inherent in the *Discovering Mathematics* series. Institutes, courses, and workshops on The Geometer’s Sketchpad and Fathom Dynamic Data Software can help you appropriately integrate these tools into your mathematics classroom. During these Professional Development Center events, you have many opportunities to share your experiences and to learn from teacher leaders and other participants. Key Curriculum Press also offers implementation workshops and custom workshops at your site, tailored to fit the needs of your teachers.


## Reading

Over the last decade or so, teachers and researchers have become aware that the ability to read mathematics and other technical material not only enhances students’ understanding of the concepts and gives them a lifetime skill, but also improves their language skills in other subjects. In *Discovering Mathematics*, students must read mathematics because they need to understand the steps of the investigations. And because you won’t be the only source of all information, they will need to read other parts of the book as well.

As you embark on teaching the course, be sensitive to some of your students’ inexperience at reading mathematics and, perhaps, English. There are several steps you might take to help them.

- **Ask students to read aloud.** If one student in the class or a group reads an instruction or problem aloud, the others may benefit from hearing it. Be careful with this, though; don’t ask students to read anything aloud that they haven’t first read silently. Ask the group to rotate readers counterclockwise to be sure everyone gets a chance.

- **Ask students to paraphrase.** After one student reads aloud, ask another student to restate the instruction or problem using different words.

On page 52 is a handout to help your students read better. You might include it in How to Be a Great Math Student, a packet you hand out at the beginning of the course.

## Communicating

Many educators stress the importance of teaching communication skills because, in the real world, people need to communicate well to be effective in their jobs. Another reason for learning to communicate well is that the process of deciding what to say or write deepens understanding of the ideas. As the old saying goes, “Writing is Nature’s way of telling us what we don’t understand.” The same holds for careful speaking.

Writing and speaking don’t necessarily deepen understanding, though, if they’re not critiqued. Work to establish in your students the habit of critiquing what they themselves are saying and writing. Encourage them not to turn in their scratch paper for a homework assignment, but to write up a careful presentation of their answer and its justification. When you grade homework or critique a presentation, ask for clarification of vague expressions or fuzzy logic. Also encourage the use of graphics in communicating, both orally and in writing. Provide access to overhead transparencies or the board as much as possible. Computer graphics programs, including programs that run on graphing calculators, allow students to incorporate graphical illustrations in oral presentations and in written reports and portfolios.

You might copy the handouts on pages 52–54 for your students as part of their How to Be a Great Math Student packet. The handouts will help them present mathematics better in writing and orally.
HOW TO READ A MATH BOOK

What part of your body works the hardest when you read?
Sure, your eyes move the most. But if your brain isn’t working, you aren’t really reading; you’re just moving your eyes.

For example, here’s a word.

reflected

If you couldn’t read at all, you might say, “Oh, pretty picture.” With some basic reading ability, you might be able to sound out the word: re-flect. With some knowledge of the English language, you might say, “Oh, reflect! That’s what a mirror does. Or when you think back on something.” If your brain is working even harder, you might add, “That writing is pretty unusual.” And if your brain is really in gear, then you might notice that the reason for the unusual writing is that each half of the word is a reflection of the other half, as if through a mirror.

When you read sentences, your brain is working on several levels as well. For example, if a sentence begins

My dog ate my

your brain begins to form an image of a hungry dog. But it may also begin to imagine the dog with papers in its mouth, anticipating that the sentence will end, as it often does, with the word “homework.” Then, if you finish the sentence and the word is indeed “homework,” your brain has performed a guess-and-check procedure similar to what you do in solving some mathematics problems. On the other hand, if the sentence ends with something like “house” or “garden,” you’ll adjust your whole imagined picture. For example, if you read “My dog ate my house,” your imagined dog would probably grow larger. And your brain might wonder, “How could that happen?”

When you read a paragraph in a mathematics book, your brain needs to work harder than your eyes. To follow the thread of the discussion, ask after every sentence or paragraph, “What questions is this answering?” To do guess-and-check on a large scale, ask yourself, “What questions could I ask about this paragraph?” “How might the book answer those questions?” Later you might read something that allows you to check your guesses.

When you begin reading an explanation or justification or proof of something, you might stop and try to write out the explanation for yourself. You may not be able to get very far, but you’ll have some questions firmly in mind. Read a sentence or two of the book and see how well you guessed. Perhaps you can finish writing out your own explanation. If not, read a little more of the book and try again.

It is a great habit to read your math book with paper, pencil, and tools (such as a calculator and patty paper) at hand. This way you’ll use your tools and your brain as you read.
DOING MATH HOMEWORK

Some people begin their math homework by turning to a clean sheet of paper in their notebooks. Then they work the problems on that and other sheets, rip them out, put their names on them, dog-ear them together, and turn them in.

Some people, but not you; you start with the back of an envelope—or maybe an advertisement, or a paper napkin, or even a small white board. That is, anything you can scribble on.

And then you scribble. You read the problems over; you reread the book and your class notes; you draw diagrams; you write down all kinds of stuff; you scratch out or erase a lot; you doodle. Nobody else will ever see these scribbles. They’re just to help your thinking.

They really do help. You’ll get ideas for how to solve the problems. To check your ideas, you do more scratching. You write down steps and see if they make sense. You write out justifications and see if they make sense when you read them as if you were someone else, like your teacher.

Then, when you’re satisfied, you turn to a clean sheet of paper and begin writing, carefully. You copy your latest thoughts from the scratch paper. Sometimes you have to pull out more scratch paper and rethink the problem. Only your best work goes onto the paper you’ll turn in. You add little notes or pictures to explain why you did what you did.

As a result, whoever reads your paper will be very impressed. They might ask, “How did this genius think so clearly from the beginning to the end of these problems?” They’re impressed because they don’t see your paper napkins!
MAKING A WRITTEN PRESENTATION OF MATH

Unless your teacher gives other instructions, your presentation will have several parts:

- The problem (or project task)
- What I/we did
- What I/we found
- What I/we think about the results

If you really want to impress yourself and the rest of the class, you might call these sections “Problem,” “Procedure,” “Results,” and “Analysis.”

Just as when you do your homework, make at least one “scribbled” copy of each section, perhaps on the computer. You may have to move things around. In their first drafts, many writers put some results in the procedure section, or they get into the analysis section before they remember some results. Don’t worry about mixing things up in your draft because you will separate the ideas as well as you can for your presentation.

Before making a final draft, look back over the instructions for the project. Make sure you have satisfied the objectives your teacher has specified. Then check your draft carefully to be sure the mathematics is accurate and your explanations are clear. Adding diagrams can often help your audience understand what you’re saying. Also emphasize what is special about your work: Did you come up with a procedure you’re really proud of? Did you have an unusually creative thought about the results?

PRESENTING MATH ORALLY

Preparing an oral presentation is much like preparing a written one—you’ll have the same sections and do the same kinds of checking. For an oral presentation, though, you need to make sure your talk is not so long that it will bore your audience. You might pay extra attention to graphics that will help keep your audience engaged. Most important, when speaking in public, remember to speak clearly, loudly, and slowly so everyone can understand what you say.

To help your audience follow your talk, you might use an overhead transparency or a computer display to show your outline. But you don’t want your displays to appear too cluttered. A good guideline is to put no more than seven lines of text on any display you expect to be read. If you want your audience to study patterns in your results, consider making copies to hand out.

Adjust your talk to your audience. If they’ve already heard talks about the same project, don’t spend so much time talking about the first section as you would if they didn’t know the problem. If the emphasis of your talk is on procedure or your brilliant analysis, make special graphics for that section.
Collaborating on a Lesson

Lesson study, or “the research lesson,” is gaining popularity in the United States as a way to help teachers work together to improve instruction. It is based on a Japanese model of professional development to which some educators attribute high student achievement in mathematics.

The lesson study process involves a group of five to seven teachers who meet frequently to plan, teach, observe, critique, and reteach lessons. The group usually focuses on only one or two lessons per year. Through this process, teachers learn all they can about how students learn the particular topic under consideration. The lessons are almost always investigation-based, like those in the Discovering Mathematics series.

In general, a lesson study follows these steps.

1. **Meet to decide a research problem.** The problem may be as specific as “How can we teach students the notion of variable?” or as general as “How can we improve enthusiasm about mathematics?” The topic might be handed down from administrators. Regardless, it is usually a question that the group feels is pressing.

2. **Plan the lesson.** It may take your group weeks to plan a single lesson. First agree on the mathematics to be taught, then decide on a problem to carry that mathematics. Here, the investigative problems in Discovering Mathematics can help you. You might, however, devote extensive discussion to adjusting the wording of the problem for your own students. Further planning includes answering the critical teaching questions on page 56. As mentioned there, the Discovering Mathematics teacher's editions can assist you with this process. You must also decide who in the group will teach the lesson, when they will teach it, and to whom. Consider having the designated teacher practice teaching the lesson to the group.

3. **Teach the lesson.** The designated teacher should teach the lesson to a class as the rest of the group observes, trying to see how well the lesson solves the problem the group started with. (Group members may need to arrange to have substitutes teach their own classes.) The observers may walk around and observe individual students or groups, but they must not interact with students. You may want to videotape the session so you can view it later.

4. **Critique and revise the lesson.** The group should meet soon after the lesson is taught, on the same day if possible, to critique the lesson (not the teacher) as a solution to the problem the group is trying to solve. Then meet a few more times to revise the lesson.

5. **Reteach and recritique.** One member of the group teaches the revised lesson to a different class. Consider inviting guests—other teachers, administrators, parents, university professors, and so on—to observe and respond to this teaching. You will probably want to prepare the guests for their roles, perhaps with forms to use for observations and critiques. All the observers should meet afterward to debrief.
6. **Report.** The group should write a report on its lesson-writing process, at least for the school audience if not for the district or a journal article. For more details about lesson study, look for links at [www.keypress.com/DM/teachersguide](http://www.keypress.com/DM/teachersguide).

## Critical Teaching Questions

You can use this pool of potential Critical Teaching Questions to reflect on or plan any lesson. The questions are divided into those that focus on task, on classroom interactions, on student learning, on teaching decisions, and on mathematics.

### Reflecting on the tasks
- How mathematically appropriate are the tasks for developing an understanding of the mathematics being taught?
- How effectively are the tasks sequenced and connected?
- Are there alternative tasks that could improve the lesson? What are they?

### Reflecting on classroom discourse and the classroom environment
- How effectively is questioning used to help students develop mathematical understanding?
- What techniques and strategies are used to orchestrate and promote student discourse and how effectively are these strategies implemented?
- What other questions could be asked to further the development of mathematical understanding?

### Reflecting on student learning
- What strategies are used to assess student understanding?
- What evidence is there that students learn the mathematics being taught?
- How do you use the evidence about student learning to diagnose and modify?
- What alternative strategies could be used to gather more or better information about student learning?

### Reflecting on the teaching decisions
- What decisions does the teacher make to achieve the goal of reaching all students?
- How are transitions made and how effectively is this done?
- How are tools and other instructional materials used to enhance the development of mathematical understanding?
- How effectively are student mistakes and misconceptions addressed?
- How effectively does the teacher determine when to clarify, explain, question, or let a student struggle?

### Reflecting on the mathematics
- What are the mathematical ideas in this lesson and how significant are they?
- How are contexts, representations, connections, and applications used to enhance the mathematics being taught?
